DØ Note 6224
Version 2
August 4, 2011

The Empty Crossing Method for Determining the DØ Luminosity

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Abstract

We describe how the DØ luminosity is measured using the empty crossing method. This method counts the number of beam crossings where there are no in-time coincidences between the north and south luminosity counters during the measurement period. Poisson statistics is then used to determine the average number of interactions, and thus the luminosity, from the fraction of empty crossings. The statistical uncertainty and systematic error resulting from the empty crossing technique are presented, with an emphasis on the high luminosity region where the number of empty crossings is small.

The DØ luminosity is derived from the number of beam crossings with north-south in-time coincidences in the DØ Luminosity Monitor (LM) that occur during the measurement period. The luminosity reported to the accelerator division is based on nominal 15s measurement period, while the luminosity used for physics analysis employs a nominal 60s measurement period (the nominal measurement period is occasionally shorter to maintain synchronization between the luminosity measurement and the state of the data acquisition system). Statistical fluctuations in the number of luminosity coincidences lead to statistical and systematic errors in individual luminosity measurements.

The luminosity calculation is performed separately for each of the 36 bunches in the Tevatron, resulting in each bunch having its own measured luminosity. We show below how this measurement is made for a bunch with a true luminosity L.

The average number of proton-antiproton interactions in a beam crossing that produce north-south coincidences is proportional to the luminosity

$$\bar{N}_{NS}(L) = \frac{\sigma_{NS} L}{f}$$

where σ_{NS} is the Luminosity Constant defined by eq. 1 in DØ Note 5945 and f is the beam crossing frequency. Similarly, we have the average number of interactions in a beam crossing that produce in-time hits in only the north (south) luminosity counter array given by

$$\bar{N}_N(L) = \frac{\sigma_N L}{f}$$

$$\bar{N}_S(L) = \frac{\sigma_S L}{f}$$

where σ_N and σ_S are the single-sided effective cross sections. In the absence of background, the single sided cross sections are those defined by eq. 3 in DØ Note 5945. Since the out-of-time background described in DØ Note 5946 is effectively an additional contribution to the single-sided effective cross section, measurements that do not employ the background unfolding procedure described in DØ Note 5904 need to use an effective single-sided cross section that is the sum of the directly produced single-sided cross section derived in DØ Note 5945 and the background cross section derived in DØ Note 5946.

The fraction of beam crossings that that are classified as empty because they do not have a north-south coincidence of in-time luminosity counter hits is given by Poisson statistics

$$F_0(L) = e^{-\bar{N}_{NS}(L)}\left(e^{-\bar{N}_N(L)} + e^{-\bar{N}_S(L)} - e^{-(\bar{N}_N(L)+\bar{N}_S(L))}\right)$$

where the first factor is the probability for having no proton-antiproton interactions giving a north-south coincidence and the terms in parenthesis give the probability for not having multiple single-sided interactions that result in a north-south coincidence.

The average number of empty crossings during a measurement period is then

$$\bar{N}_0(L) = N_{Live}\, F_0(L)$$

where N_{Live} is the number of live beam crossings in the measurement period. In the results below, we will ignore the small fraction of beam crossings that are not counted as live beam crossings (due to having the halo veto condition) and take $N_{Live} = fT$ where T is the measurement period.

There will be a binomial distribution for the actual number of empty crossings observed in different measurement periods. Since we are particularly interested in the high-luminosity behavior where the number of empty crossings is small, the distribution for the observed number of empty crossings is well approximated by a Poisson distribution

$$P(n_0) = \frac{n^{\bar{N}_0(L)}}{n!} e^{-\bar{N}_0(L)}$$

where $P(n_0)$ is the probability of observing n_0 empty crossings.

For each measurement period, we calculate the measured luminosity L_m corresponding to the observed number of empty beam crossings by numerically solving the equation

$$n_0 = N_{Live}\, e^{-\sigma_{NS} L_m(n_0)/f} \left(e^{-\sigma_N L_m(n_0)/f} + e^{-\sigma_S L_m/f} - e^{-(\sigma_N + \sigma_S) L_m/f} \right).$$

For the case where no empty crossings are observed, the solution of the above equation yields an infinite measured luminosity. In this case, we calculate the luminosity to be the value that would be found if there had been one empty crossing observed. We find that this treatment for those measurement periods with no empty crossings provides a graceful transition to the saturation region where the luminosity is no longer accurately measured.

The average measured luminosity is given by

$$\bar{L}_m = P(0) L_m(1) + \sum_{n_0=1}^{fT} P(n_0)\, L_m(n_0)$$

where the first term accounts for the special handling described above when there are no empty crossings.

Typically, we are interested in the total luminosity obtained by summing the luminosity from the 36 beam bunches. If all 36 bunches had the same luminosity, then the total luminosity would be 36 times the bunch luminosity and the RMS spread of the total luminosity would be a factor of 6 times the RMS for a single bunch since each bunch measurement is statistically independent. In practice, there are typically few percent variations among the bunch luminosities. While these bunch-to-bunch variations are accounted for in the DØ luminosity measurement, the results below illustrate the behavior of the measured total luminosity under the assumption that all bunches have the same bunch luminosity. Including typical bunch-to-bunch variations would not significantly affect these results.

The relation between the average measured luminosity and the true luminosity for measurements periods of 15s and 60s is shown in Fig. 1. Due to the non-linear behavior in the empty crossing probability at high luminosity, the average measured luminosity systematically exceeds the true luminosity before entering the saturation region, where the luminosity asymptotically approaches the value $L_m(1)$ that is assigned to bunches with <2 empty beam crossings.

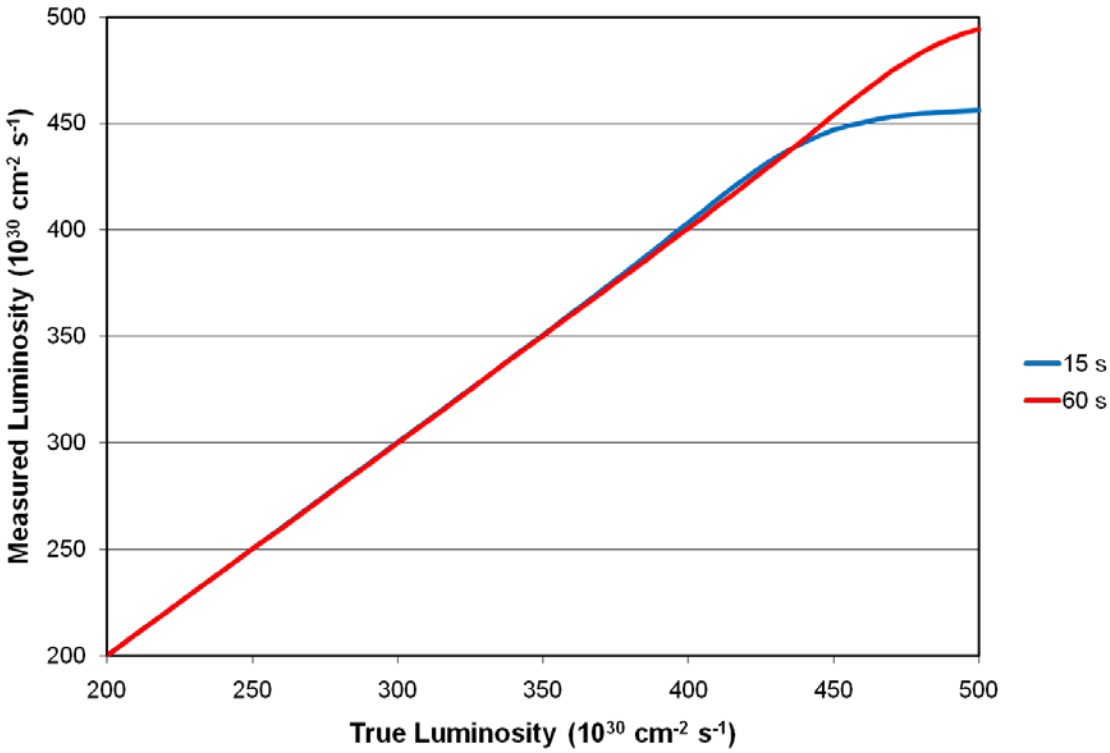

Figure 1: Average measured luminosity versus the true luminosity for 15 s and 60 s measurement periods.

Table 1 illustrates how the non-linear behavior of the empty crossing probability and the special treatment of the case where there are no empty crossing in the measurement period lead to non-linear behavior for the average measured luminosity. In this example, the true luminosity is 420×10^{30} cm^{-2}s^{-1}, the measurement period is 15 s, and the average number of empty crossings in the measurement period is 3.05. The exponential decrease in the measured luminosity as the number of observed empty crossings increases results in an average measured luminosity of 424.8×10^{30} cm^{-2}s^{-1}, which is 1.1% higher than the true luminosity.

Table 1: Illustration of the non-linearity in the luminosity measurement showing the probability, measured luminosity, and contribution to the average luminosity as a function of the observed number of empty crossings. For this illustration, the true luminosity is 420×10^{30} cm^{-2}s^{-1} and the measurement period is 15 s.

n_0	$P(n_0)$	L_m ($\times 10^{30}$ cm^{-2}s^{-1})	$P(n_0)$ * L_m ($\times 10^{30}$ cm^{-2}s^{-1})
0	0.047	456.9	21.7
1	0.145	456.9	66.1
2	0.220	434.0	95.7

3	0.224	420.5	94.2
4	0.171	411.0	70.1
5	0.104	403.6	42.0
6	0.053	397.5	21.0
>6	0.036	-	14.0
Sum	1	-	424.8

Figure 2 shows the mean deviation between the average measured luminosity and the true luminosity as a function of the true luminosity. The largest positive deviation occurs when the average number of empty crossings is ~3. This occurs at a luminosity of $420\times10^{30}\ cm^{-2}s^{-1}$ for a 15 s measurement period, where the average number of interactions with a north-south coincidence is ~12 and the fraction of empty crossings is ~4 ppm. For a 60 s measurement period, an average of 3 empty crossings occurs at a luminosity of $420\times10^{30}\ cm^{-2}s^{-1}$, where the average number of interactions with a north-south coincidence is ~13 and the fraction of empty crossings is ~1 ppm.

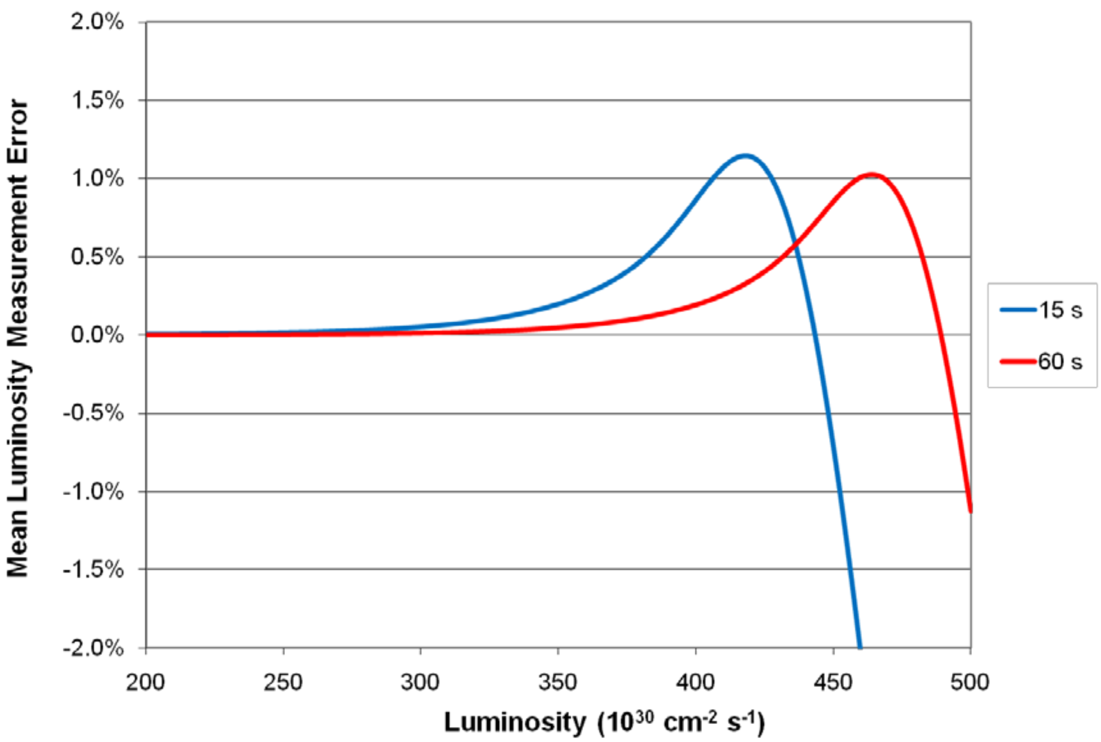

Figure 2: Mean deviation between the average measured luminosity and the true luminosity for 15 s and 60 s measurement periods.

The RMS width of the measured luminosity for a single bunch is given by

$$\sigma_{L_m}^2 = P(0)(L_m(1) - \bar{L}_m)^2 + \sum_{n_0=1}^{fT} P(n_0)\,(L_m(n_0) - \bar{L}_m)^2$$

where the first term accounts for the special treatment of the case where there are no empty crossings. The statistical uncertainty in the total luminosity measurement for 36 bunches is shown in Fig. 3. The statistical uncertainty in the luminosity measurement is <0.1% for luminosities in the range $(1.5 - 250)\times10^{30}$ cm^{-2} s^{-1} for the 15 s measurement period and $(0.4 - 310)\times10^{30}$ cm^{-2} s^{-1} for the 60 s measurement period (at low luminosities, it is important to use a binomial, rather than Poisson, distribution for the number of empty crossings). The RMS width reaches a maximum of ~0.7% when the average number of empty crossings is ~3. Further increases in the true luminosity push the luminosity measurement into the saturation region where an increasing fraction of luminosity measurements report the maximum possible measured luminosity $L_m(1)$, leading to a decrease in the RMS width.

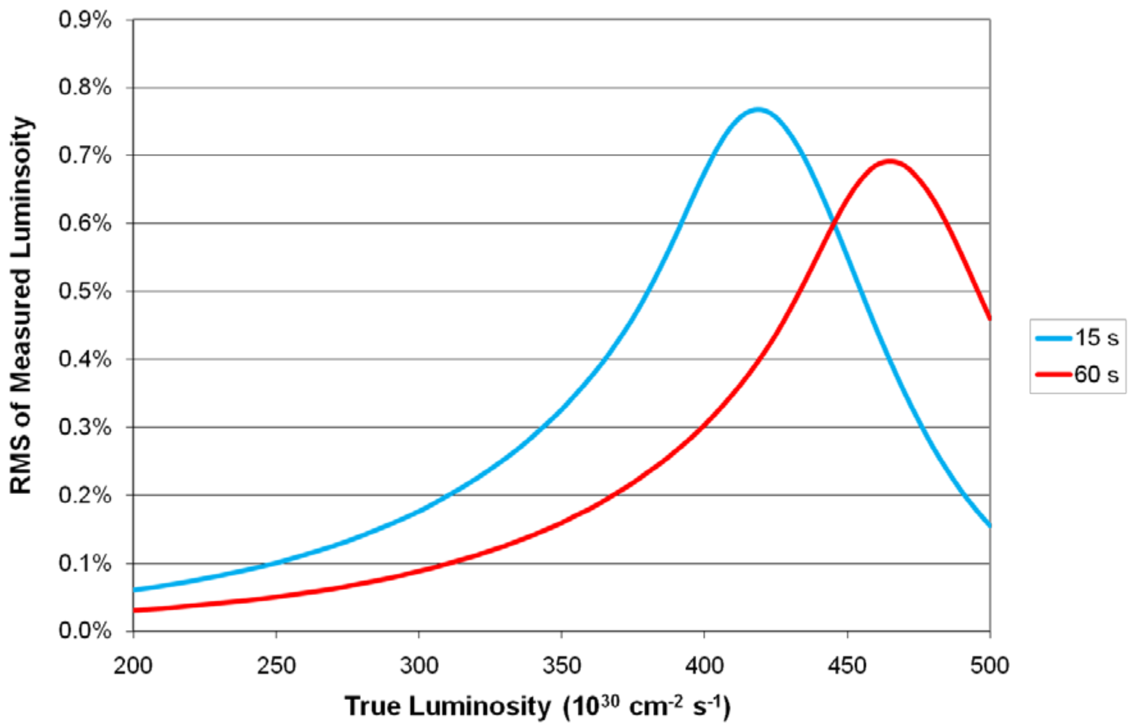

Figure 3: RMS width of the measured luminosity for 15 s and 60 s measurements periods.

One might be surprised that the luminosity can be measured with better than 1% statistical uncertainty when the average number of empty crossings is only 3. At high luminosity, the statistical error is approximated by

$$\frac{\sigma_L}{L} \approx \frac{1}{\sqrt{N_B}} \frac{1}{\bar{N}_{NS}} \frac{\sigma_{R_0}}{R_0}$$

where R_0 is rate of empty crossings, N_B is the number of beam bunches, and we have ignored the small contribution to the empty crossing rate from multiple single-sided interactions. Thus, the large statistical error on the empty crossing rate when there are an average of 3 empty crossings ($3^{-½}$ or 58%) is reduced by a factor of 6 due to the 36 independent bunch measurements and an additional factor of ~12 at 420×10^{30} cm^{-2} s^{-1} due to the exponentially falling empty crossing rate to yield a precision in the total luminosity of better than 1%.

A key requirement for the empty crossing method to work at high luminosity is that beam crossings with in-time north-south coincidences are not misclassified as empty crossings. With empty crossing rates in the part-per-million range at peak luminosities, this misclassification probability must be well under 1 ppm for typical beam crossings. Fortunately, such beam crossings are easily recognized in the DØ LM since a typical beam crossing produces in-time hits in most or all of the 24 luminosity counters, whereas our requirement is ≥1 in-time hit. Because early hits in a large number of counters can mask the presence of in-time hits, a halo veto has been implemented to exclude beam crossings with more than 6 early hits from the luminosity calculation. Timing distributions are monitored and the LM TDCs are re-calibrated as needed to ensure that the timing distributions are well centered within the timing window. Tests of the digital counting electronics have proven this system to be robust with no evidence of misclassification.

In summary, the systematic and statistical uncertainties associated with using the empty crossing method have been studied. At high luminosity there is a small bias in the measured luminosity that peaks at ~1% when the number of empty crossings in the measurement period is ~3 (420×10^{30} $cm^{-2}s^{-1}$ for a 15 s measurement period, 465×10^{30} $cm^{-2}s^{-1}$ for a 60 s measurement period). Further increases in luminosity push the measurement into the saturation region where the luminosity is no longer accurately measured. The statistical uncertainty in the luminosity measurement is below 1% for all luminosities.