Design of the Booster transverse damper.

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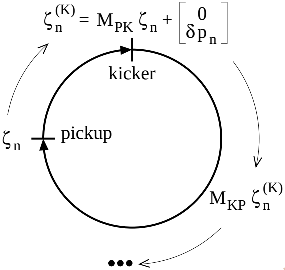
1. Damper System

Transverse beam motion can be described in terms of betatron state-vector

$$\zeta(s) \stackrel{\text{def}}{=} [q(s), p_q(s)]^{\mathsf{T}}$$

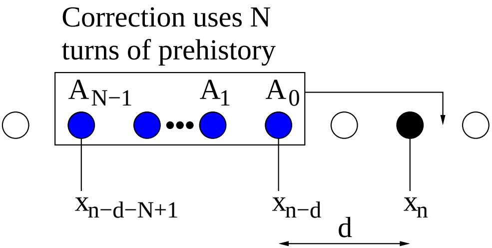
(where $q = x, z$ is one of the canonical transverse coordinates and p_q is the corresponding conjugate momentum dq/ds)



2. Digital Filter

In most general form the filter which is based on a prehistory of **N** turns and applied **d** turns later can be written as:

$$\delta p_n \propto \sum_{k=0}^{N-1} A_k x_{n-d-k}.$$



3. Choice of coefficients

1. Notch Filter

In general the digital filter should not be sensitive to the equilibrium orbit offset relative to the geometrical center of pickup

$$x_n = \alpha = \text{const}$$

in order to provide the best work of the damping algorithm. This condition is usually called a notch filter and simply gives an additional constraint

$$\sum_{k=0}^{N-1} A_k = 0.$$

Sum rearrangment

The notch condition allows one to rearrange the sum and reduce number of independent coefficients:

$$\sum_{k=0}^{N-1} A_k x_{n-d-k} = \sum_{k=0}^{N-2} B_k (x_{n-d-k} - x_{n-d-k-1}).$$

where

$$B_i = \sum_{k=0}^{i} A_k$$

2. Filter sensitivity to the linear change of the orbit in time

In addition to the notch filtering one can make the filter to be non-sensitive to the linear orbit change

$$x_n \propto \beta n$$

which gives

$$\sum_{k=0}^{N-2} B_k = 0.$$

3. Generalization

This result can be generalized for K-th order polynomial

$$x_n^{\text{eq}} = \alpha + \beta n + \ldots + \gamma n^K,$$

where at least $(K+2)$ coefficients are needed and they are given by the solution of matrix equation

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & (K+1)^0 \\ 0 & 1 & 2 & \cdots & (K+1)^1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & 2^K & \cdots & (K+1)^K \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ \vdots \\ A_{K+1} \end{bmatrix} = 0,$$

which solution is related to the binomial coefficients

$$A_k = (-1)^k C_k^{K+1}, \qquad k = 0, \ldots, K+1.$$

4. Is it enough?

- **Whether the choice of coefficients is optimal?**
- **Why do not we use let say 100 turns of prehistory for the corrections?**

4. Beam dynamics

The canonical transformation to the so-called normalized coordinates $(p, q) \to (\mathcal{P}_q, \eta_q) \to z = \eta - i\mathcal{P}$:

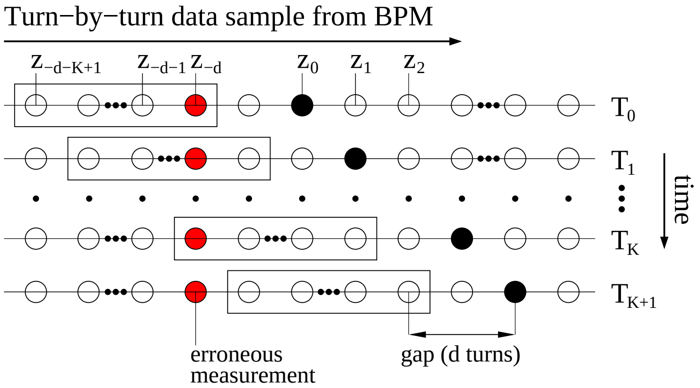
$$\begin{aligned} \eta_q(s) &= q(s)/\sqrt{\beta(s)}, \\ \mathcal{P}_q(s) &= p_q(s)\sqrt{\beta(s)} + q(s)\,\frac{\alpha(s)}{\sqrt{\beta(s)}}, \end{aligned}$$

simplify the motion to the simple rotation, and allow to write recurrent relation which describes the beam dynamics

$$z_{n+1} = e^{i\mu_2}\left[e^{i\mu_1} z_n - ig\sum_{k=0}^{\mathcal{K}-1} A_k\left(\Re\, z_{n-d-k} + \delta\eta_{n-d-k}\right)\right].$$

5. Emittance growth

Each erroneous measurement will contribute K turns $\rightarrow$ the number of coefficients should be minimized.



6. Damping

Damping rate can be easily calculated for the case of the small gain g:

$$g_d = i\,\frac{g}{2}\,e^{-i(\mu_1+\mu_0 d)}\sum_{k=0}^{\mathcal{K}-1} A_k e^{-i\mu_0 k},$$

which gives an additional constraint

$$\Im\left(g_d\right) = 0$$

in order to maximize $\Re\left(g_d\right)$.

7. Conclusion

Joint solution of matrix equation for A_k along with damping constraint $\Im\left(g_d\right)$ gives:

$$
\begin{array}{r|ll}
N=0 & A_0: & \sin\left(1.5\mu_0+\delta\mu\right) \\
 & A_1: & -\sin\left(1.5\mu_0+\delta\mu\right)-\sin\left(0.5\mu_0+\delta\mu\right) \\
 & A_2: & \sin\left(0.5\mu_0+\delta\mu\right) \\
\hline
N=1 & A_0: & \cos\left(2\mu_0+\delta\mu\right) \\
 & A_1: & -2\cos\left(2\mu_0+\delta\mu\right)-\cos\left(\mu_0+\delta\mu\right) \\
 & A_2: & \cos\left(2\mu_0+\delta\mu\right)+2\cos\left(\mu_0+\delta\mu\right) \\
 & A_3: & -\cos\left(\mu_0+\delta\mu\right) \\
\hline
N=2 & A_0: & \sin\left(2.5\mu_0+\delta\mu\right) \\
 & A_1: & -3\sin\left(2.5\mu_0+\delta\mu\right)-\sin\left(1.5\mu_0+\delta\mu\right) \\
 & A_2: & 3\sin\left(2.5\mu_0+\delta\mu\right)+3\sin\left(1.5\mu_0+\delta\mu\right) \\
 & A_3: & -\sin\left(2.5\mu_0+\delta\mu\right)-3\sin\left(1.5\mu_0+\delta\mu\right) \\
 & A_4: & \sin\left(1.5\mu_0+\delta\mu\right)
\end{array}
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Thank you for attention!