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Monte Carlo estimate of impact parameter distributions from the diffusion coefficients measured with collimator scans

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I. INTRODUCTION

Transverse beam halo diffusion rates were measured in the Tevatron and at the LHC using collimator scans [1–3]. These measurements were interpreted using a diffusion model of collimation [4, 5]. From the diffusion coefficients as a function of amplitude, one can calculate the distribution of impact parameters, i.e. the depth at which particles impinge on the collimators. These distributions can significantly affect the efficiency of a collimator system.

Particles are generated near a collimator and their trajectories are propagated around the machine including the diffusion processes measured in the LHC. Particles of increasing emittance may miss a collimator due to betatron oscillations. When a particles does reach the limiting aperture, its coordinates and turn number are recorded. A similar Monte Carlo technique was used in Ref. [4]. The results are compared with analytical estimates of the average impact parameter given in Ref. [4] and with the assumptions commonly used for the LHC collimation system design and performance evaluation [6]. A scaling law for the dependence of the average impact parameter on the diffusion coefficient and on the collimator parameters is suggested.

II. METHOD

According to the diffusion model, the evolution of the particle density in action space $f(J,t)$ is described by the following equation:

$$\partial_t f = \partial_J (D \partial_J f), \tag{1}$$

where $D(J)$ is the diffusion coefficient. The coordinate x_k of a particle at turn k can be expressed in terms of action J_k and phase θ_k:

$$x_k = \sqrt{2 J_k \beta} \cdot \cos(\theta_k), \tag{2}$$

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where β is the lattice amplitude function at the collimator. We add diffusion to an uncoupled linear machine, so that action and phase can be calculated turn by turn as follows:

$$\begin{aligned} J_{k+1} &= J_k \cdot \left(1+\sqrt{2R}\cdot \xi_k\right) \\ \theta_{k+1} &= \theta_k + 2\pi Q, \end{aligned}$$

where $R = D\tau/J^2$ is the adimensional diffusion coefficient per turn, τ is the revolution period, ξ is a Gaussian-distributed random variable with zero mean and unit standard deviation, and Q is the machine tune.

A number N_p of particles is randomly generated and followed for a maximum of N_t turns. Particles are initially generated with a uniform distribution in phase, $dN/d\theta = 1/(2\pi)$, and with a decreasing triangular distribution in action dN/dJ, from a minimum action J_m to the action corresponding to the collimator gap $J_c = x_c^2/\beta$ (Figure 1). The minimum action J_m depends on the diffusion coefficient D and on the tracking time $N_t \cdot \tau$:

$$(J_c - J_m)^2 \gg D \cdot N_t \cdot \tau. \tag{3}$$

The value of J_m is chosen so that the distribution contains all the particles that may hit the collimator in a given tracking time. The triangular shape aims at reproducing a linearly decreasing beam tail. This is a reasonable assumption given the following two facts: (a) any tail in the vicinity of the collimator can be expanded in Taylor series, of which the linear component is the first approximation; (b) because the particle flux ϕ (losses at the collimator) is given by the product of the diffusion coefficient $D(J_c)$ and the slope of

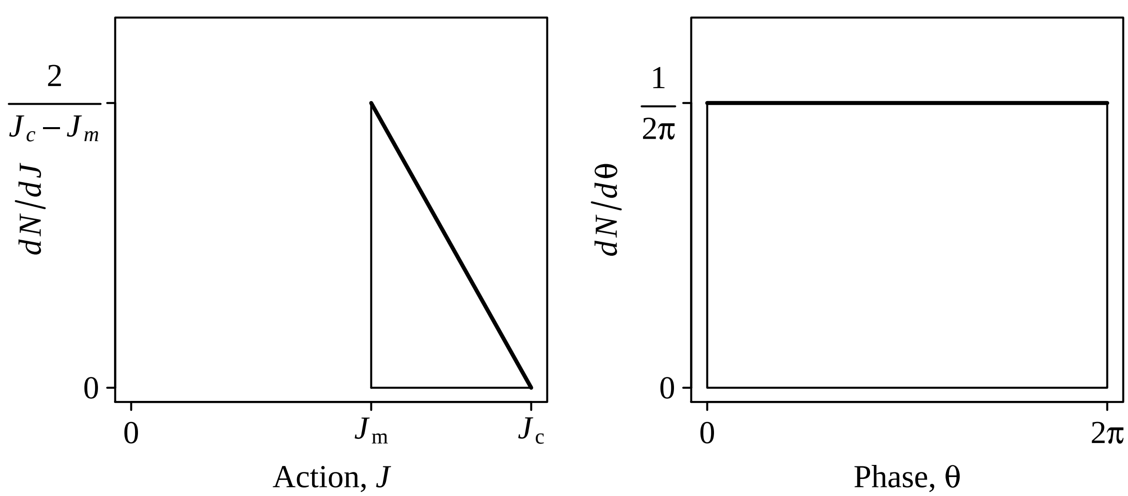

FIG. 1. Illustration of initial action and phase distributions of generated particles.

the distribution function $\partial_J f(J_c)$, $\phi = -D \cdot \partial_J f$, a linear tail produces a constant beam lifetime for a given collimator position.

If at turn $k = n$ a particle's position x_n exceedes the collimator half gap x_c for the first time, the turn number n and the impact parameter $b \equiv x_n - x_c > 0$ are recorded.

III. RESULTS

A few cases were chosen to explore the dependence of the impact parameter on the diffusion coefficient. They are based on the LHC measurements in the horizontal and vertical planes, shown in Figure 2 [2, 3]. A summary of simulation parameters is reported in Table I. Cases A–D represent the horizontal plane, where

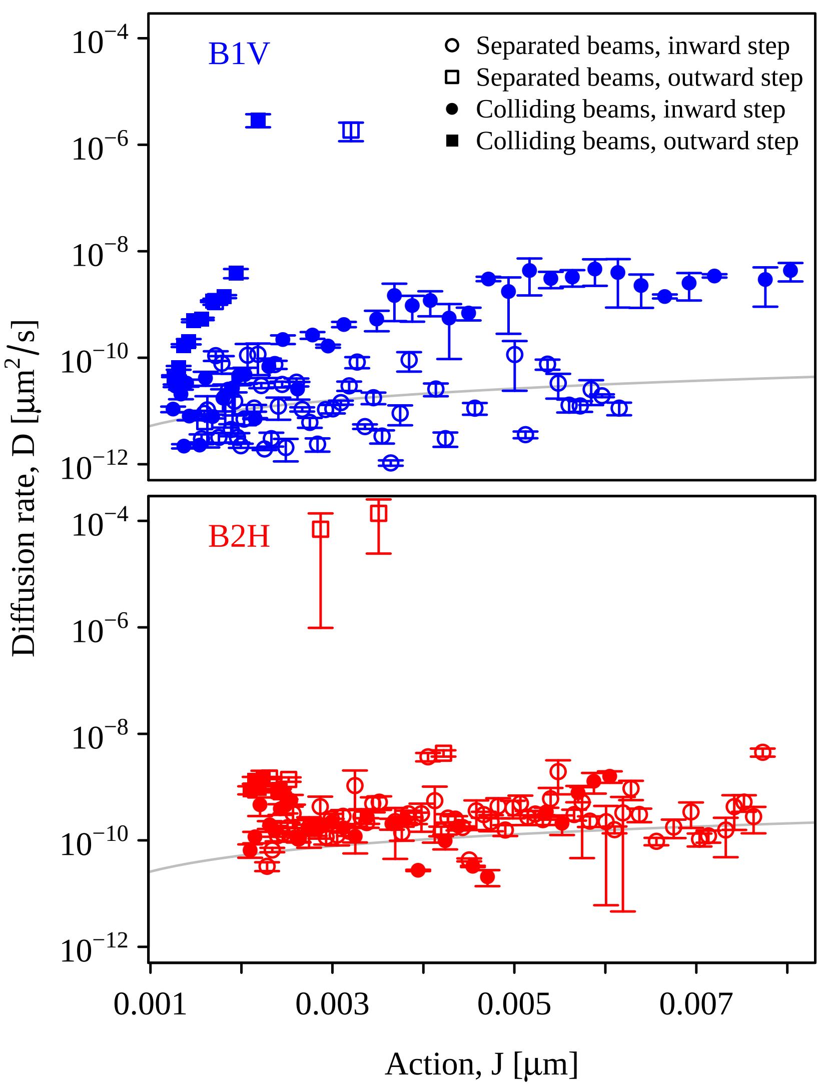

FIG. 2. Measurements of diffusion coefficients in the LHC (22 June 2012) [2, 3].

TABLE I. Summary table of simulation parameters and results.

Case	N_p	N_t	τ [μs]	Q	β [m]	D [μm^2/s]	J_c [μm]	J_m [μm]	$\langle b\rangle$ [μm]	$\langle b\rangle_{\text{ana}}$ [μm]
A	16384	2048	88.9	0.31	150	$1.3e-10$	0.002	0.00199	0.122	0.175
B	16384	2048	88.9	0.31	150	$2.3e-10$	0.004	0.00398	0.114	0.179
C	16384	2048	88.9	0.31	150	$3.3e-10$	0.006	0.00598	0.111	0.183
D	16384	2048	88.9	0.31	150	$6.5e-10$	0.008	0.00797	0.136	0.22
E	16384	2048	88.9	0.32	78	$1.2e-11$	0.002	0.002	0.0182	0.0488
F	16384	2048	88.9	0.32	78	$4.7e-11$	0.005	0.00499	0.0227	0.064
G	16384	2048	88.9	0.32	78	$3.4e-11$	0.002	0.00199	0.0315	0.074
H	16384	2048	88.9	0.32	78	$9.7e-10$	0.004	0.00396	0.12	0.23
I	16384	2048	88.9	0.32	78	$3.6e-09$	0.006	0.00592	0.19	0.344
J	16384	2048	88.9	0.32	78	$3.6e-09$	0.008	0.00792	0.162	0.315

little variation in the diffusion coefficient was observed. Cases E and F represent the vertical plane with separated beams. Cases G–J are chosen to reproduce vertical measurements with colliding beams, where the largest variation in diffusion coefficient was observed.

For each simulated case, Figure 3 shows the distribution of impact parameters, with the typical decreasing 'exponential' shape. The average impact parameter $\langle b\rangle$ of each distribution is tabulated in Table I. In Figure 4, one can see the distribution of impact turn numbers. As expected, after a few hundred turns of transient, the loss rate reaches an equilibrium. Because of the appropriate choice of the parameter J_m, the halo population is not depleted before the end of the simulation.

The dependence of the average impact parameter on action and on diffusion coefficient is plotted in Figure 6. An analytical formula was suggested in Ref. [4], as revised in Ref. [7]:

$$\langle b\rangle_{\text{ana}} = \frac{2}{3}\sqrt{\frac{2}{\pi}}\left[(1.109)\,x_c\left(\frac{D\cdot\tau}{J^2}\right)^{2/5}\right] = (1.109)\cdot\xi, \tag{4}$$

where we have defined a 'scaling parameter' ξ. A comparison between the average Monte-Carlo impact parameter $\langle b\rangle$ and the average impact parameter from the analytical estimate $\langle b\rangle_{\text{ana}}$ is shown in Table I. The Monte Carlo estimates may be biased towards low values due to the transient time needed to reach the equilibrium particle distribution, although the average impact parameter does not seem to depend on the impact turn number (Figure 5).

The scaling described by Eq. 4 is only approximately followed by the Monte Carlo calculations (Figure 6). Instead, for these calculations, the following scaling is good to about 1.3% (Figure 6):

$$\langle b\rangle = \sqrt{2\beta J}\left(\frac{D\tau}{J^2}\right)^{1/2}\sqrt{\frac{\beta}{L}} \equiv \frac{\zeta}{\sqrt{(L=18\text{ m})}}, \tag{5}$$

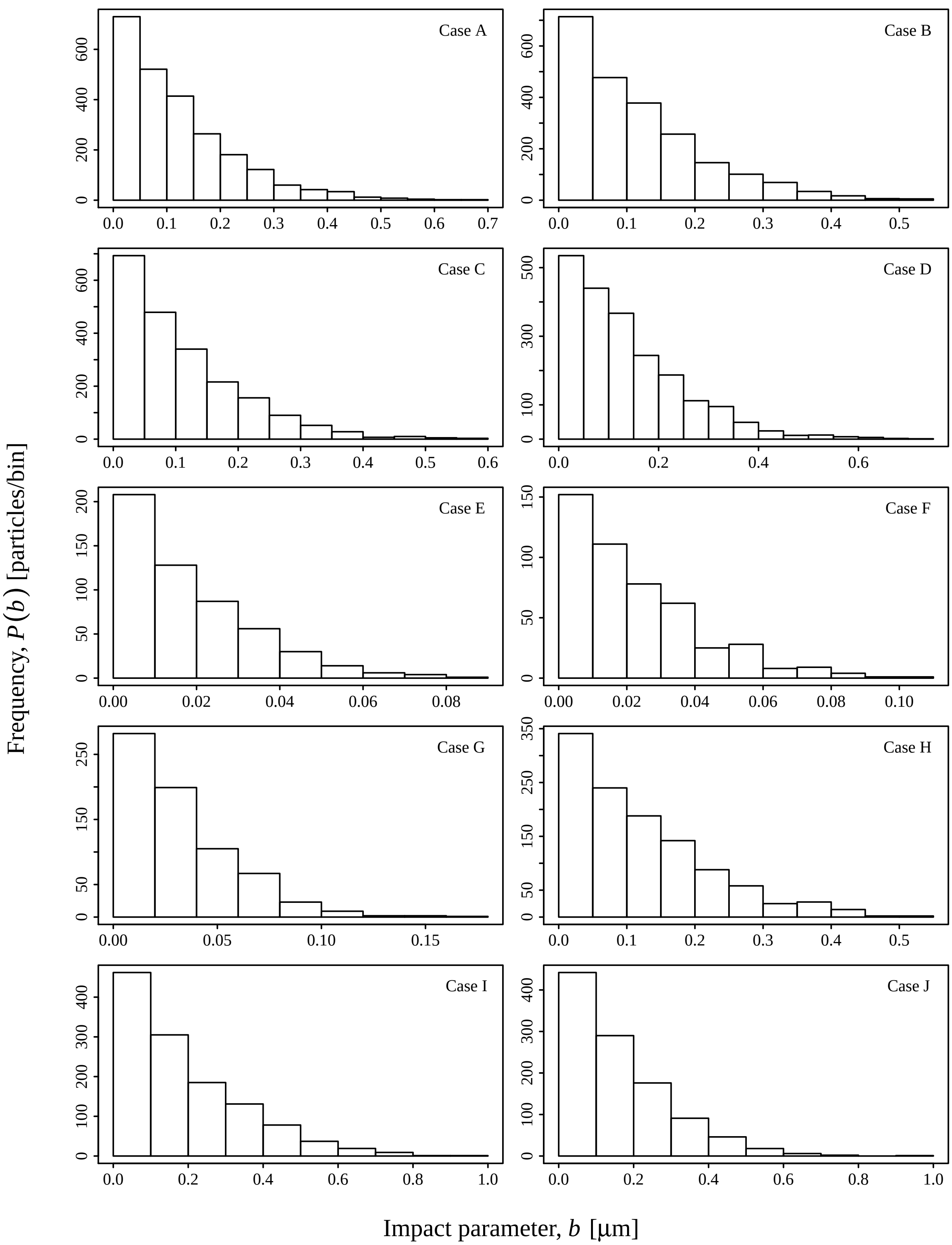

FIG. 3. Distribution of impact parameters for each of the simulated cases.

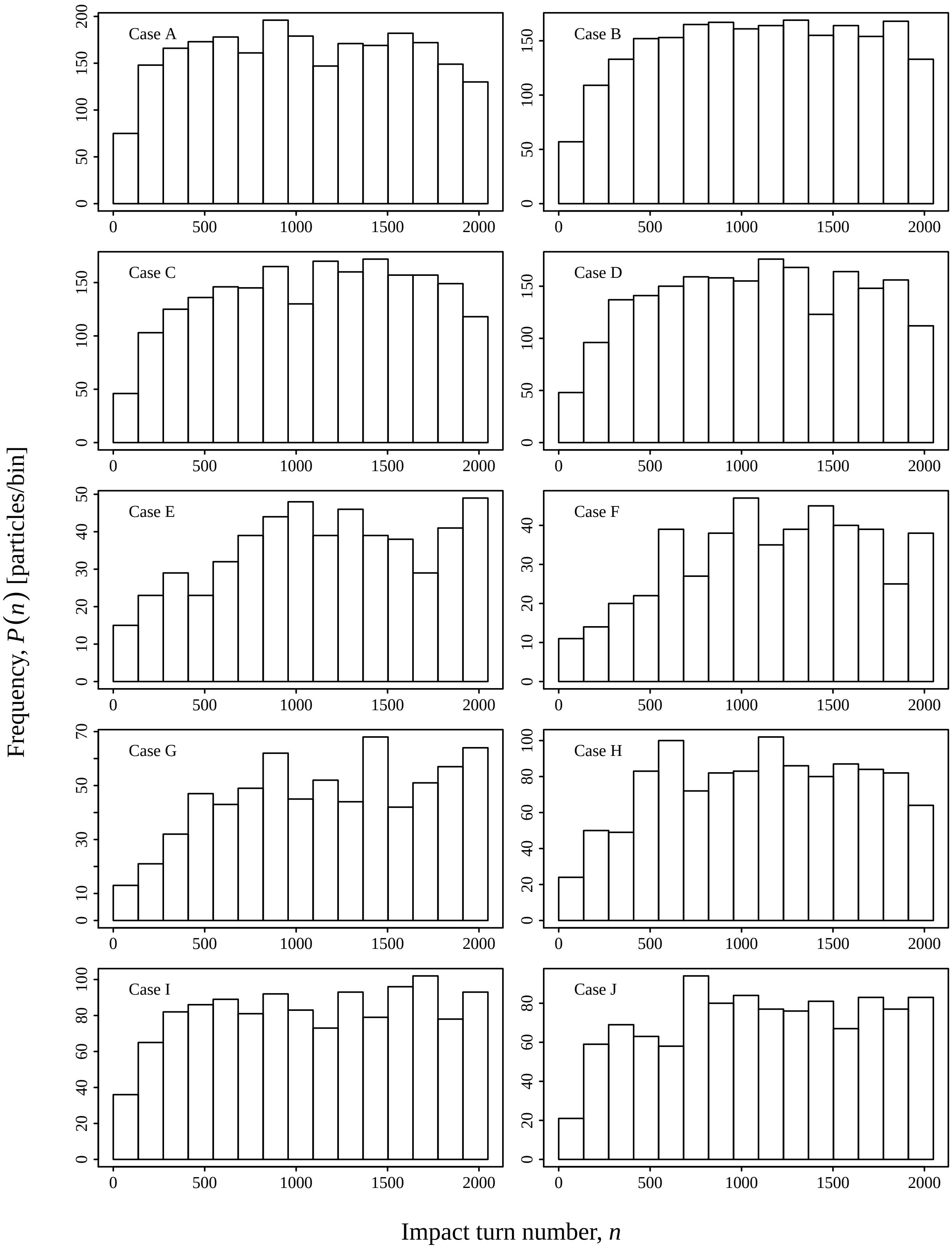

FIG. 4. Distribution of impact turn numbers for each of the simulated cases.

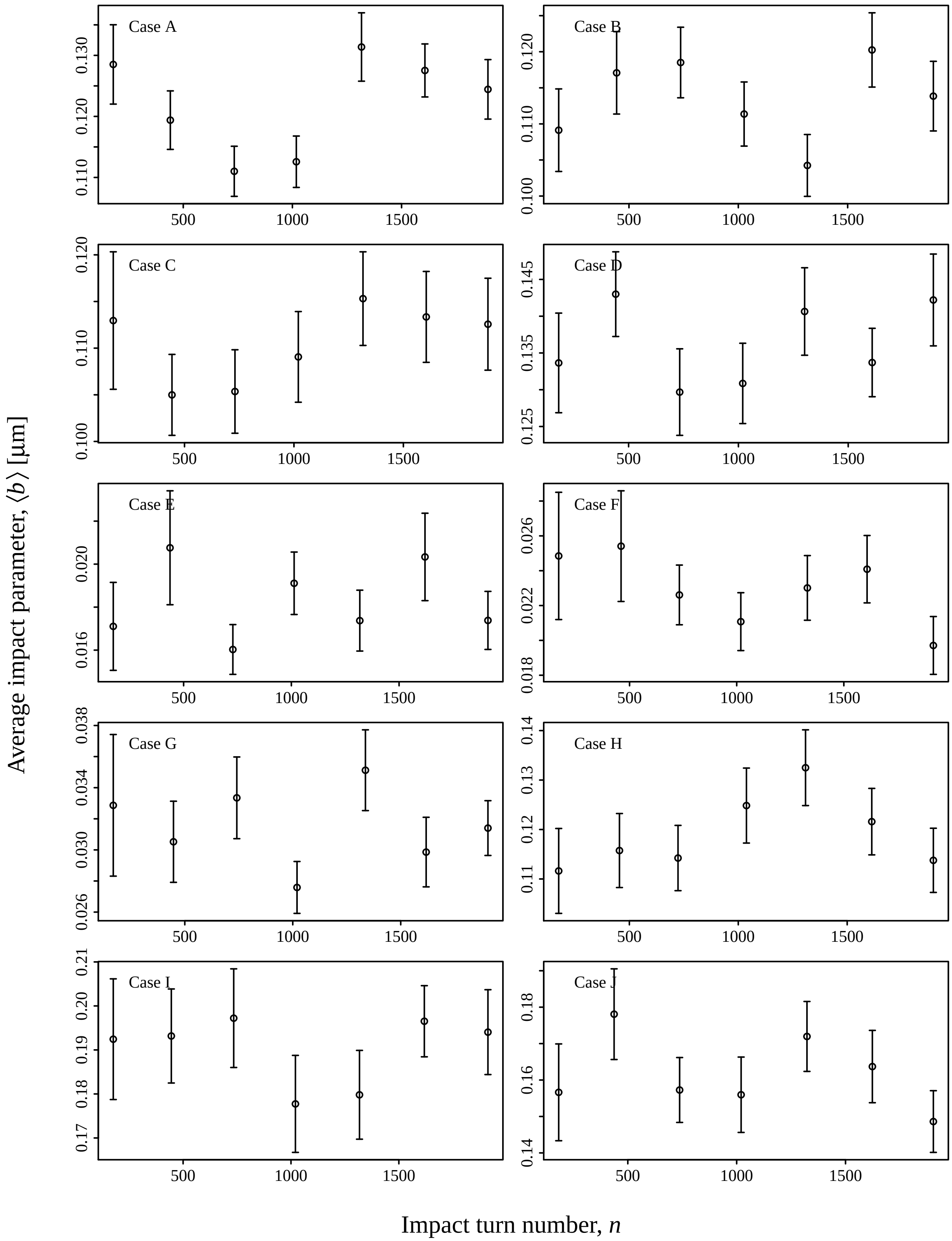

FIG. 5. Dependence of the average impact parameter on the turn number.

where we defined the scaling parameter ζ and we introduced an arbitrary length L, independent of the simulation parameters. The first factor, $\sqrt{2\beta J}$, represents the scaling with amplitude in configuration space, whereas the second factor, $\sqrt{D\tau/J^2}$, reflects the physics of diffusion. The third factor, $\sqrt{\beta/L}$, is phenomenological and we don't have a physical explanation for it, except to note that it probably depends on multi-turn collimation effects and on the betatron tune of the machine.

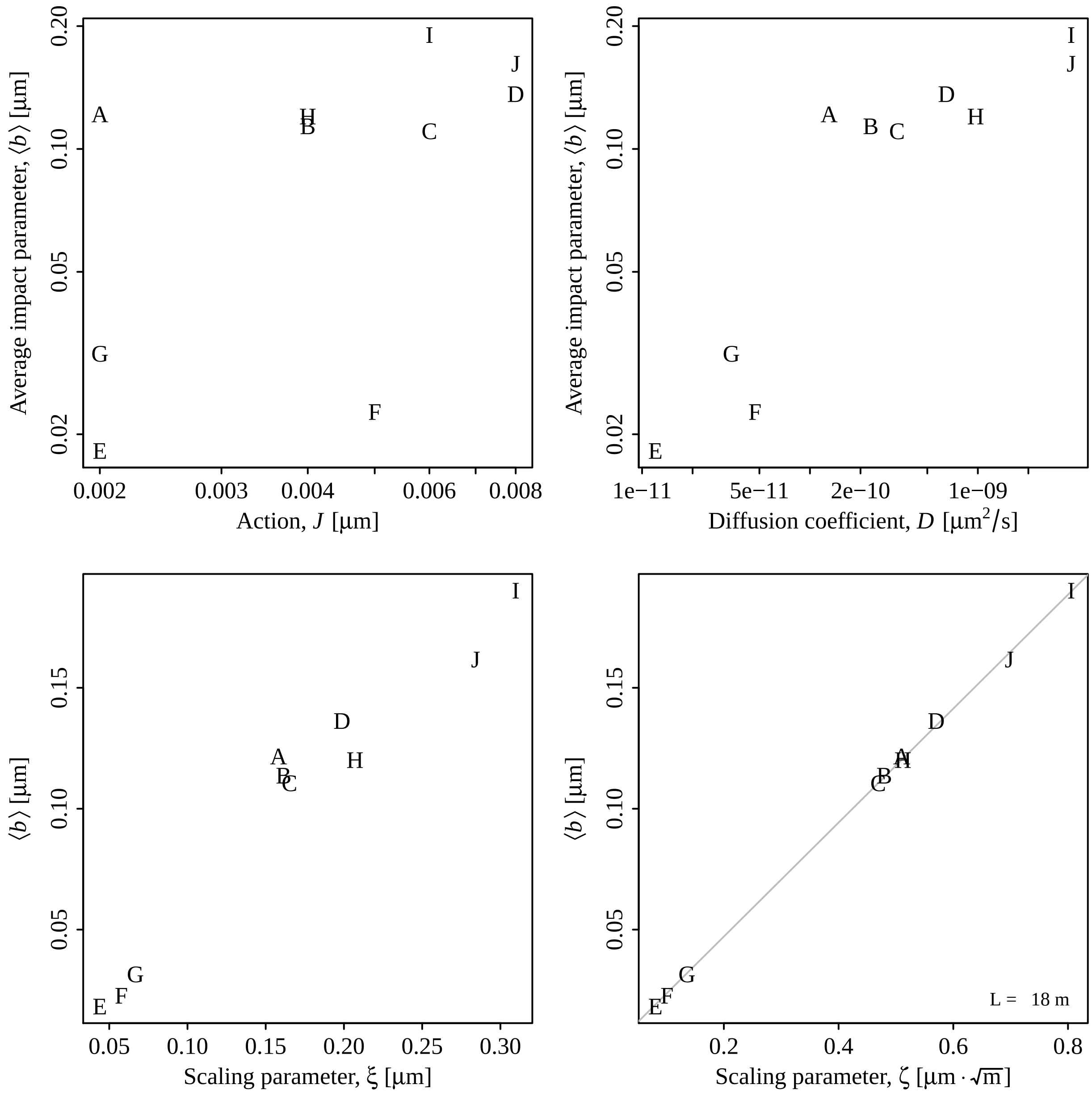

FIG. 6. For each of the cases described in Table I, the average impact parameter is plotted as a function of action (top left), diffusion coefficient (top right), and as a function of the scaling parameters described by Eq. 4 (bottom left) and Eq. 5 (bottom right).

The impact parameters found here are somewhat smaller but still compatible with the ones used in Ref. [6] to study the performance of the LHC collimation system. Of course, the numbers obtained here should be considered a lower limit: in a real machine, many effects contribute to increasing the impact parameters for a given value of the diffusion coefficient, such as transverse coupling, synchrotron oscillations in dispersive regions, and orbit jitter. Nevertheless, these estimates provide an important connection between beam dynamics (halo diffusion) and machine performance (impact parameters and collimation efficiency).

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