Development of alternative method for Booster TBT data Analysis

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August 31, 2012

Abstract

This is the summary of the work I performed under supervision of Eliana Gianfelice-Wendt during my stay at Fermilab in the Accelerator Physics Centre within the summer student program PARTI 2012. The task was to find and realize a minimization algorithm for tune identification aiming to improve results of Continuous Fourier Transform.

1 Introduction

48 Beam Position Monitors (BPMs) are distributed around the Fermilab Booster ring to measure beam deviation from design orbit in both transverse planes. Turn-by-turn data acquired along two ramps while the beam is kicked every 500 turns either in the horizontal or in the vertical plane are analyzed by Booster Console Application B38. The tunes are computed through Fourier Transform.

At first data from all BPMs are combined under the assumption that phase advance does not differ from theoretical values. The phased sum has the form:

$$\tilde{z}_n \equiv \Sigma_{k=1}^{M} z_n^{(k)} e^{-i\mu_z^{(k)}} = A\, e^{i[2\pi n\nu_0+\psi_0]}$$

where M is the number of BPMs and $z_n^{(k)}$ is either the horizontal or vertical beam position measured by the *k-th* monitor at *n-th* turn. If the signal is damped the phased sum is

$$\tilde{z}_n^d = \tilde{z}_n e^{-nd}$$

with d real positive. The actually computed Fourier transform coefficients are

$$Z(\nu) = \frac{1}{N}\Sigma_{n=1}^{N} \tilde{z}_n e^{-nd} e^{-i2\pi n\nu} = \frac{1}{N}\Sigma_{n=1}^{N} \tilde{z}_n e^{-i2\pi n(\nu - id/2\pi)}$$

B38 finds peak frequency and damping in the plane of kick from the Fourier spectrum. The corresponding component is subtracted from the original signal and the difference is again Fourier analyzed.

At this point there are two candidate tunes for each plane of kick: $\nu_{x,I}$, $\nu_{x,II}$, $\nu_{y,I}$, $\nu_{y,II}$, where $\nu_{z,I}$ and $\nu_{z,II}$ are the tunes corresponding to mode I and mode II in the plane of kick *z*, *z* being either *x* or *y*. If the distance between $\nu_{x,I}$ and $\nu_{y,I}$ is larger than CFT resolution, we assume $\nu_{x,0} = \nu_{x,I}$ and $\nu_{y,0} = \nu_{y,I}$. Otherwise we compare the high of the second peak normalized with the high of the first one and if $|A_{x,II}|/|A_{x,I}| > |A_{y,II}|/|A_{y,I}| \Rightarrow \nu_{x,0} = \nu_{x,II}$ and $\nu_{y,0} = \nu_{y,I}$;

while if $|A_{x,II}|/|A_{x,I}| < |A_{y,II}|/|A_{y,I}| \Rightarrow \nu_{x,0} = \nu_{x,I}$ and $\nu_{y,0} = \nu_{y,II}$.

This recipe works in most of the cases. But in presence of strong noises or when the mode with lowest damping rate dominates the spectra of oscillations in both planes CFT fails to identify tunes correctly. So there is the need for a method improving CFT tune identification.

2 Alternative algorithm of TBT data analysis

It has been suggested to fit the signal by the sum of two exponential functions:

$$F^{x}{}_{n} \;=\; a_x e^{i\ 2\pi n\nu_x + \varphi_x\ - n d_x} + b_y e^{i\ 2\pi n\nu_y + \psi_y\ - n d_y}$$

$$F^{y}{}_{n} \;=\; a_y e^{i\ 2\pi n\nu_y + \varphi_y\ - n d_y} + b_x e^{i\ 2\pi n\nu_x + \psi_x\ - n d_x}$$

where a_x, a_y, d_x, ν_x, ψ_x, φ_x, a_y, b_y, d_y, ν_y, ψ_y, φ_y are real fit parameters.

The figure of merit may be written as:

$$F_n = [\mathfrak{R}(F^{x}{}_{n} - \widetilde{\boldsymbol{x}}_{\boldsymbol{n}})]^2 + [\mathfrak{T}(F^{x}n - \widetilde{\boldsymbol{x}}_{\boldsymbol{n}})]^2 + \frac{A_{x0}{}^2}{A_{y0}{}^2}\{\mathfrak{R}[F^{y}{}_{n} - \widetilde{\boldsymbol{y}}_{\boldsymbol{n}}]^2 + \mathfrak{T}[F^{y}{}_{n} - \widetilde{\boldsymbol{y}}_{\boldsymbol{n}}]^2\}$$

where $\widetilde{\boldsymbol{x}}_{\boldsymbol{n}}, \widetilde{\boldsymbol{y}}_{\boldsymbol{n}}$ are experimental data. Weight $w = \frac{A_{x0}{}^2}{A_{y0}{}^2}$ is introduced to avoid overwhelming motion in one of the two planes by the other. The amplitudes $A_{x0}{}^2$ and $A_{y0}{}^2$ are obtained from CFT analysis.

So the task was to find a minimization algorithm for function F_n and write a subroutine for B38. The data processing will be done by B38. The starting values of parameters and phased sum of experimental values will be passed from the main program.

In general it is expected that $a_x > b_y$ and $a_y > b_x$. If such inequalities are not satisfied it may be that the two modes are swopped.

3 Nelder-Mead algorithm

Preliminary tests using Nelder-Mead method (or downhill simplex method) done by Yuri Alexahin with Mathematica were encouraging. Simplex method does not use gradients so it can be applied to noisy and non-smooth functions.

The ASA047 library was used for realizing the method. ASA047 is a C++ library which seeks to minimize a scalar function of several variables using the Nelder-Mead algorithm, by R ONeill (Original FORTRAN77 version by R ONeill; C++ version by John Burkardt).

4 Results, off-line test

Method was tested in off-line program with TBT data (1 ping, 64 turns) recorded from Fermilab Booster. By default Nelder-Mead method has no variable constraining, so during test minimization algorithm may find function minimal value with parameters which have no physical meaning, like negative amplitudes, phases greater $|\pi|$ by absolute value and, for the Booster, tunes out of the range [0.5;1].

A work around is that the routine computing the objective function returns infinity in case variables lie outside the wished range. This modification leads to rapid increase of number of iterations and sometime to instability with respect to changes in starting parameter values.

The second idea was to re-compute parameter values once the minimum is found by following the rules below

- if amplitude is negative than it can be replaced by its absolute value and a π added to the phase,
- if for instance phase φ_x is out of range than new $\varphi_x = \text{atan2}\ (a_x*\sin(\varphi_x), a_x*\cos(\varphi_x))$,
- if for instance tune |q| is larger than 1, then we drop the integer part, while if -0.5<q< 0.5 then we replace q with 1- |q|.

After parameters are re-computed, minimization is repeated as a check. Also this modification was tested for stability from initial point deviation. It was found that when changing initial point by

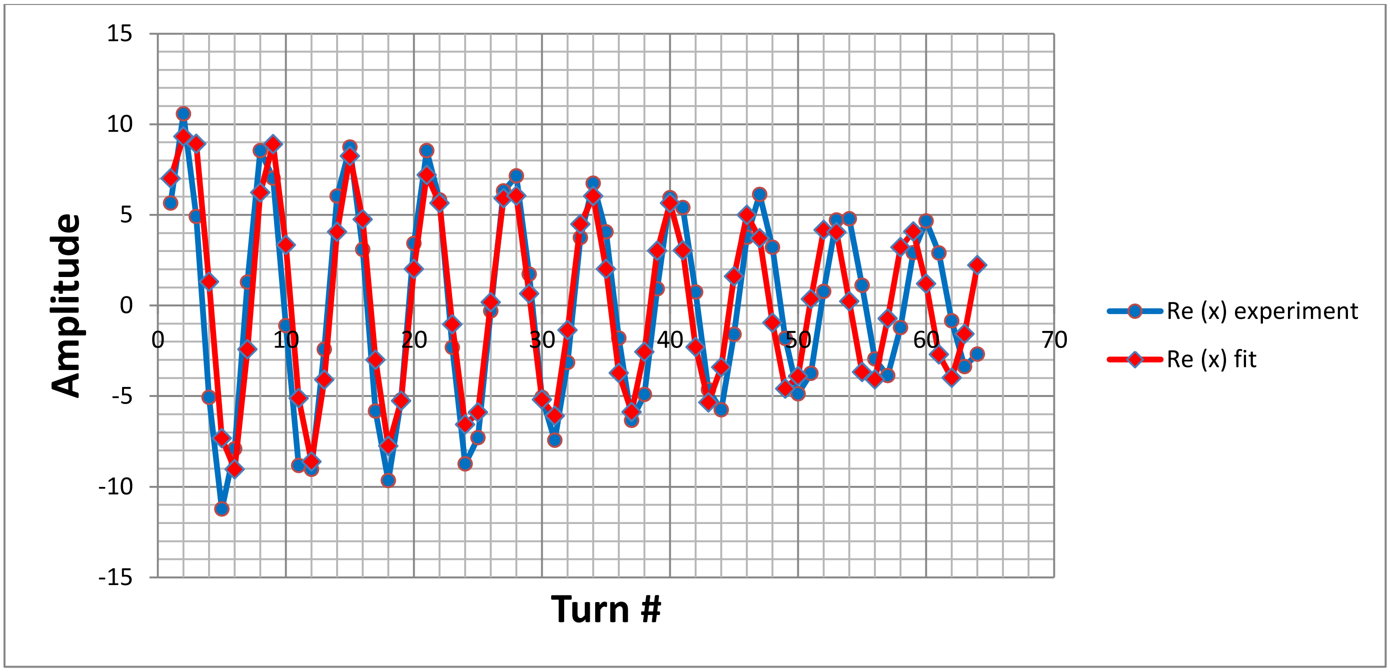
- 30% Amplitudes,
- 40% Phases,

**where atan2 is a function of two arguments, atan2(S,C) gives an angle which satisfies S=sin(angle) and C=cos(angle) and angle will be inside [-π;π];*

- 100% Damping,
- 30% Tunes,

final tunes values for minimal point do not change.

For parameters deviation more than above tunes final values do not change by more than 2%. Thus 2-nd modification was found appropriate for this kind of minimization. Plots of fitting function and experimental data were compared.



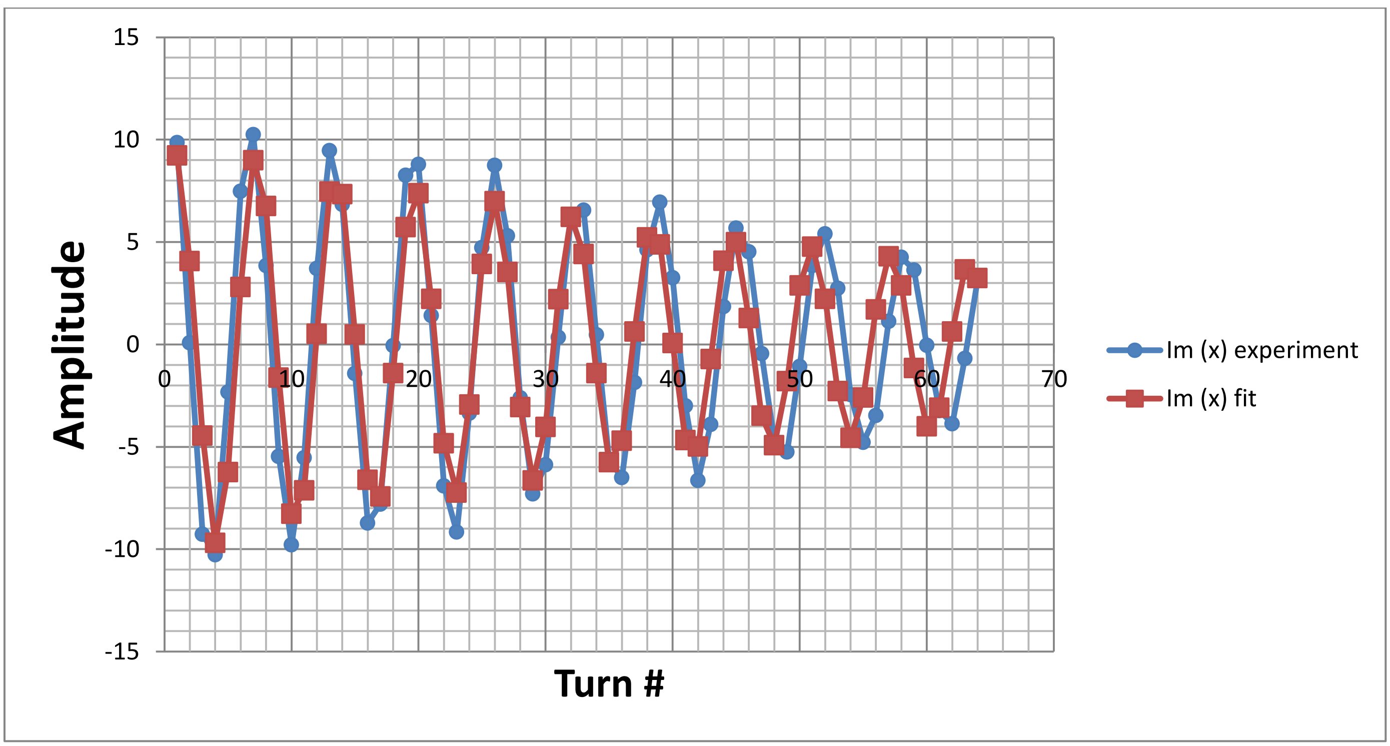
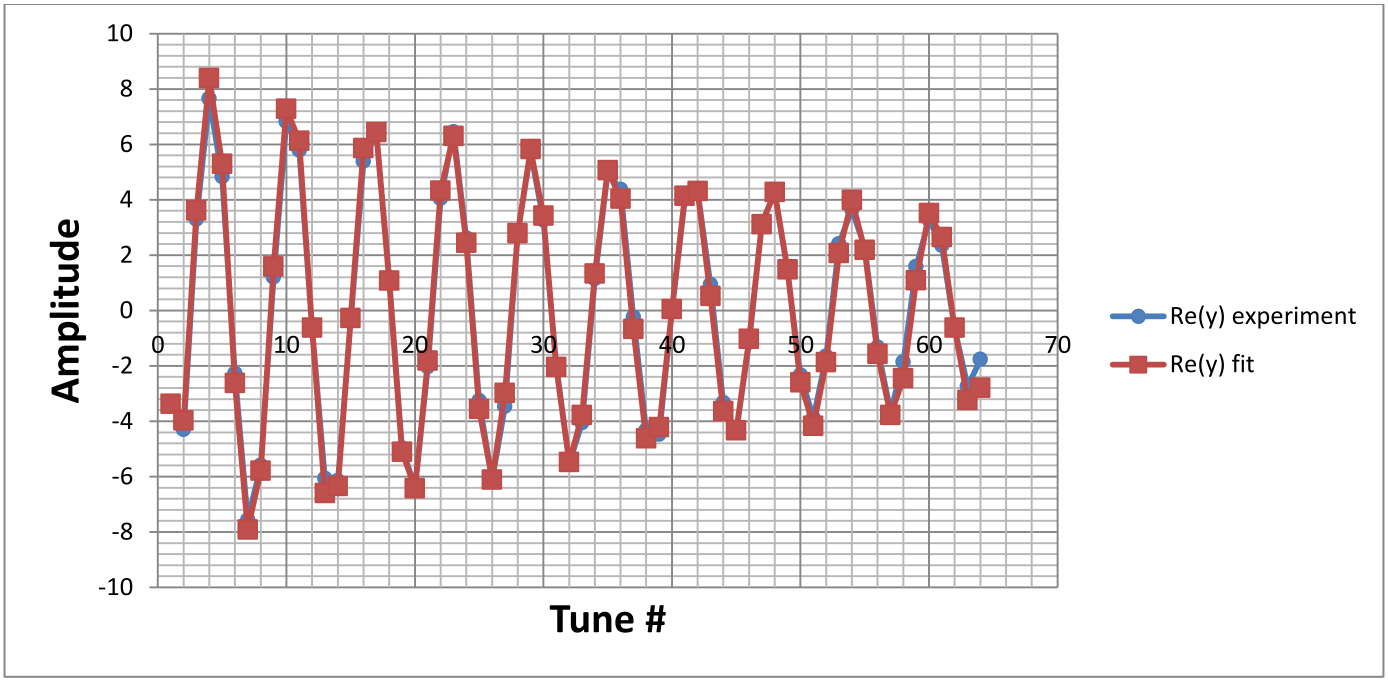


Figure1. Comparison of experimental data and fitting function for x-plane.



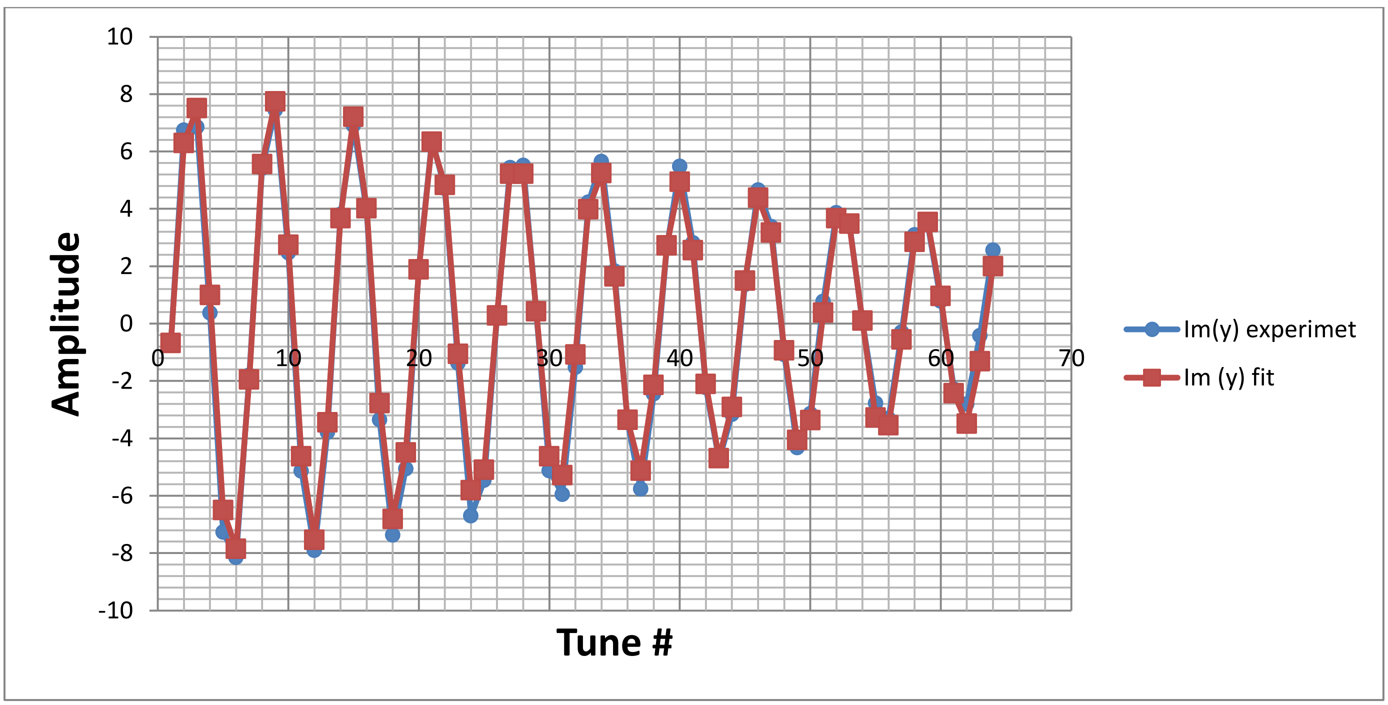


Figure 2. Comparison of experimental data and fitting function for y-plane.

Thus algorithm was found to suit for minimization of fitting function and stable w.r.t. the initial point deviation.

5 Results, application to B38

Off-line program was embedded in B38 as subroutine and tested with data recorded from Booster for 40 pings (20 000 turns). Subroutine was tested in some different regimes. Spectra were compared with CFT ones.

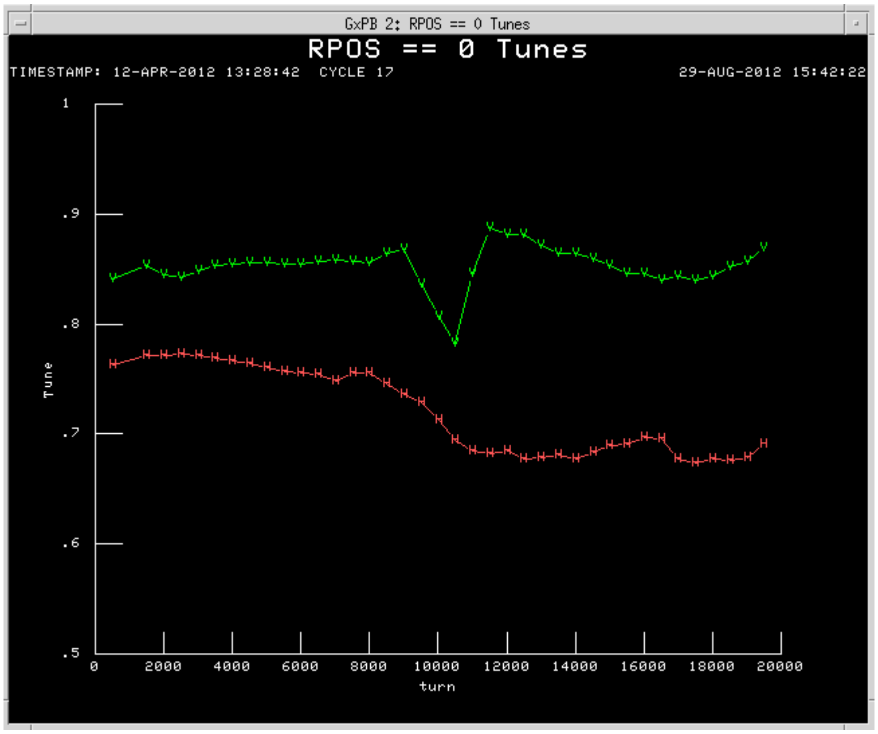


Figure 3. CFT spectrum for tunes q_x and q_y.

In first case spectra were obtained by the original Nelder-Mead method minimization, without any modifications. The second used variable restriction. And the third considered tune swopping, e.g.

- if $a_x \times a_y > b_x \times b_y$ then $qx^{true} = q_x$ and $q_y^{true} = q_y$;
- but when $a_x \times a_y < b_x \times b_y$ then $q_x^{true} = q_y$, $q_x^{true} = q_y$.

So it can be noted that variable constraining correct tune identification for two points. Tune swopping corrects variables identification for 7 points. Yet in the end 4 points of the horizontal spectrum are not correctly identified.

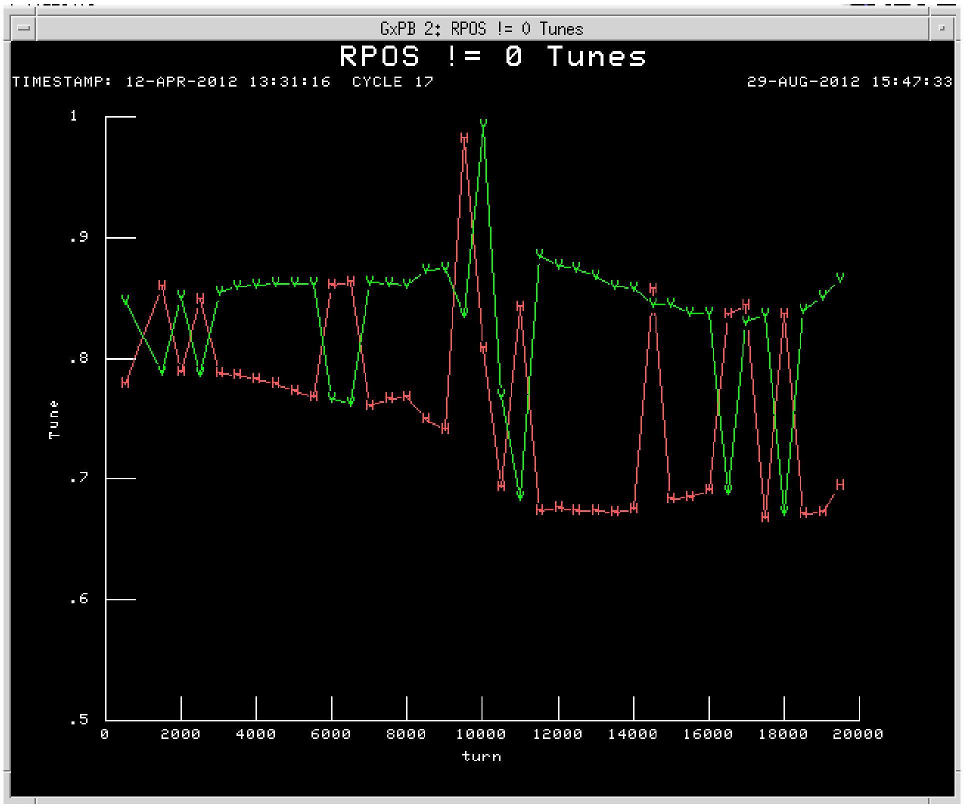


Figure 4. Spectra of fitted tunes with only Nelder-Mead minimization.

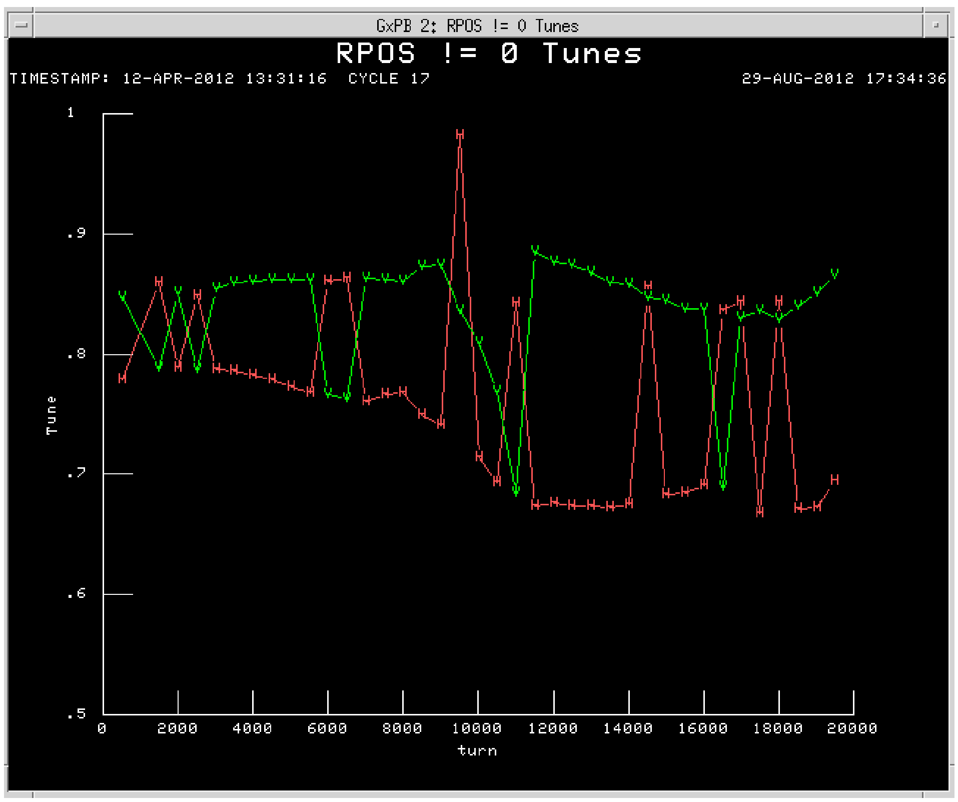


Figure 5. Spectra of fitted tunes with parameters restriction.

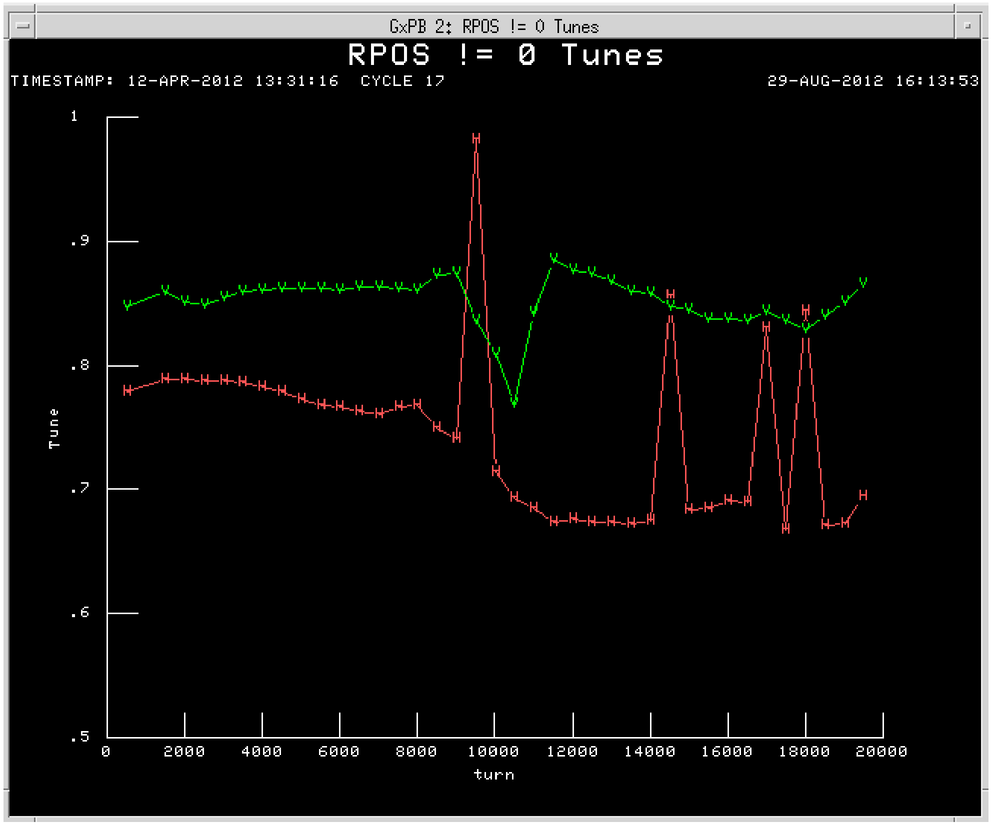


Figure 6. Spectra of fitted tunes with variable restriction and tune switching considering.

8 Summary

A minimization algorithm for computing the Fermilab Booster tunes from TBT data has been tested and embedded into B38. The method seems not more suitable than the already existing CFT for robust tune identification. Further studies are necessary for understanding whether it may be improved.

Acknowledgements

I thank Eliana Gianfelice-Wendt, William Marsh and Yuri Alexahin for supervising and guidance.

Literature

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