Luminosity Limitations for Colliders Based on Plasma Acceleration

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Preface

- There are 2 proposals for plasma based colliders
  - (1) SLAC and (2) LBL
- Both proposals claim that they can achieve a collider operation with luminosity 10^{34} $cm^{-2}s^{-1}$ or above
- However an examination of presented parameters shows that they are inconsistent with limitations coming from fundamental principles of physics
  - We also could not find how the present paradigm can be changed so that the required luminosity can be achieved
- Separately, each of below limitations can be mitigated, in principle, to an acceptable level
  - However they cannot be satisfied simultaneously
- We knowingly do not consider technical limitations
  - which are also present but, potentially, can be overcame with technology development

Preface (continue)

- The following limitations were considered
  - Limitations on the beam emittance and momentum spread coming from the SR radiated in the final focus lenses/quads
  - Emittance growth due to multiple scattering in the plasma and/or residual gas
  - Pinching/expulsion of plasma electrons by electric field of positron/electron bunch
  - Pinching/expulsion of plasma ions (in the bubble regime) by electric field of electron/ positron bunch
  - Impact ionization of ions by bunch electric field if heavy ions are used to mitigate ion pinching by electrons in the bubble regime
  - Beamstrahlung on heavy ions in the bubble regime

- The hollow beam channel was proposed to overcame these problems. However it has additional limitations
  - First, it is unclear how to create a μm radius hollow channel free of ions and gas
  - Second, achieving transverse stability in the presence of very large transverse impedance of the plasma channel requires considerable momentum spread and very strong focusing which can be achieved with plasma focusing only. Such focusing cannot be supported in the hollow channel. External focusing will be 3-4 orders of magnitude weaker than the required value
    - Note that in the case of plasma focusing a few percent tune spread is required for transverse beam stability
- Allocated time does not allow detailed discussion of all limitations (some details are in the backup slides)
  - We are open to offline discussion

Final Focus and Beam Emittance Limitations

- Assume
  - Axial symmetric focusing with chromaticity suppressed by η_F
  - Chromatic contribution to σ^* is equal to betatron beam size

$$\sqrt{\varepsilon\beta^*} \approx \eta_F \sqrt{\frac{\varepsilon}{\beta^*}} F \frac{\Delta p}{p} \quad \Rightarrow \quad \frac{\Delta p}{p} \approx \frac{\beta^*}{\eta_F F}$$

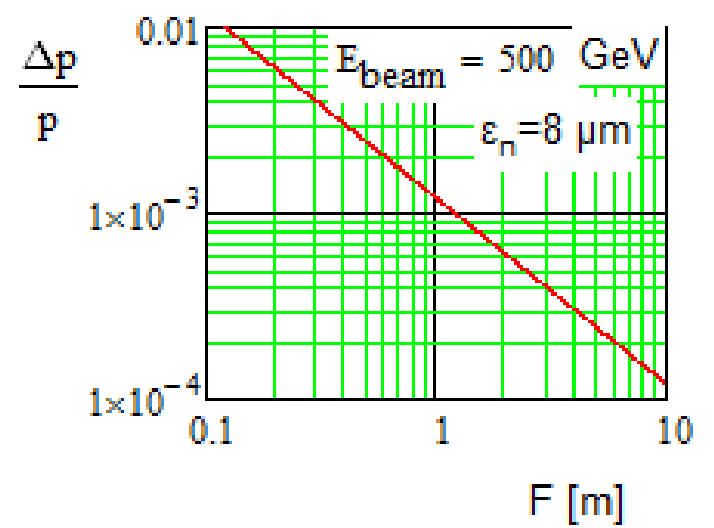
  - SR from the FF lens changes particle momentum by Δp for $r=\sigma$
- Then

$$\left.\frac{\Delta p}{p}\right|_{SR} \approx \frac{2}{3}\frac{e^4 B^2 \gamma L_{lens}}{m_e{}^3 c^6}, \quad L_{lens} \approx F\,, \quad B = Gr \approx GF\sqrt{\frac{\varepsilon}{\beta^*}}\,, \quad \frac{1}{F} \approx \frac{eGL_{lens}}{mc^2\gamma} \quad \Rightarrow \quad \left.\frac{\Delta p}{p}\right|_{SR} \approx \frac{2}{3}\frac{r_e\gamma^3}{F}\frac{\varepsilon}{\beta^*}$$

  - Equalizing $\Delta p/p$ and $\Delta p/p_{SR}$ one obtains limitation on the beam emittance

$$\varepsilon_n \equiv \varepsilon\gamma \le \sqrt[3]{\frac{3\sigma^{*4}}{2r_e\eta_F}} \quad \text{where} \quad \sigma^* = \sqrt{\varepsilon\beta^*} \quad \Rightarrow \quad \frac{\Delta p}{p} \le \frac{\gamma}{F}\sqrt[3]{\frac{2r_e\sigma^{*2}}{3\eta_F{}^2}}$$

- Thus, SR from FF limits beam emittance and momentum spread.
  - For σ^* = 10 nm & η_F = 0.01 we have ε_n < 8 μm, $\Delta p/p = 10^{-3}$ for F=1 m
    - Emittance reduction allows $\Delta p/p$ increase so that to bring it to a manageable level



Final Focus and Beam Emittance Limitations (2)

- Note that this estimate is extremely optimistic
  - Only one axial symmetric lens
    - It will be worse with quads
    - Making plasma with required linearity is questionable
      - Pinching excludes plasma lens usage
  - SR considered at 1σ (need 2.5-3σ)
  - SR in chromaticity compensation section is negligible
- To get acceptable values of momentum spread we further will assume the following parameters (for 500 GeV & σ^*=10 nm)
  - $F \approx 1$ m, $\varepsilon_n < 0.1$ μm, $\sigma_p \equiv \Delta p/p < 10^{-2}$
  - This value of emittance coincides with LBL proposal
    - Later, to avoid problems with beam loading, they claimed that it can be strongly increased
    - We do not see how it can be consistent with the above limitation

Emittance Growth due to Multiple Scattering

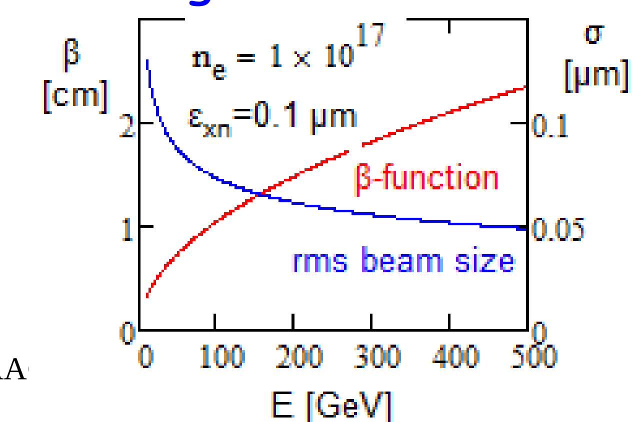
- The multiple Coulomb scattering creates emittance growth in the course of acceleration. In a fully-ionized plasma it is

$$\frac{d\varepsilon_n}{d\gamma} = \frac{2\pi Z(Z+1) r_e^2 n_i \beta_f(\gamma)\Lambda_c}{\gamma(d\gamma/ds)}, \quad \varepsilon_n = \beta\gamma\varepsilon$$

- Assume that an accelerator operates in the blow-out regime
  - ⇨ focusing is supported by plasma density and the beta-function is: $\beta_f(\gamma) = \frac{\sqrt{2\gamma}}{k_p}, \quad k_p = \frac{\omega_p}{c}$
  - ⇨ Accelerating gradient is: $\frac{d}{ds} mc^2\gamma = \frac{4\pi e^2 Z n_i}{k_p}\sin\theta, \quad \sin\theta \approx 0.5 - \text{accelerating phase}$
  - ♦ That yields the emittance increase for acceleration to γ_{fin}:

$$\Delta\varepsilon = \frac{Z\Lambda_{ci} + \Lambda_{ce}}{\sin\theta} r_e \sqrt{2\gamma_{fin}}, \quad \Lambda_{ci} \approx 20, \quad \Lambda_{ce} \approx 2$$

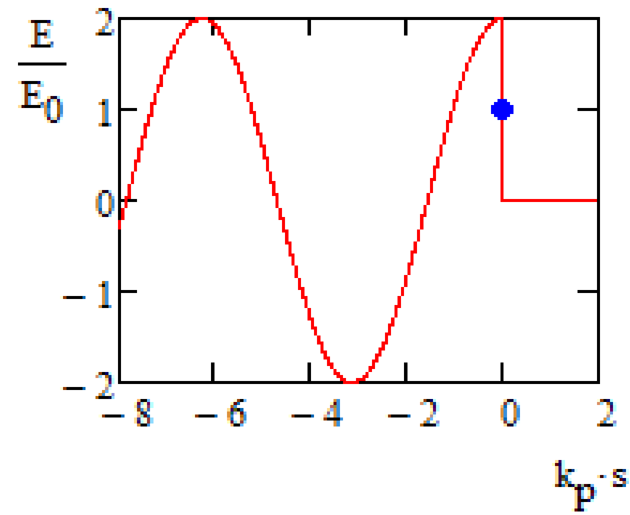
- For E_{fin}=500 GeV it yields negligible growth: $\Delta\varepsilon_n = 2\cdot 10^{-4}$ μm
  - ♦ However it is achieved due to very large plasma focusing: $G=3\cdot 10^8$ G/cm for $n_e=10^{17}$ cm^{-3}
    - It is 4 orders of magnitude above present achievements with SC magnets
    - It results in a sub-micron beam size



Bunch Deceleration and Wake in Plasma

- Well-known 3-step solution for a particle:
  1. Collective plasma response at large ρ
  2. Particle interaction with independent plasma electrons at small ρ
  3. Combining two approaches one obtains:

$$\left(\frac{dE}{ds}\right)_0 \approx \frac{4\pi n_e Z^2 e^4}{m_e v^2} \ln\left(\frac{\rho_{max}}{\rho_{min}}\right), \qquad \rho_{max} \approx 1.123 \frac{v}{\omega_p}, \quad \rho_{min} = \frac{Ze^2}{m_e v^2}.$$

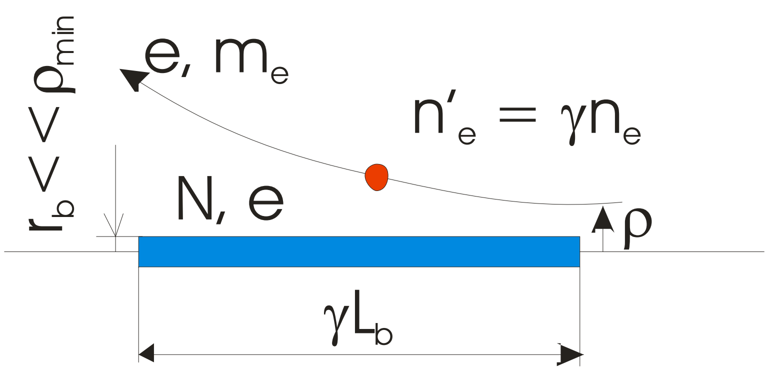


Logarithmic approximation:

it works well as long as $\rho_{max} \gg \rho_{min}$

- Longitudinal wake is $\frac{dE}{ds} = \left(\frac{dE}{ds}\right)_0 2\cos(k_p s)$

- Similar approach can be used for a bunch
  - Field screening and ρ_{max} will stay the same if the bunch is short, $L_b \ll k_p^{-1}$
  - ρ_{min} will be modified because of finite bunch length
    - For small ρ electric field of plasma electrons can be neglected
  - Deceleration will be growing from bunch head to its tail



Bunch Deceleration and Wake in Plasma (2)

- In logarithmic approximation an integration yields:

$$\frac{dE}{ds} \approx \frac{4\pi n_e N_e e^4}{mc^2} L_c F_g(s), \quad F_g(s) = 2\int_s^{\infty} \cos\left(k_p(s-s')\right) f(s')\,ds', \quad \int_{-\infty}^{\infty} f(s)\,ds = 1$$

$$L_c = \begin{cases} \ln\left(1+\dfrac{\sqrt{2}c}{\omega_p \rho_{\min}}\right), & \text{electrons}, \\ \ln\left(1+\dfrac{2.15\sqrt{2}c}{\omega_p \rho_{\min}}\right), & \text{positrons}, \end{cases} \qquad \rho_{\min} = \sqrt{\frac{2\sqrt{2}N_e e^2 \sigma_s}{\sqrt{\pi} m_e c^2}}$$

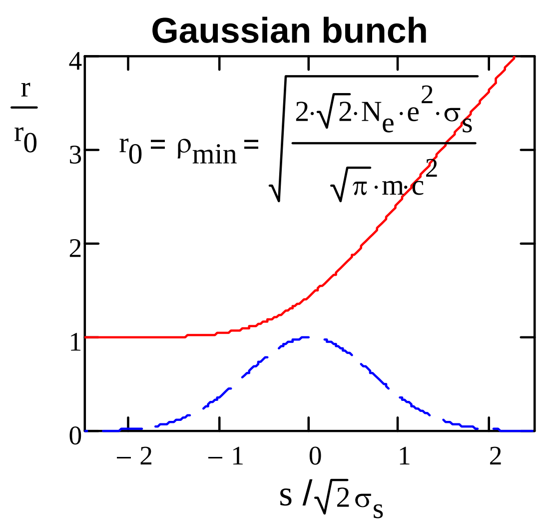
1 is added to logarithm makes equation working for $\rho_{min} \sim \rho_{max}$. Good coincidence with numerical simulations[*]

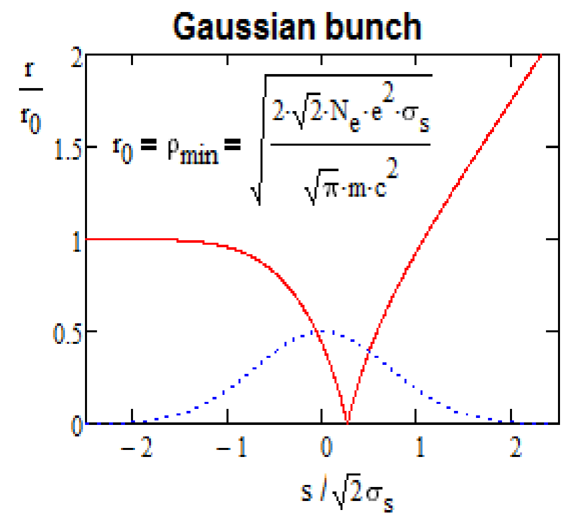
By definition $\delta\rho \approx \rho_0$ @ $\rho_0 = \rho_{min}$

- The contribution for $\rho < \rho_{min}$ is small
- The longitudinal wake is close to a step function:

$$W(s) \propto \theta(s)\cos(k_p s)$$

  - strictly speaking it is dependent on the bunch density distribution but it makes only logarithmic correction





Plasma electron scattering on the bunch with impact parameter ρ_{min} : left – e-, right – e+

[*] W. Lu, et.al.,"Limits of linear plasma wakefield theory for electron or positron beams" Physics of plasmas 12, 063101 (2005)

Bunch Deceleration and Wake in Plasma (3)

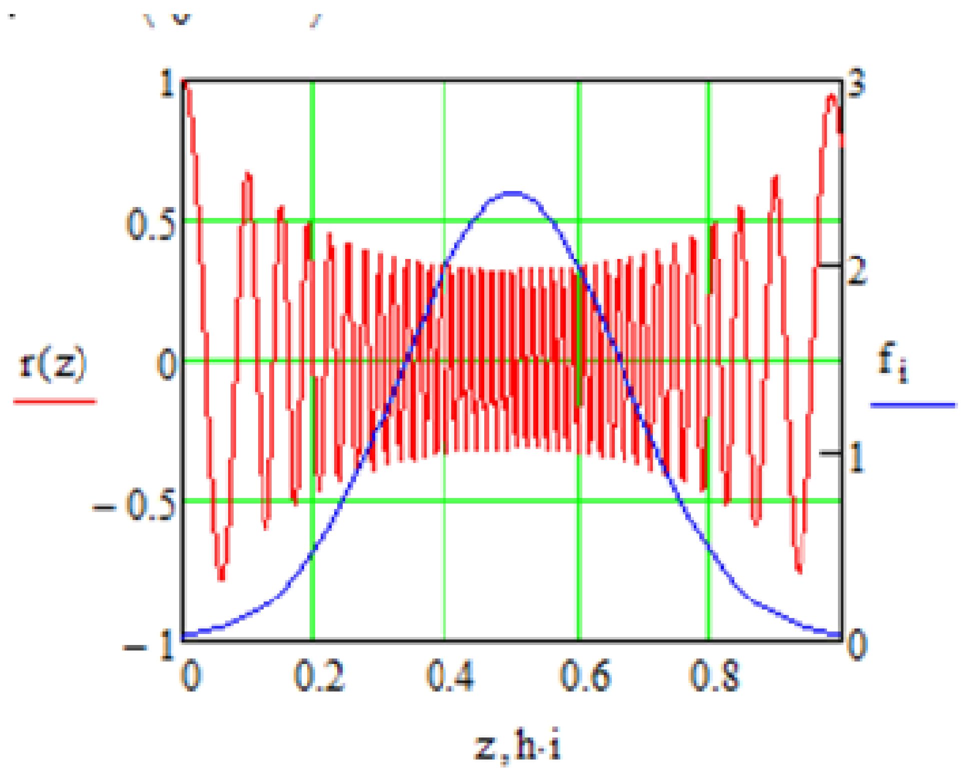
- Longitudinal wake function is obtained in a logarithmic approximation
  - It is justified only for $\rho_{min} < \rho_{max}$
- If bunch radius is larger than ρ_{min} but is still much smaller than ρ_{max} it should be used instead of ρ_{min}
- Thin electron bunch ($\sigma_{\perp n} \le \rho_{min}$) leaves behind a cavity with radius of $\sim\rho_{min}$ oscillating at plasma frequency
- The transition to the blowout regime happens when the bunch intensity increase increases ρ_{min} to $\approx\sqrt{\rho_{\max}\sigma_s}$
  - Then, the deceleration achieves its maximum field of about

$$E_{\max} = \frac{4\pi e n_e}{k_p} = 30.2\frac{\text{GV}}{\text{m}}\sqrt{\frac{n_e}{10^{17}\,\text{cm}^{-3}}}$$

- The same equation describes bunch deceleration in the blow out regime
  - $L_c \approx 2$ ($\ln\left(1+\rho_{\max}/\rho_{\min}\right)$, $\rho_{\min} \approx r_{cavity}$, $\rho_{\max} \approx r_{cavity} + 1/k_p$, $r_{cavity} = 1/k_p$)
- The above equations describe bunch deceleration comparatively accurate for the parameters of plasma colliders

Collapse of electrons inside of the positron bunch

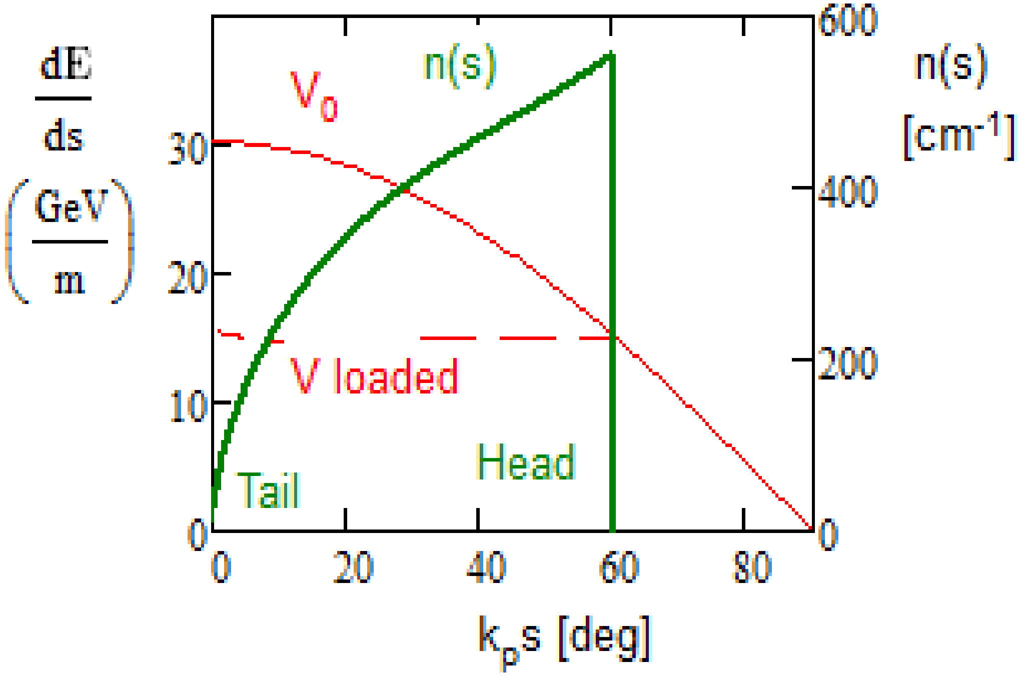
- Collapse of plasma electrons to the positron bunch center makes comparatively little correction to the beam deceleration but greatly affects the beam focusing coming from plasma
  - Focusing is dependent on coordinate and is strongly non-linear
    ⇨emittance growth
  - That prevents acceleration of positrons in the linear regime
  - Blow-out regime does not have electrons on the axis but cannot be used for positrons due to their strong defocusing from ions
    - Using electrons for restoring focusing of positrons has a problem with collapse of electrons to the center as it was discussed above



A trajectory of a plasma electron inside of the positron bunch for $\rho_0 \ll \rho_{min}$

Bunch Shaping

- Shaping bunch profile can significantly reduce accelerating voltage variations along the bunch
  - Growth of accelerating voltage is compensated by growth of decelerating force along the bunch
- The total bunch length is $\mathrm{acos}\left(V_{leaded}/V_0\right)$ (60 deg. for 50% loading)
- There are open questions
  (1) How such shape can be created and
  (2) how accurate can it be for the required bunch brightness
- An increase of beam loading reduces accelerating gradient



Longitudinal bunch density and loaded accelerating voltage for 50% beam loading

Plasma Focusing

- Using Maxwell equation one obtains that magnetic field is zero in the absence of external currents

$$\mathrm{rot}\,\mathbf{B} = \frac{4\pi \mathbf{j}}{c} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \quad \mathbf{j}_\omega = \frac{n_e e^2 \mathbf{E}_\omega}{i\omega m_e} \quad \Rightarrow \quad \mathrm{rot}\,\mathbf{B}_\omega = \frac{i\omega}{c}\left(1 - \frac{{\omega_p}^2}{\omega^2}\right)\mathbf{E}_\omega = 0 \quad \Rightarrow \quad \mathbf{B} = 0$$

- In the case of quasi-linear regime this statement is not perfectly correct. However electric field makes major part of focusing

$$F_r = eE_r = -2\pi e^2 \delta n\, r - \frac{r}{2}\frac{\partial E_z}{\partial z}$$

- In the blowout regime focusing has contributions from E & B fields partially compensated. That leaves contribution from ions only

$$F_r = e\left(E_r + B_\theta\right) = -2\pi e^2\, n_i\, r$$

- Hollow plasma channel problem (suggested as a remedy from pinching)
  - ♦ An absence of charge near the axis removes its focusing
  - ♦ For laser driven plasma where an excitation wave propagates below c an absence of E&B compensation results in minor defocusing for both charges. It changes along the bunch.

Transverse Wake

- There is no transverse wake in uniform plasma
  - However focusing of trailing particles do exist (detuning wake)
- Beam acceleration perturbs plasma density and creates accelerating channel and, consequently, transverse wake
- For small beam size ($\sigma_{b\perp} \ll c/\omega_p$) the wake field is nearly uniform in transverse plane
  - The wake-function grows almost linearly
  - In logarithmic approximation it is

$$W_\perp = 2\left(\frac{\omega_p}{c\,\sigma_\perp}\right)^2 \left(\frac{\Delta n}{n}\right)_e (s-s')\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \xrightarrow{c/\omega_p=\sigma_\perp} 2\left(\frac{\Delta n}{n}\right)_e \frac{s-s'}{\sigma_\perp{}^4}\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right)$$

$\sigma_\perp$ is the rms size of plasma channel

- Comparing it with focusing strength of plasma channel we obtain at the bunch end

$$\frac{E_{wake}(s=L_b)}{E_{plasma\_foc}} \approx \frac{2L_b}{\sigma_\perp}\frac{(dE/ds)_{loss}}{(dE/ds)_{\max}}\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right)$$

where $(dE/ds)_{loss}$ – average energy loss in plasma
$(dE/ds)_{max}$ - maximum accelerating field for given plasma density

Main Features of SLAC& BNL Proposals

	LBNL [1,3]	SLAC [2]
Plasma parameters		
Regime of acceleration	Quasi-linear	Bubble
Excitation type	Laser beam	Electron beam
$\Delta n/n$	~ 0.3	1
n_e, cm^{-3}	10^{17}	$2\cdot10^{16}$
Wave length, $2\pi/k_p$, μm	105	234
E_{max}, GV/m	30	13.5
Loaded dE/dx, GeV/m	5	7.6
Beam parameters (colliding bunches)		
Final beam energy, GeV	500	
Luminosity (geometr.), $cm^{-2}s^{-1}$	$1.9\cdot10^{34}$	
Repetition rate, kHz	15	
Particles per bunch, $N_e=N_p$	$4\cdot10^9$	10^{10}
Rms. norm. emittance, $\varepsilon_y/\varepsilon_x$, nm	100/100†	10^4/35
Beta at IP, β_x^*/β_y^*, mm	1†	11/0.1
Rms bunch length, μm	**1 ?**	20

†Emittance is not presented in the most recent publication

Issues with the LBNL Proposal

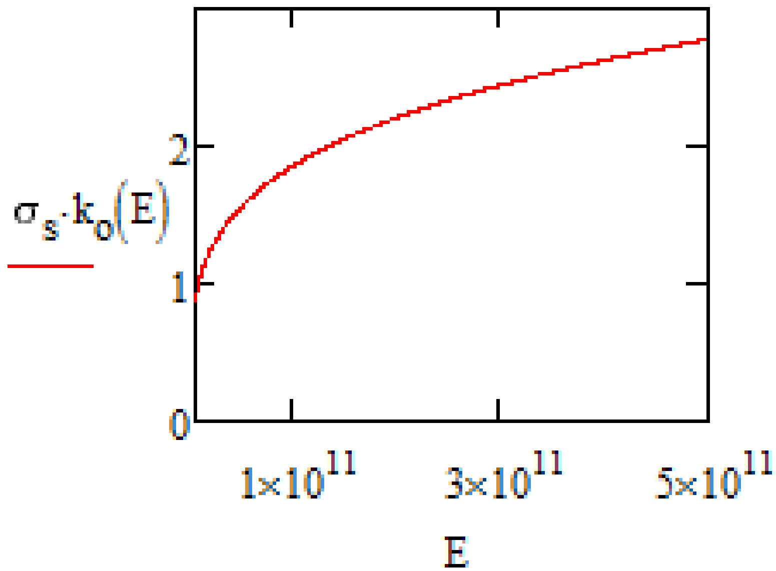
- RMS bunch length is only 3.4 deg. (1 μm)
  - It is incompatible with small energy spread at large beam loading
    - 50% beam loading => total bunch length of 60 deg. (~15-20 deg. rms)
- Bunch population is at the beam loading limit
    - Average deceleration force (~30GeV/m) exceeds accelerating gradient (~10 GeV/m)
- Pinching of plasma electrons by a positron bunch and their repulsion by an electron bunch destroys transverse focusing
  - An increase of transverse bunch size suggested as a remedy does not look as a possibility for the required small emittances due to emittance growth supported by multiple scattering
- Large number of sections will require incredible control for the optics match between sections

Issues with the SLAC Proposal

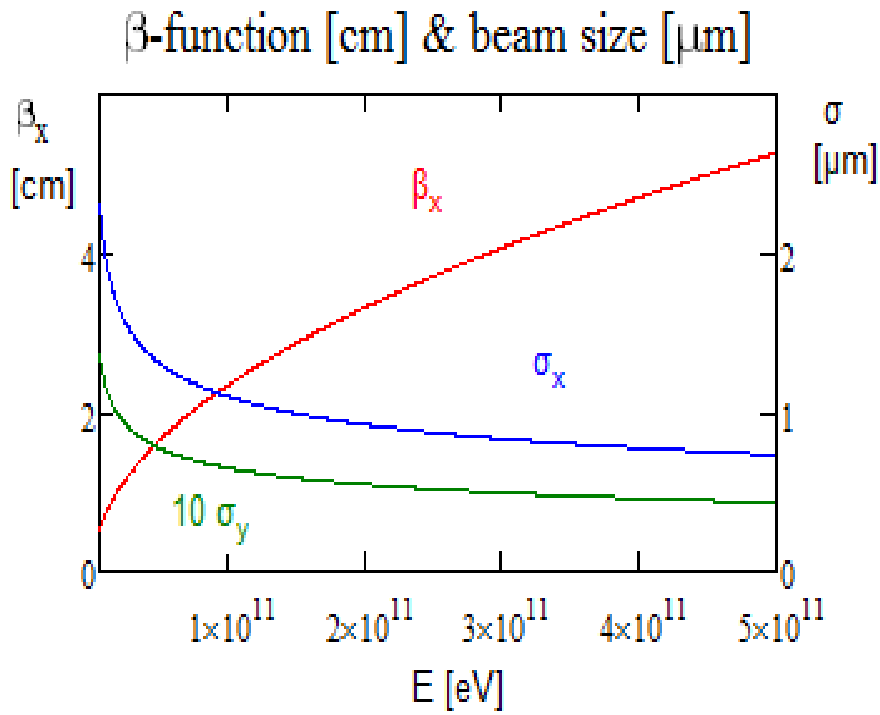
- Overall looks as a carefully thought-through proposal
- However:
  - There is no clear understanding of how positrons can be accelerated in the blowout regime
    - There are already a lot of publications and it is rather improbable that more work will open the way for acceleration of positron beam with required brightness
    - Presence of electrons on the axis required for transverse focusing will amplify the beam deceleration and pinching of electrons
  - Pinching of plasma protons completely destroys transverse focusing for electrons
    - Addressing ion collapse requires an increase of emittances or decrease of particle number in the bunch with consecutive decrease of luminosity

      $\Rightarrow$ It creates a possibility for $\gamma-\gamma$ collider with compromised luminosity

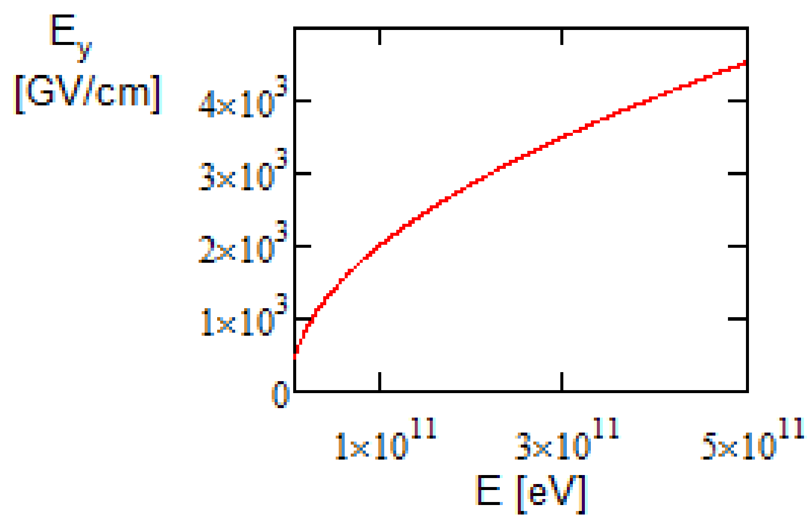
Issues with the SLAC Proposal (continue)



Phase advance of plasma ions oscillations in the field of electron bunch



- Impact ionization by bunch field is not negligible problem
  - Compare to el. field at a_0 in the hydrogen atom is ~6 GV/cm
- It prohibits a usage of heavy atoms in plasma



Conclusions

- Fair discussion with SLAC and LBNL proponents of e+e- collider based on plasma acceleration is required
- Beam interaction with plasma puts severe limitations on efficiency of the energy transfer from plasma to beam and the emittance conservation
  - As far as we can presently judge an acceleration of "a collider quality" beam required to achieve luminosity comparable to 10^{34} $cm^{-2}s^{-1}$ is not supported by the present proposals
- No doubts Plasma acceleration presents
  - interesting scientific subject
  - can find good application in a number of fields
  - But it hardly can be a valuable tool for future $e^{+}e^{-}$ colliders of TeV energy scale

References

[1] C. B. Schroeder, E. Esarey, and W. P. Leemans, Phys. Rev. ST Accel. Beams 15, 051301 (2012). See also a comment to this paper by V. Lebedev and S. Nagaitsev

[2] C. B. Schroeder et al., Phys. Rev. ST Accel. Beams 13, 101301 (2010).

[3] E. Esarey, C. B. Schroeder, and W. P. Leemans, Review of Modern Physics 81, 1229 (2009).

[4] Valeri Lebedev & Sergei Nagaitsev "Luminosity Limitations for Colliders Based on Plasma Acceleration" http://beamdocs.fnal.gov/AD-public/DocDB/ShowDocument?docid=4354

Backup slides

Objective

- An active discussion on the R&D for plasma accelerators in application to e+e- colliders in the TeV energy range
  - Two competing proposals:
    - beam-based from SLAC/UCLA
    - and laser-based from LBNL
- Figure of merit
  - Luminosity $$L = \frac{fN^2}{4\pi\sigma_x\sigma_y} = \frac{P_{beam}}{4\pi E_b}\frac{N}{\sigma_x\sigma_y}$$
  - $N/\sigma_x\sigma_y$ is limited by disruption and beamstrahlung
  - Energy efficiency (P_{beam}/P_{total}) is the primary issue
  - Beam emittances and momentum spread need to be small enough to allow focusing in the desired spot
- Our main goal is to discuss the collider applications of plasma-based technology.
  - The plasma acceleration is an interesting research topic. This technology may have other applications

Introduction

- A design for a conventional e+e- collider of up to 3 TeV exists! A plasma-based concept is being offered to the HEP community as a cost-saving proposal (or as an after-burner).
  - The proponents argue that plasma-based colliders could cost less because they could be made shorter and with fewer components;
  - We know from existing cost estimates that the overall length is just one of many factors going into the facility cost. Others include complexity, total power, risk of new untested technologies, R&D costs, etc.
- Here we consider only the first principles limitations on the plasma colliders luminosity
  - Implying that technology limitations can be overcome in future
- To compete with ILC or CLIC designs, a plasma-based concept needs to achieve a luminosity of ~2×10^{34} at ~1 TeV c.m.
- If acceleration of collider quality positron bunch is unfeasible then the γ-γ collider (NOT e-e-) can be interesting alternative.

Single Particle Deceleration and Wake in Plasma

- Well-known three-step solution:
  1. **Collective plasma response at large impact parameters**
     - Solution of Maxwell equations with $\varepsilon = 1 - \omega_p{}^2/\omega^2 + i\delta\ ,\quad \delta \to 0$
       - o diverges at small ρ where perturbation theory does not work
  2. **Particle interaction with independent plasma electrons**
     - Energy transfer to a single electron => Deceleration rate

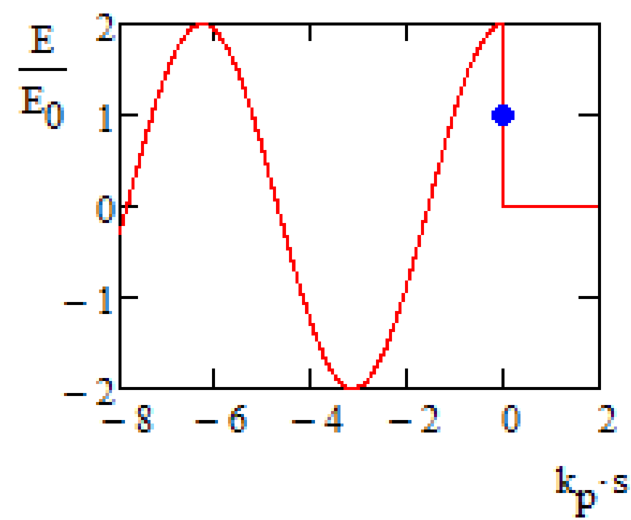
       ⇒ diverges at large ρ where screening of particle field by plasma needs to be taken into account
  3. **Combining two approaches for m >> m_e one obtains:**

$$\left(\frac{d\text{E}}{ds}\right)_0 \approx \frac{4\pi n_e Z^2 e^4}{m_e \text{v}^2} \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right), \qquad \rho_{\max} \approx 1.123\frac{\text{v}}{\omega_p}, \quad \rho_{\min} = \frac{Ze^2}{m_e \text{v}^2}.$$

Logarithmic approximation : it works well as long as $\rho_{\max} \gg \rho_{\min}$

- Longitudinal wake is

$$\frac{d\text{E}}{ds} = \left(\frac{d\text{E}}{ds}\right)_0 2\cos\left(k_p s\right)$$

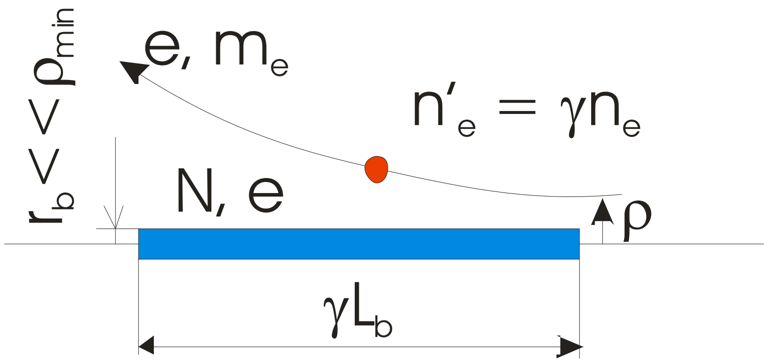


- At ρ>ρ_{min} (plasma pert. theory works)

$$\mathbf{B} = 0, \quad \mathbf{E} = -\nabla\phi, \quad \delta n \equiv n_i - n_e = 0$$

Deceleration of Ultra-relativistic Bunch

- For sufficiently small number of particles the bunch deceleration is not much different from single particle
  - Field screening and ρ_{max} will stay the same if the bunch is short, $L_b \ll k_p^{-1}$
  - ρ_{min} will be modified because of finite bunch length
  - Deceleration will be growing from bunch head to its tail
- For small impact parameters we can neglect collective electric field of plasma on plasma electron motion



  - Then in the beam frame the plasma electric field is:

$$eE' = -e\frac{d\varphi'}{ds'} = 4\pi n'_e e^2 \frac{d}{ds'}\left(\int_0^{\rho_{max}} \rho \ln\left(\frac{r(s',\rho)}{\rho}\right) d\rho\right)$$

where $r(s',\rho)$ is particle trajectory for impact parameter ρ

    - We assume zero ⊥ beam size (actually any size < ρ_{min} is small enough)
  - In logarithmic approximation an integration yields:

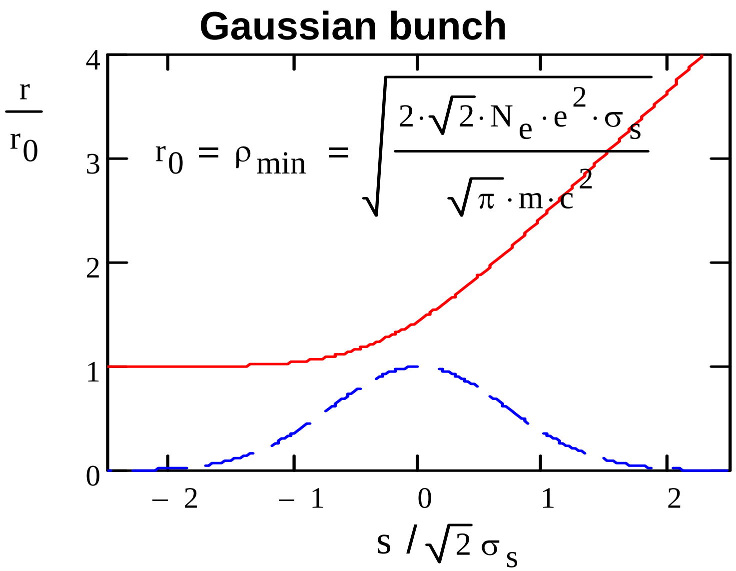
$$\frac{d\mathrm{E}}{ds} \approx \frac{4\pi n_e N_e e^4}{mc^2} \mathrm{L}_c F_g(s), \quad F_g(s) = 2\int_s^{\infty} \cos\left(k_p(s-s')\right) f(s') ds', \quad \int_{-\infty}^{\infty} f(s) ds = 1$$

Coulomb Logarithm and Minimum Impact Parameter

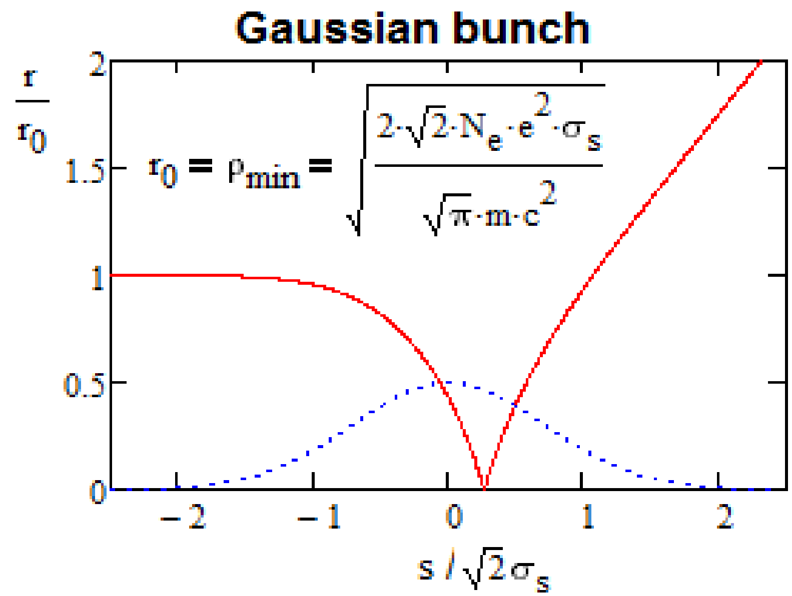
$$L_c = \begin{cases} \ln\left(1+\dfrac{\sqrt{2}c}{\omega_p \rho_{\min}}\right), & \text{electrons}, \\ \ln\left(1+\dfrac{2.15\sqrt{2}c}{\omega_p \rho_{\min}}\right), & \text{positrons}, \end{cases} \qquad \rho_{\min} = \sqrt{\frac{2\sqrt{2}N_e e^2 \sigma_s}{\sqrt{\pi} m_e c^2}}$$

1 is added to logarithm makes equation working for $\rho_{min} \sim \rho_{max}$. Good coincidence with numerical simulations[*]!

Electrons



Positrons



Plasma electron scattering on the bunch with impact parameter ρ_{min}

- Trajectory shape ρ/ρ_{min} depends only on ρ_0/ρ_{min}
  - Definition of ρ_{min}: $\delta\rho \equiv (\rho - \rho_0) \approx \rho_0$ for $\rho_0 = \rho_{min}$
- The contribution for impact parameters $\rho < \rho_{min}$ is small

[*] W. Lu, et.al.,"Limits of linear plasma wakefield theory for electron or positron beams" Physics of plasmas 12, 063101 (2005)

Deceleration of Electron Bunch

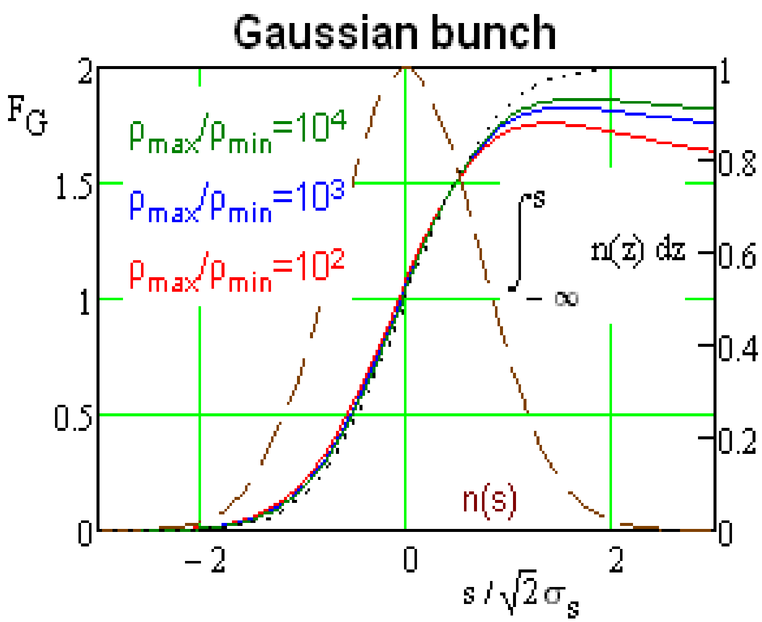
- The longitudinal wake is close to a step function:

$$W(s) \propto \theta(s)\cos(k_p s)$$

  - strictly speaking it is dependent on the longitudinal density distribution in the bunch but it makes only logarithmic correction

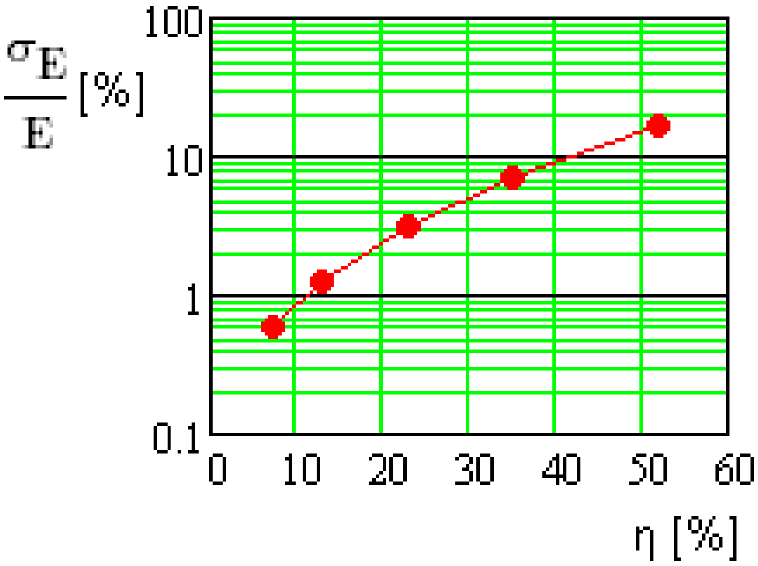
- For Gaussian bunch(σ_s<< $1/k_p$)

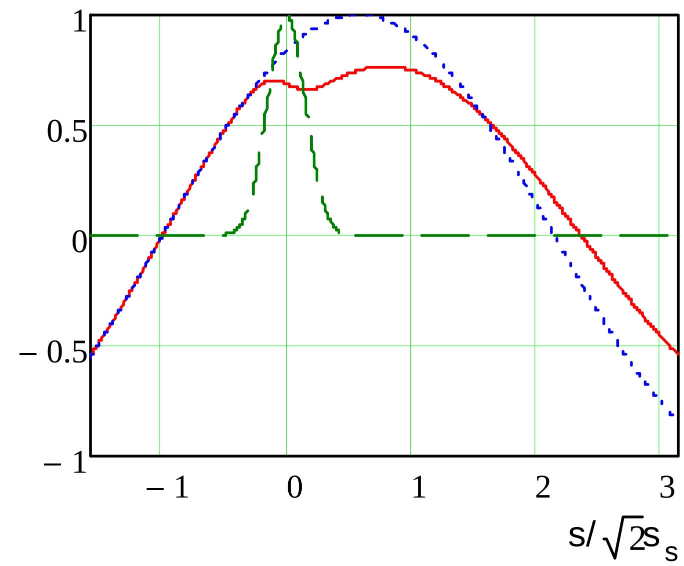
$$\frac{d\mathrm{E}}{ds} \approx \frac{4\pi n_e N_e e^4}{mc^2} \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) F_G\left(\frac{\rho_{\max}}{\rho_{\min}}, \frac{s}{\sqrt{2}\sigma_s}\right), \quad F_G(X,s) \approx \frac{2-\mathrm{erfc}(s)}{2\ln X} \ln\left(\frac{2X^2}{\sqrt{\pi}s\left(2-\mathrm{erfc}(s)\right)+\exp\left(-s^2\right)}\right)$$

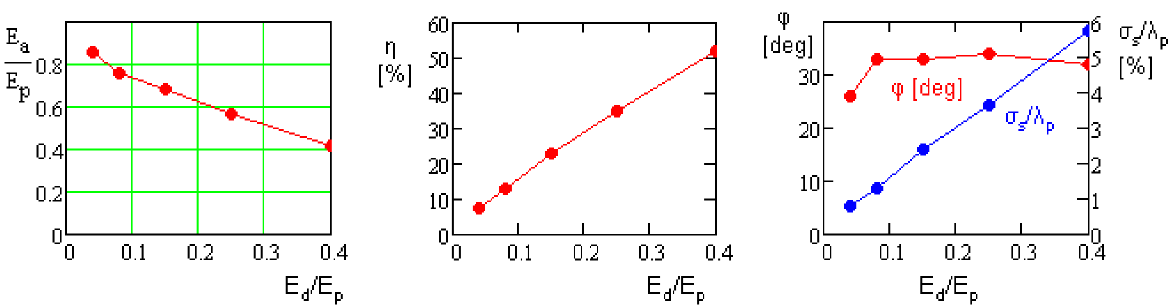


$$\mathrm{erfc}(s) = \frac{2}{\sqrt{\pi}} \int_s^\infty e^{-x^2} dx$$

Energy Spread and Acceleration Efficiency of Gaussian Bunch







Longitudinal electric field with and without bunch field; $\Delta E/E_p$=0.15, $\phi=33^0$,, σ_s/lp=0.024 => E_{acc}/E_p=0.68.

E_p - amplitude of plasma accelerating electric field

E_d - decelerating electric field in the bunch center

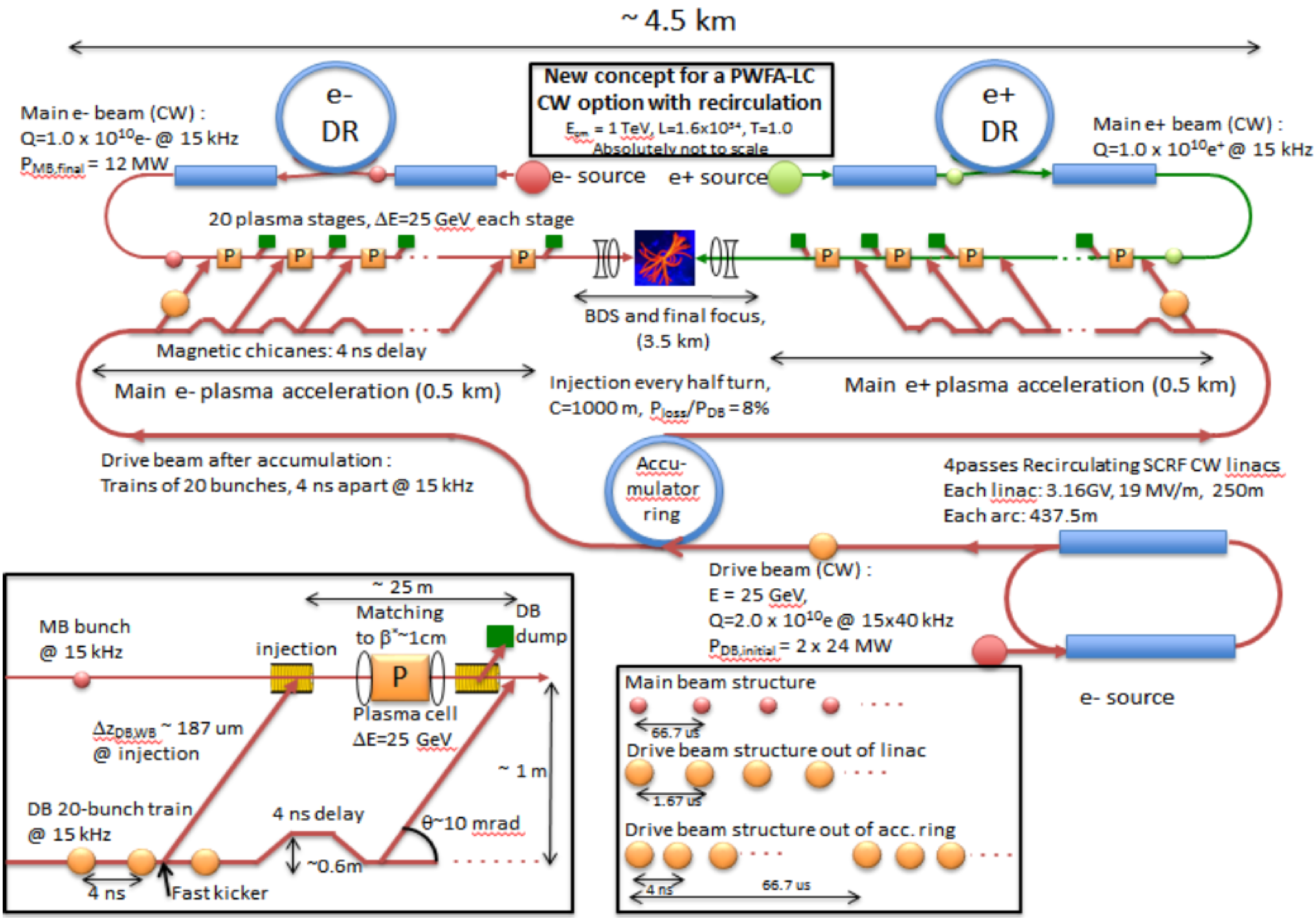
η - percentage of energy transferred from plasma to beam

σ_E/E – rms energy spread in accelerated beam

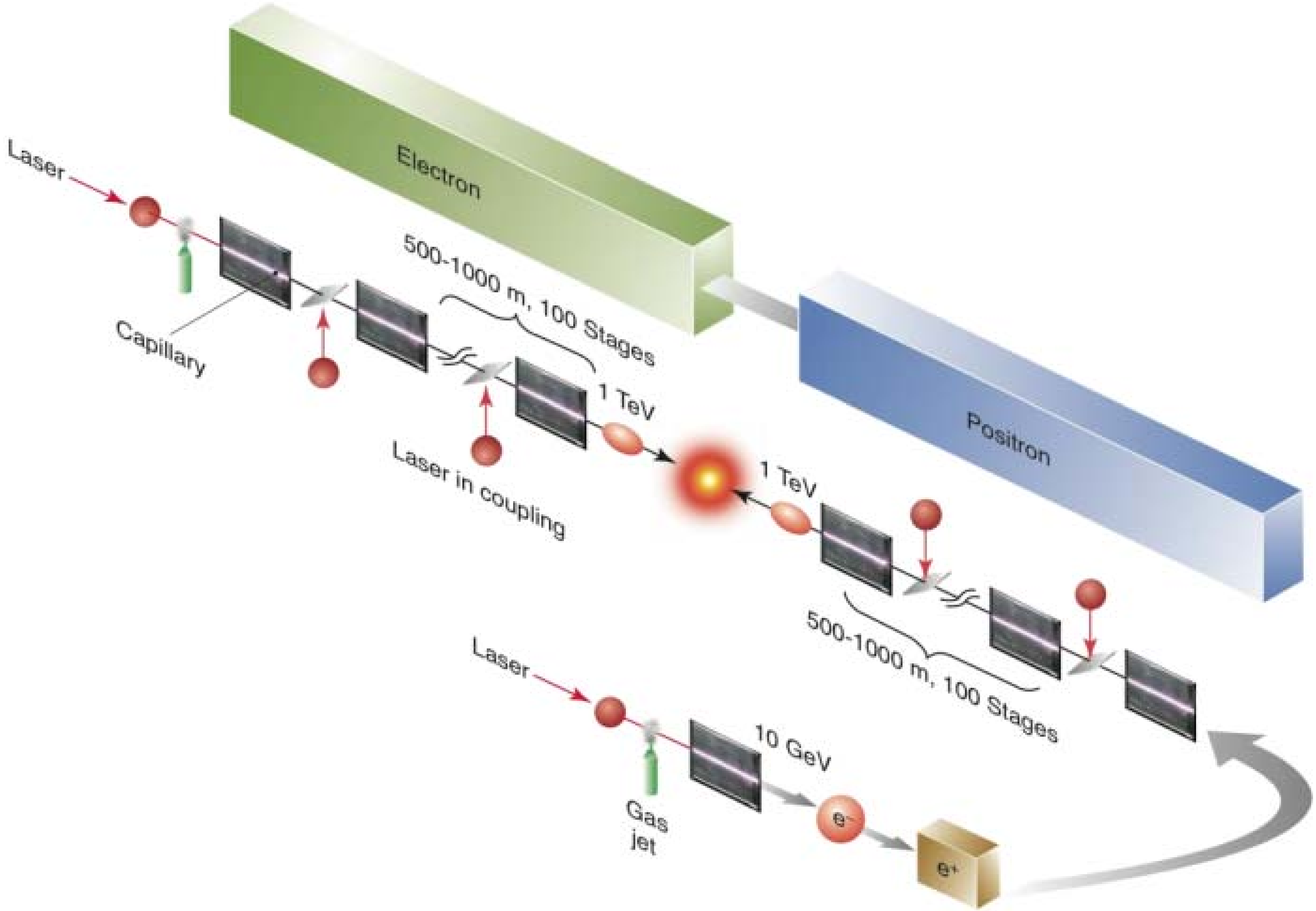
E_a - average accelerating field

- Small energy spread is required to transfer the beam from one accelerating section to another and to focus the beam in IP
  - For 1% rms energy spread only ~9% of plasma energy can be transferred to the beam
    - ±2.5% total spread is a huge number

SLAC Proposal for e^+e^- Collider (Ref. [2])



LBNL Proposal



Picture is from [3]: Wim Leemans for the ICFA-ICUIL Joint Task Force, "White Paper of the ICFA-ICUIL Joint Task Force – High Power Laser Technology for Accelerators", ICFA BD (2011)