On Beamstrahlung Limit on the Ultimate Energy of e+e- Colliders

Vladimir Shiltsev

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The beam-beam interaction at IP of a e+e- collider produces radiation (beamstrahlung) that generates energy spread in the oncoming beams, background for the detectors and results in the loss of the HEP measurements precision. The effects scale up with energy and at some point become so severe that all traditional advantages of e+e- colliders (collisions of point-like particles) disappear and instead, the effective collisions occur over a wide COM energy spectrum and with all kind of particles (e+, e-, photons, pairs, etc) becoming almost as "poorly defined" as collisions of compound particles, like protons or ions.

The physics of the beamstrahlung is well understood and a comprehensive theory is developed long ago [1, 2]. The *L* spectrum can be parameterized in terms of parameter Υ, a measure of the field strengths (mean EM field strength in the beam rest frame normalized to the Schwinger critical field), n_γ - the average number of photons radiated per particle in the act of collision, and the energy loss due to the beamstrahlun δ_E (sometimes also marked as δ_B) [2]:

$$\Upsilon \approx \frac{5}{6}\frac{r_e^2\gamma N}{\alpha\sigma_z(\sigma_x+\sigma_y)} \quad n_\gamma \approx \frac{2r_e\alpha N}{\sigma_x+\sigma_y}\frac{1}{\sqrt{1+\Upsilon^{2/3}}}$$

$$\delta_B \approx \frac{5}{4}\frac{\alpha\sigma_z\Upsilon^2}{\lambda_c\gamma}\frac{1}{(1+(1.5\Upsilon)^{2/3})^2}$$

where r_e, α, and λ_c are the classical electron radius, the fine structure constant, and the Compton wavelength and γ and N are the beam energy and the number of particles per bunch. Now, in general, the width of the L spectrum is described by δ_E, and that spread has been consistently taken into account while optimizing the main parameters of any linear e+e- facility proposal [3, 4] as it poses one of the major limitations on the maximum achievable luminosity.

The luminosity can be directly expressed via maximum tolerable δ_E [3], e.g. in the so called "transition regime" of modest beamstrahlung fields (Υ~0.2-200), it does not depend on the bunch length σ_z :

$$L \sim \frac{6.45\delta_E}{4\pi\alpha r_e\gamma\sigma_y}\left(\frac{P}{mc^2}\right) = 1.5\times10^{34}\frac{P[\mathrm{MW}]\delta_E}{E_0[\mathrm{TeV}]\sigma_y[\mathrm{nm}]}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

while in the "quantum regime" with Υ>200 :

$$L \sim \frac{1.95}{4\pi\alpha^2\sigma_y}\sqrt{\frac{\delta_E^3}{r_e\sigma_z\gamma}}\left(\frac{P}{mc^2}\right) = 5\times10^{34}\frac{P[\mathrm{MW}]}{\sigma_y[\mathrm{nm}]}\sqrt{\frac{\delta_E^3}{E_0[\mathrm{TeV}]\sigma_z[\mu\mathrm{m}]}}.$$

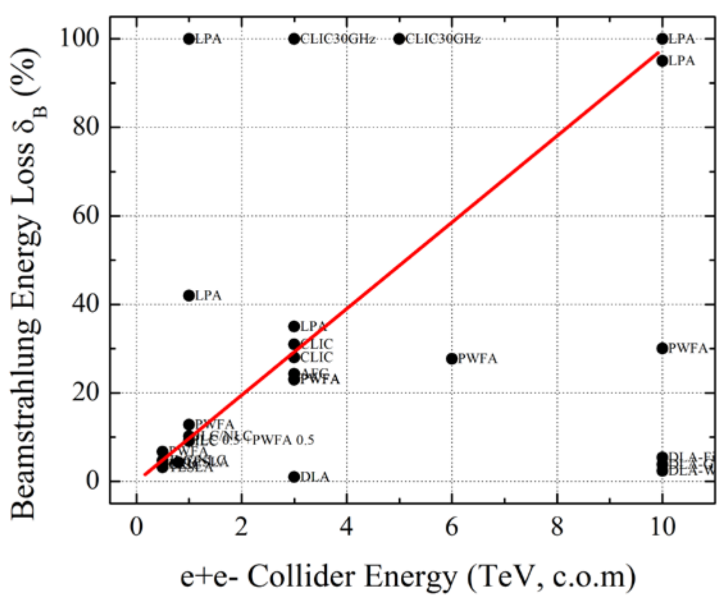
Telnov [3] concludes that from very basic principles it is impossible to go beyond 3-5 TeV com energies in any *e+e-* colliders of any (then) foreseeable type because of simple arguments : "…the number of accelerated particles is limited by total AC power which is proportional to the beam power P. Due to the dependence of cross sections on the energy as $\sigma \propto 1/E^2$ the luminosity should increase as E^2, as a result the required transverse beam sizes at TeV energies should be very small. Beams with small sizes have very strong fields that lead to large radiation losses during beam collisions (beamstrahlung). This effect does not allow us to use beams with simultaneously small horizontal and vertical beam sizes (σ_x, σ_y) (only very flat beams) and to get the required luminosity the beam power should be additionally increased. This leads to the "energy crisis" at the beam energy of about $2E_0 \sim 5$TeV…In the $\gamma\gamma$ mode of operation… only somewhat higher energies are possible due to conversion of high energy photons to $e+e-$ pairs in the field of the opposing beam (coherent pair creation)."

Table below summarizes the beamstrahlung situation for all the e+e- collider concepts considered by now and, in essence, shows general concurrence with Telnov's analysis. The first several lines are given for the concepts which were thoroughly developed and heavily reviewed, and corresponding studies concluded in comprehensive report at the level of CDR. The rest of the Table is for the very novel, sometimes speculative approaches (e.g., beam- or laser plasma wakefield acceleration, DLA, etc) and the cited data represent proponent's understanding and hopes.

	Ref, Year	**Energy COM TeV**	L, e34	L 1% , e34	**dE/E_B, %**	n_gamma	Y
CLIC30GHz	[5], 1997	**3**	9	0.9	**100**	1.6	6
		5	10	0.9	**100**	1.7	13
JLC/NLC	[6], 2001	**0.5**	2	1.1	**4.7**	1.3	0.11
		1	3.4	1.5	**10.2**	1.3	0.29
CLIC	[3], 2003	**0.5**	1.4		**3.8**	0.7	0.3
		3	10.3		**31**	2.3	8.1
CLIC	[7], 2012	**3**	5.9	2	**28**	2.1	1.8
TESLA	[8, 3], 2003	**0.5**	3.4	2.2	**3.2**	2	0.06
		0.8	5.8	3.8	**4.3**	1.5	0.09
ILC	[9], 2013	**0.5**	1.5	0.9	**3.9**	1.7	0.03
		1	4.9	2.2	**9.1**	2	0.09
DLA-Woodpile	[10], 2012	**10**	204		**2.4**		
DLA-Fiber	[10], 2012	**10**	409		**5.4**		
DLA-Grating	[10], 2012	**10**	282		**3.8**		
DLA	[11], 2013	**3**	3.2	2.8	**1**	0.1	0.001
DLA	[12], 2014	**3**	3.2		**1**		
AFC	[13], 2013	**3**			**24.3**		

PWFA	[11], 2013	**3**	6.3	3.8	**23**	1.1	3.5
PWFA	[14], 2013	**6**	8.8	3.1	**27.7**	1.1	9.9
ILC 0.5 +PWFA 0.5	[15], 2014	**1**	2.6	1.3	**9.3**	0.7	0.52
PWFA	[15], 2014	**0.5**	2.1	1.1	**6.7**	0.73	0.24
		1	3.1	1.6	**12.8**	0.88	0.68
		3	6.3	2.5	**23.1**	1.05	3.5
		10	10.5	3.1	**30**	1.14	21.4
LPA	[10], 2012	**1**	2		**42**	1.4	180
		1	2		**100**	10	180
		10	200		**95**	3.2	18000
		10	200		**100**	22	18000
LPA	[11], 2013	**3**	7.6	2.6	**35**	1.2	371

One can see that for any acceleration technology, the beamstrahlung is serious issue which becomes severe beyond 3 TeV com energy (when δ_E exceeds 20-40%). Figure further illustrates the point.



The only technique that offers low δ_E at very high energies is DLA (Direct Laser Acceleration) which offers potential reduction of the bunch change by a factor of ~10^5 compared to other schemes due to employment of very high rep-rate laser sources of tens to hundreds of MHz. Without questioning feasibility of such sources even in the future – see discussion in Ref. [10] – one can only note that the method can promise only (comparatively) very low accelerating gradients in the range of few 100's to 1000 MV/m, therefore the collider facilities are 10 to 50 km long and, usually, require very high wall plug electric power of several 100's of MW.

As concluded by many, the beamstrahlung is not a limiting factor for a muon collider [16], and novel fast acceleration of muons can offer a way to economical ultra-high energy muon colliders with the energy reach of 100 TeV – 1 PeV [17].

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