POWER CONTROL IN PRE-EXCITED MAGNETRONS

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Principle of operation (simplified)

Magnetron is a self-exciting coherent oscillator, converting DC into RF and generating at the cyclotron frequency, \( \omega_c = \frac{eH}{m_e c} \).

\[
F_L = eE + \frac{e}{c} \left[ \mathbf{v} \times \mathbf{H} \right]
\]

In the diode: \( v_{dr} = \frac{E \cdot c}{H} \).

For cylindrical geometry: \( E \approx \frac{U_a}{r_a - r_c} \).

At \( v_{\phi n} = v_{dr} \):

\[
U_H = \frac{\pi (r_a^2 - r_c^2)}{n \cdot c} f_n H.
\]

\( U_H \)-Hartree voltage

\[
\frac{V}{H^2} = \frac{e r_a^2}{8 mc^2} \left[ 1 - \left( \frac{r_c}{r_a} \right)^2 \right]^2
\]

When \( U_a \geq U_H \), the Cherenkov synchronism between \( v_{dr} \) and \( v_{\phi n} \) is fulfilled.
Trajectories and phase focusing in magnetrons

At $U_a > U_H$ the transported charge causes coherent Cherenkov generation greatly magnified by cavities.

▶ Magnetrons are most efficient RF sources in wide range of power and frequency
**Motion of charge in a magnetron at \( H > H_c \).**

At a plane geometry of a plane magnetron (planotron) the motion of electrons may be described as:

\[ \ddot{z} + i\Omega \dot{z} = f(z, z^*, t), \]

The equation has an exact solution if \( f(z, z^*, t) = Cz \):

\[ z_0 = \alpha \exp(-i\Omega_1 t) + \beta \exp(-i\Omega_2 t), \quad \text{where: } \Omega_2 = \frac{\omega_c = (eH)/(m_e c)}{\beta} \]

\( \beta \) is radius of the rotating electron, \( \Omega_1 \) is azimuthal velocity of center of electron orbit, \( \alpha \) is radius of rotation of the center of electron orbit. Motion of center of electron orbit represents motion of charge.

The static electric field, \( E \), between coaxial cathode and anode is determined by the applied voltage, \( U \), as:

\[ E(r) = \frac{U}{r} \ln \left( \frac{r_2}{r_1} \right). \]

Complex acceleration caused by this field is:

\[ f(z, z^*, t) = \frac{e}{m} \frac{U}{\ln(r_2/r_1) \cdot z^*} \]

The drift velocity depends on the radius:

\[ v(r) = \mathbf{v} \cdot (r/r) \]

In a synchronous wave (\( \exp(-i(n \varphi + \omega t)) \), \( \omega = n\Omega_1 \)), pre-exciting the magnetron:

\[ u(r) = \mathbf{u} \cdot \left( r/r \right) \]

Thus synchronism can be fulfilled only for charge at \( r = \bar{r} \).
Motion equations for charge in origin of synchronous wave

\[
\begin{aligned}
\dot{r} &= \omega \frac{r^2}{r} \varepsilon \phi_1(r) \cos(n\varphi') \\
n\dot{\varphi}' &= -\omega \frac{r^2}{r} \left( \frac{d\phi_0}{dr} + \varepsilon \frac{d\phi_1}{dr} \sin(n\varphi') \right)
\end{aligned}
\]

Equations for charge motion in origin of the rotating synchronous wave:

\[
\begin{aligned}
\varphi' &= \varphi + \omega \cdot t/n, \\
\varepsilon &= \frac{\tilde{E}_c}{E_c} = \frac{\tilde{E}_c \cdot r_c}{\ln(r_a/r_c)} / U,
\end{aligned}
\]

Potential \( \Phi_1 \) represents RF field, potential \( \Phi_0 \) represents static electric field in the rotating origin.

- Motion of charge towards anode is possible in the interval \((-\pi/2 < n\varphi' < \pi/2)\)
- At small \( \varepsilon \) in points of “rest”, where \( d(n\varphi')/dt=0 \), the phase focusing of charge into “spokes” is impossible. These points don’t allow motion of charge towards anode.
- An increase of \( \varepsilon \) eliminates the points of “rest”, this allows phase focusing and drift of charge towards anode, i.e. the magnetron current.

Function \( n\dot{\varphi}'/(\omega) \) vs. the radius of the charge trajectory, \( r \), at \( r_c =5 \text{ mm}, r_a/r_c = 1.5, \sqrt{r/r_c} =1.2, n\varphi'=\pi/2 \)
Experimental tests: lowering of magnetron start up voltage

The magnetron assembly to study the power control.

The measurements in pulsed mode were performed at 2.7 MHz shift of the frequency locking the magnetron.

Pulsed cathode voltage at minimum generated power, traces 1, 3, 5, and at power of 450 ±8 W, traces 2, 4, 6 at the following powers of the frequency-locking signal, $P_{\text{Lock}}$: 12 W, traces 1 and 2, 27.4 W, traces 3 and 4, 53.9 W, traces 5 and 6, respectively. Minimum amplitude of the free running magnetron is $3,737\pm9\ V$. 

Experimental setup to measure the magnetron current, voltage, spectrum, output power, power of the pre-exciting (frequency-locking) signal and phase noise.
Lowering of magnetron start up voltage and current

Lowering of magnetron start up voltage in percent vs. the power of signal, $P_{\text{Lock}}$, pre-exciting the magnetron.

Measured allowable range of the power control of the magnetron vs. the pre-exciting power.

Traces of the measured pulsed cathode current at minimum generated power and at power of $450 \pm 8$ W at various power of the frequency-locking signal, $P_{\text{Lock}}$. 
Dots with error bars: dependence of the magnetron pulsed power on the magnetron current vs. power of pre-exciting signal. Solid lines: linear fits of the measured data.

**Magnetron power control by control of the current**

<table>
<thead>
<tr>
<th>Magnetron power, W</th>
<th>300</th>
<th>330</th>
<th>365</th>
<th>394</th>
<th>425</th>
<th>450</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{\text{Mag}}), W</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>(P_{\text{Lock}}), W</td>
<td>52.9</td>
<td>54</td>
<td>54.1</td>
<td>54.1</td>
<td>54.1</td>
<td>54.1</td>
</tr>
</tbody>
</table>

Phase noise, deg. (rms) | 1.4 | 1.3 | 1.4 | 1.0 | 1.3 | 1.0 |

Measured by the phase detector stochastic noise at various magnetron power. Calibration is 2.4 mV/degree.
**Experimental tests in CW mode**

The measurements in CW mode were performed at the frequency locking the magnetron approximately equal to average frequency at free run at magnetron power of ~800 W.

Power of the frequency-locked 1.2 kW CW magnetron vs. the magnetron current at the locking power of ≈50 W.