OTRI experiment

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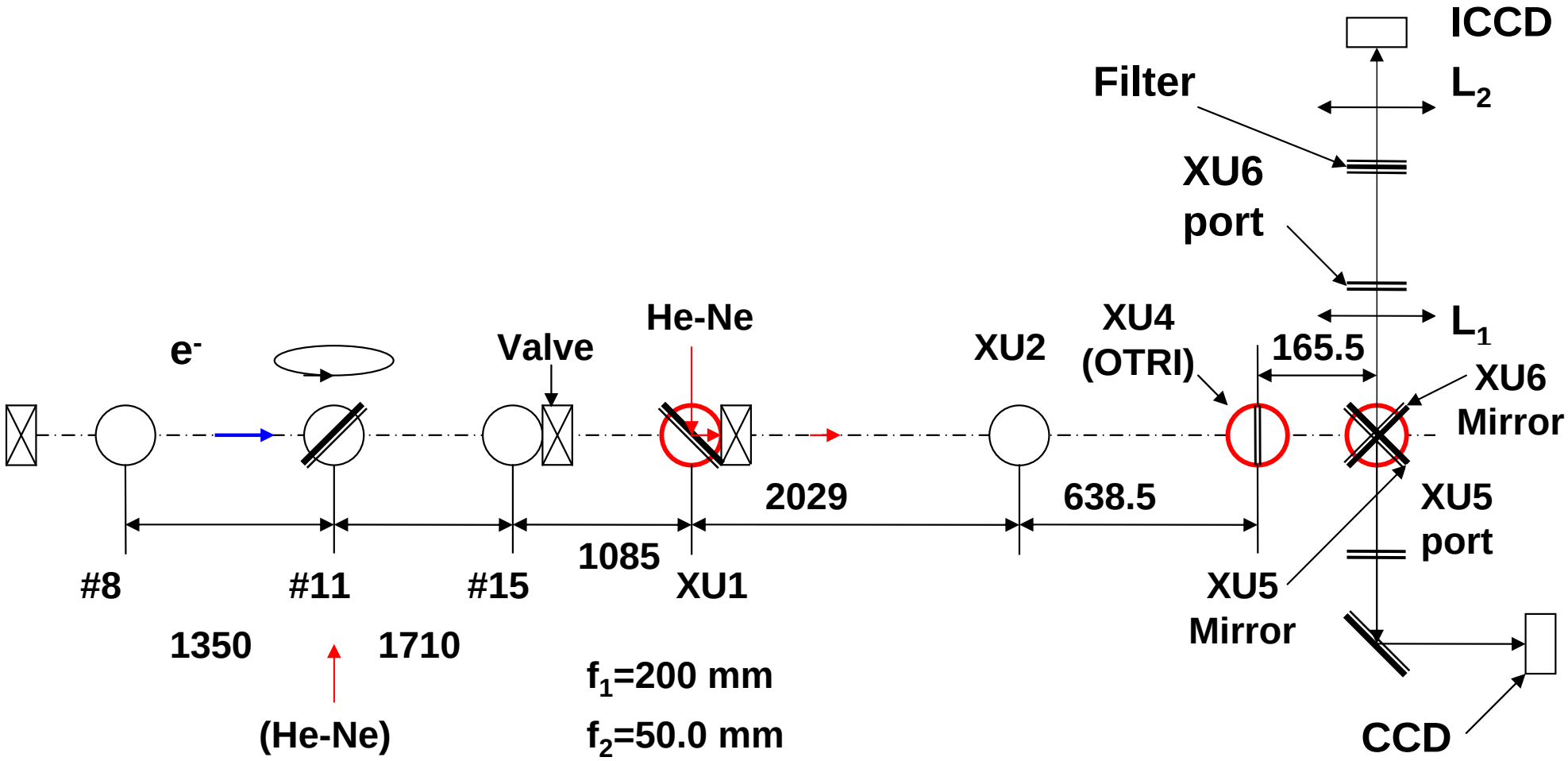
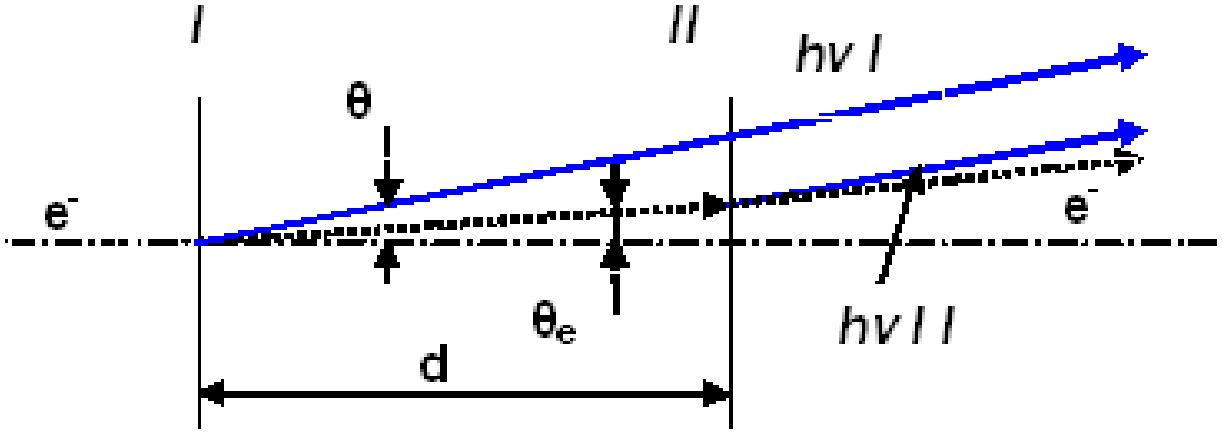


Experimental setup

The angle of inclination of the screens to the beam axis is of $\pi/2$



In this case the expression for the phase shift is:

$\Delta\chi = \chi_1 - \chi_2$, where:

$\chi_1 = kd/\cos\theta$,

$\chi_2 = kd/\beta\cos\theta_e$.

Using $k = 2\pi/\lambda$ the phase shift can be written:

$$\Delta\chi = 2\pi d[1/\cos\theta - 1/\beta\cos\theta_e]/\lambda$$

Here: θ_e is the scattering angle. The average scattering angle projection θ_p is expressed as:

$$\sqrt{\langle\theta^2\rangle} = \theta_P \cong \frac{13.6MeV}{\beta\cdot cp} z\cdot\sqrt{\frac{x}{X_0}}\cdot\left[1+0.038\ln\left(\frac{x}{X_0}\right)\right]$$

Here X_0 is the radiation length of the screen material, x is the OTR-screen thickness. The X_0 can be found as:

$$X_0 := \frac{716.4\cdot A}{Z\cdot(Z+1)\ln\left(\frac{287}{\sqrt{Z}}\right)}$$

Here: A and Z are atomic weight and Z-number of the screen material respectively.

For mica ($K_2O\cdot Al_2O_3\cdot SiO_2$): <A>=23.3; <Z>=11.5 $\rho = 2.7\ g/cm^3$, so for **10 μm OTR-screen** $x = 2.7\cdot10^{-3}$ g/cm^3; θ_p***=5.623 mrad***.

$E := 16 \quad MeV \qquad E1 := 1.02E \quad MeV$

$$I(\theta) := \beta^2 \cdot \frac{\sin(\theta)^2}{\left[1 - \beta^2 (\cos(\theta))^2\right]^2} \qquad I_1(\theta) := \beta^2 \cdot \frac{\sin(\theta)^2}{\left[1 - \beta 1^2 (\cos(\theta))^2\right]^2}$$

The OTR-screens are made from **10 μm mica**.

$$\lambda := 0.633 \cdot 10^{-3} \ mm \qquad d := 1 \ mm \qquad \Delta\chi_m(\theta) := 2\pi \cdot \frac{d}{\lambda} \cdot \left(\frac{1}{\cos(\theta)} - \frac{1}{\beta \cdot \cos(\theta_m)} \right)$$

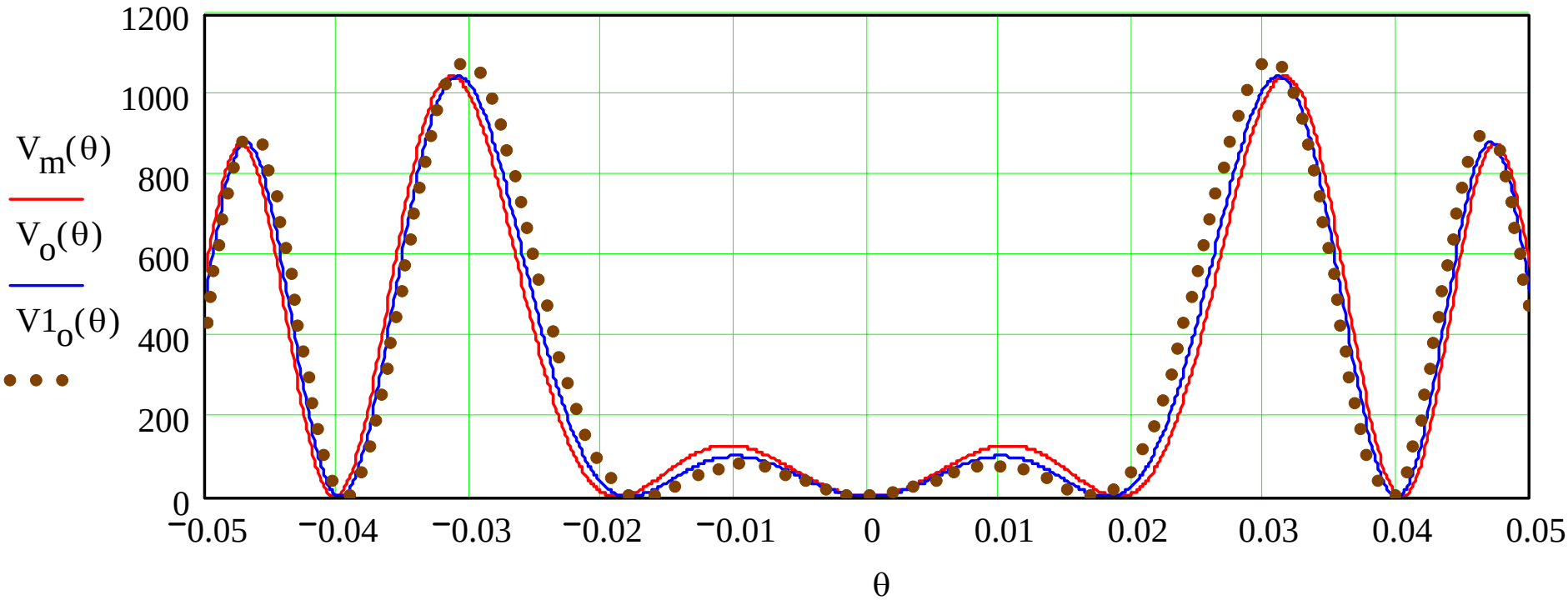
$$\Delta\chi_o(\theta) := 2\pi \cdot \frac{d}{\lambda} \cdot \left(\frac{1}{\cos(\theta)} - \frac{1}{\beta} \right) \qquad \Delta\chi 1_o(\theta) := 2\pi \cdot \frac{d}{\lambda} \cdot \left(\frac{1}{\cos(\theta)} - \frac{1}{\beta 1} \right)$$

The intensity of the interfering flat waves ***hv I*** and ***hv II*** having equal amplitudes can be expressed as: $U(\theta) = 2\Im(\theta)\left[1 + \cos\Delta\varphi(\theta)\right]$.

[1] $V_m(\theta) := 2I(\theta)\cdot\left(1 + \cos\left(\Delta\chi_m(\theta)\right)\right)$ $V1_o(\theta) := 2I_1(\theta)\cdot\left(1 + \cos\left(\Delta\chi1_o(\theta)\right)\right)$

$V_o(\theta) := 2I(\theta)\cdot\left(1 + \cos\left(\Delta\chi_o(\theta)\right)\right)$

Here: $V_o(\theta)$ is the interference caused by non scattered electron; $V_m(\theta)$ is the interference caused by the electron scattered on θ_m angle; $V1_o(\theta)$ is the interference caused by the electron having the energy of E1. In this estimation was neglected in reflectance from mica. One can see that difference in fringes caused by difference in the beam energy (2%) is more than difference caused by scattering.



Interference of the waves with different amplitudes b_1 and b_2 gives the wave with amplitude of B:

$$B^2 = b_1^2 + b_2^2 + 2b_1 b_2 \cos \Delta\chi$$

[2] Interference of the ***hv I*** photon reflected from ***II*** mica and first foil with ***hv II*** photon:

$$\Delta\chi_{o2}(\theta) := 2\pi \cdot \frac{d}{\lambda} \cdot \left(\frac{3}{\cos(\theta)} - \frac{1}{\beta} \right) \qquad n_m := 1.59 \qquad r_m := \frac{n_m - 1}{n_m + 1} \qquad r_m = 0.228$$

The intencity of the reflected from ***II*** and ***I*** mica foils photon is ~ r_m^2 relatively of intencity of the ***hv II*** (or ***hv I***) photon, and one can use the approximation:

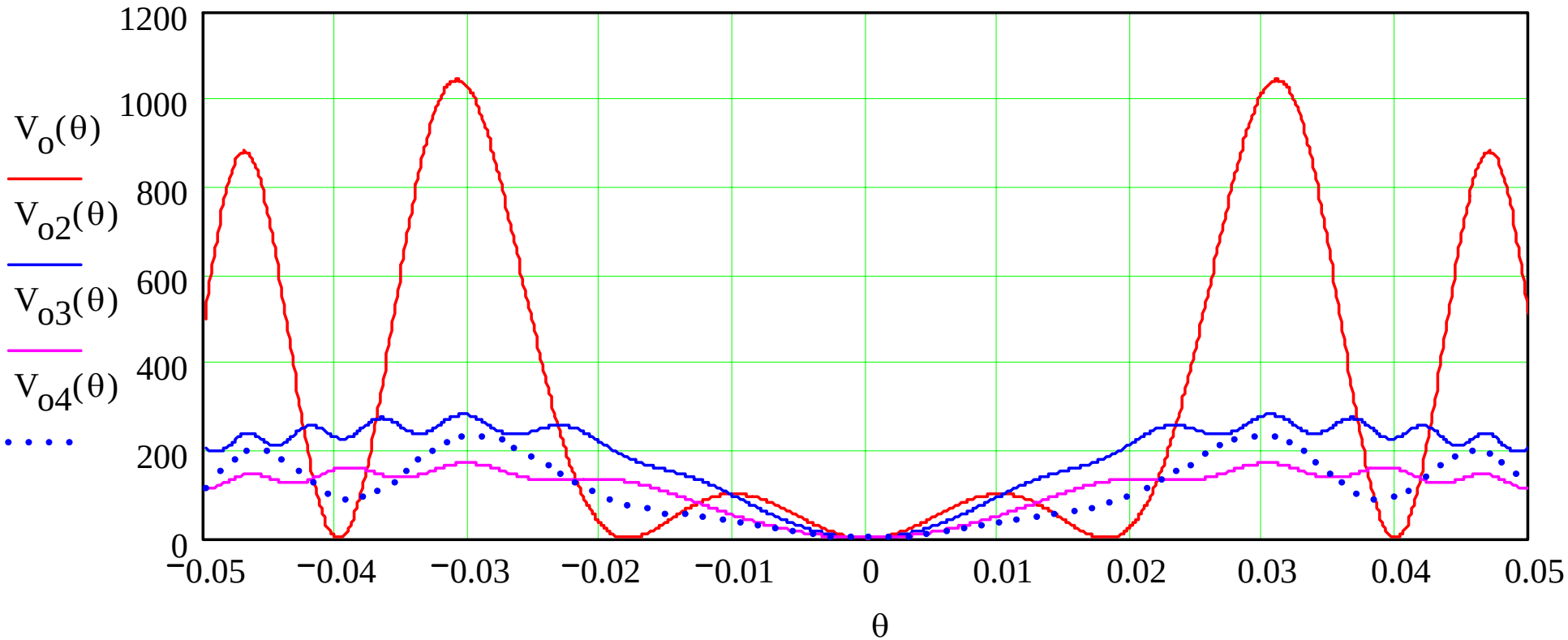
$$V_{o2}(\theta) := I(\theta) \cdot \left[1 + 2r_m^{\,2} \cdot (1 - r_m) \cdot \cos\left(\Delta\chi_{o2}(\theta)\right) \right]$$

[3] Interference of the of the ***hv I*** photon reflected from mica and first foil with itself; reflection of the interference wave is also taken into account.

$$\Delta\chi_{o3}(\theta) := 2\pi \cdot \frac{d}{\lambda} \cdot \left(\frac{2}{\cos(\theta)} \right) \qquad V_{o3}(\theta) := (1 - r_m)^2 I(\theta) \cdot \left(1 + 2r_m^{\,2} \cos\left(\Delta\chi_{o3}(\theta)\right) \right)$$

[4] Interference of the of the ***hv I*** photon with reflected by first foil ***hv II*** backward photon:

$$\Delta\chi_{o4}(\theta) := -2\pi\cdot\frac{d}{\lambda}\cdot\left(\frac{1}{\cos(\theta)} + \frac{1}{\beta}\right) \qquad V_{o4}(\theta) := \left(1 - r_m\right)^2 I(\theta)\cdot\left(1 + r_m{}^2 + 2r_m\cos\left(\Delta\chi_{o4}(\theta)\right)\right)$$



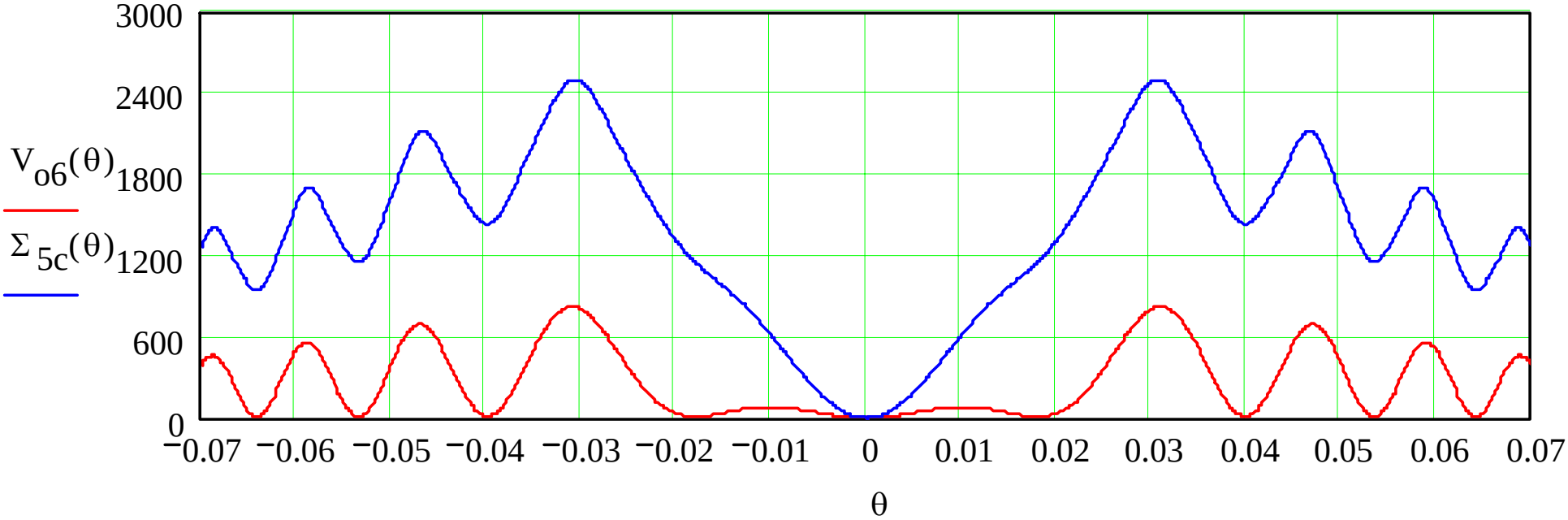
[5] Interference of the forward photons on both interfaces of the mica gives:

$$t := 10^{-3}\ \text{cm} \qquad \Delta\chi_{o5}(\theta) := 2\pi\cdot\frac{t}{\lambda}\cdot\left(\frac{n_m}{\cos(\theta)} - \frac{1}{\beta}\right) \qquad V_{o5}(\theta) := 2\,I(\theta)\cdot\left(1 + \cos\left(\Delta\chi_{o5}(\theta)\right)\right)$$

[6] Interference of the forward ***hv I*** photon and forward ***hv II*** photon considering the reflection on ***II*** mica sheet :

$$V_{o6}(\theta) := I(\theta)\cdot\left[1 + (1 - r_m)^2 + 2\cdot(1 - r_m)\cdot\cos\left(\Delta\chi_o(\theta)\right)\right]$$

$$\Sigma_{5c}(\theta) := V_{o6}(\theta) + V_{o2}(\theta) + V_{o3}(\theta) + V_{o4}(\theta) + V_{o5}(\theta)$$



Estimation of the OTR photon flux

$q_{bunch} := 1 \cdot 10^{-9}$ C

$n_{ph.bunch} := \dfrac{q_{bunch}}{1.6 \cdot 10^{-19} \cdot 137}$

$n_{ph.bunch} = 4.562 \times 10^{7}$

$\lambda := 633$ nm

$\Delta\lambda := 10$ nm

$n_{ph.band} := n_{ph.bunch} \cdot \dfrac{\Delta\lambda}{\lambda}$

$n_{ph.band} = 7.207 \times 10^{5}$

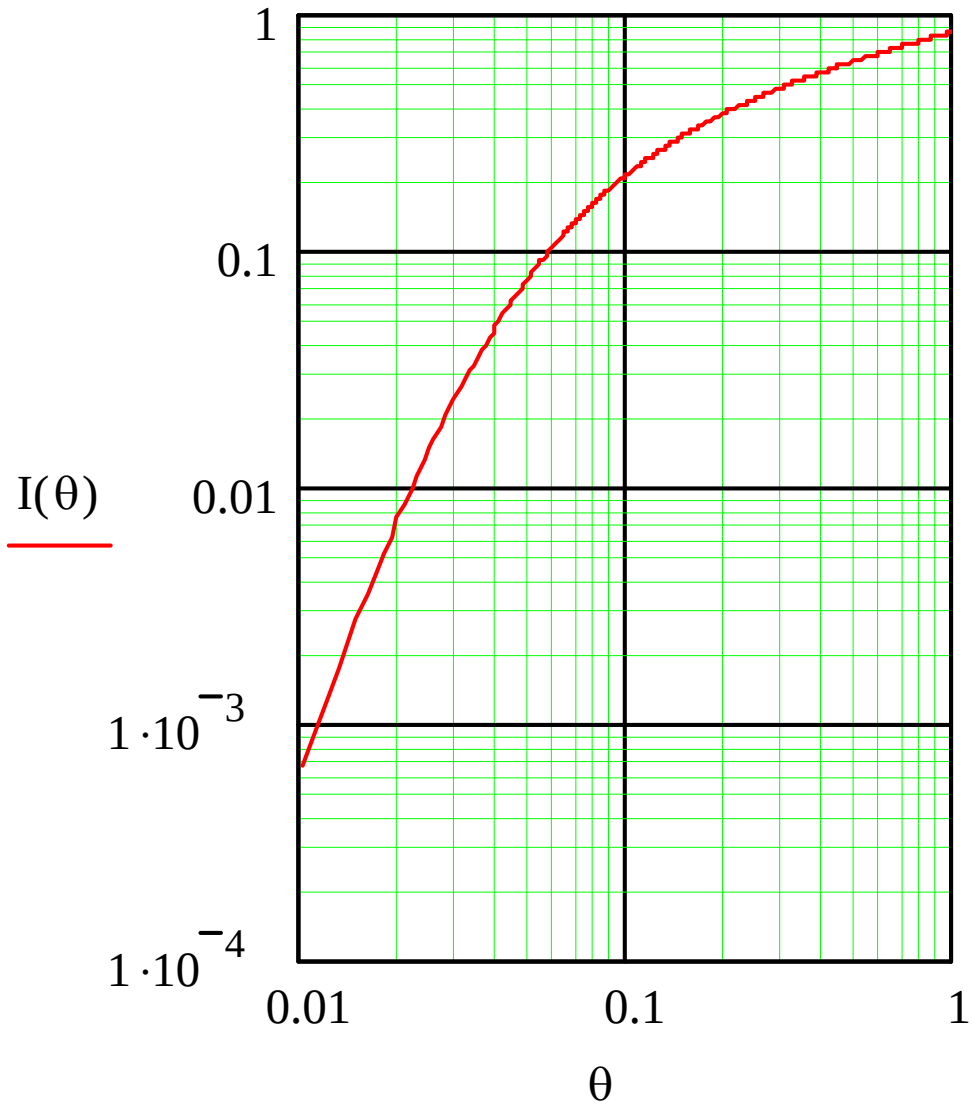
$$\Im(\omega,\theta) \approx \frac{e^2 \beta^2 \sin^2\theta}{\pi^2 c \cdot (1-\beta^2\cos^2\theta)^2}$$

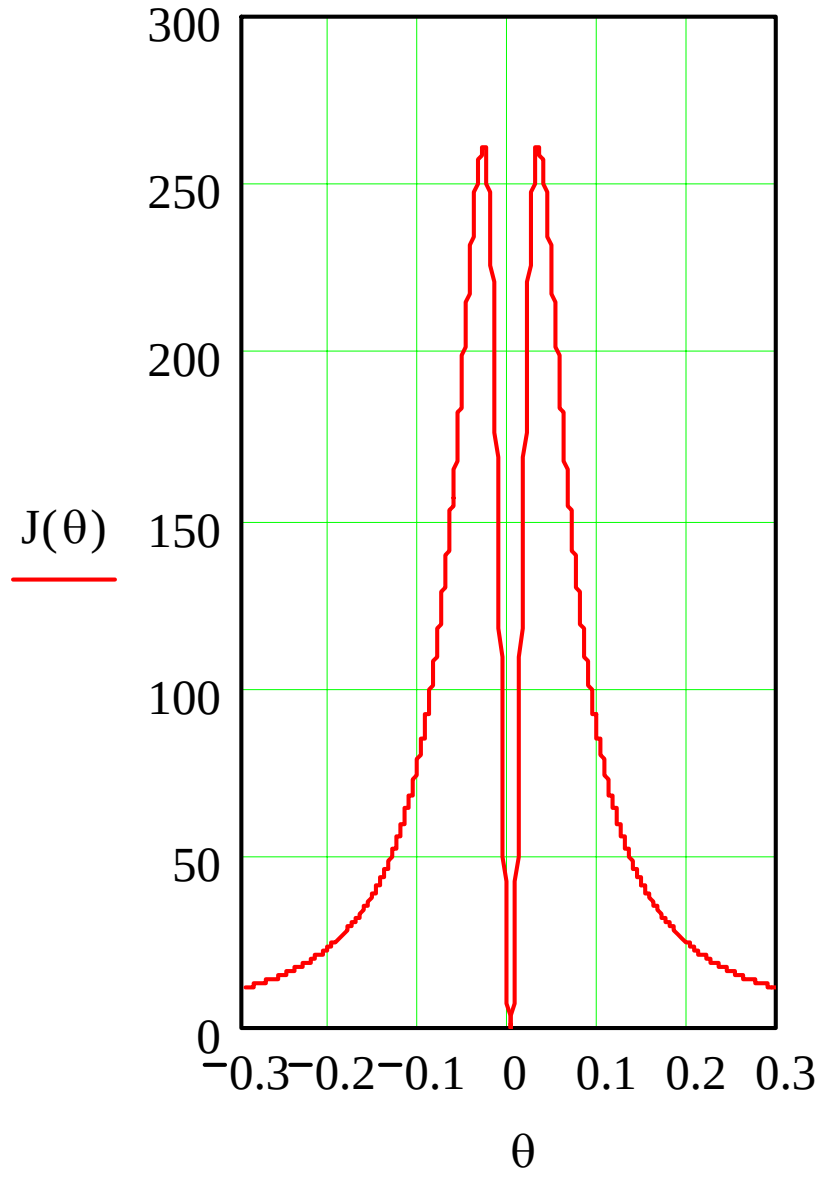
$\gamma = 32.311$

$$J(\theta) := \frac{\beta^2 \cdot (\sin(\theta))^2}{\left[1-(\beta \cdot \cos(\theta))^2\right]^2}$$

$$I(\theta) := \frac{2 \cdot \pi \cdot \int_0^{\theta} J(\theta) \cdot \sin(\theta)\, d\theta}{2 \cdot \pi \cdot \int_0^{\frac{\pi}{2}} J(\theta) \cdot \sin(\theta)\, d\theta}$$

In our case $\theta \sim 0.1$ ***rad***. For such angle the optical efficiency of the considered OTR system is ~ 0.21, so number of detected photons $n_{d.bunch} \sim 1.54 \cdot 10^5$. Considering reflecting properties of mica one can estimate output radiation in one order less, i.e.estimate real photons number per bunch as $n_{d.bunch} \sim 1.5 \cdot 10^4$. So, using picture field of 100x100 pixels and magnification of 10^3 expected ***number of photons per pixel should be*** $1.5 \cdot 10^3$.This value is more than the sensitivity threshold for ordinary CCD.


1
0.1
0.01
1·10⁻³
1·10⁻⁴
I(θ)
0.01
0.1
1
θ


300
250
200
150
100
50
0
J(θ)
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
θ

Cherenkov radiation from mica (estimations) for visible light

Tamm formula $$\frac{d^2 \cdot N}{d\Omega \cdot d\lambda} = \frac{-\alpha \cdot n_m \cdot l^2}{\lambda^3} \cdot \sin(\theta)^2 \cdot \left(\frac{\sin(k\pi)}{k\pi}\right)^2$$

Here:

$$d\Omega = 2\pi \sin(\theta) \cdot d\theta$$

$$k = \frac{l}{\beta\lambda}\left(1 - \beta n \cdot \cos(\theta)\right)$$

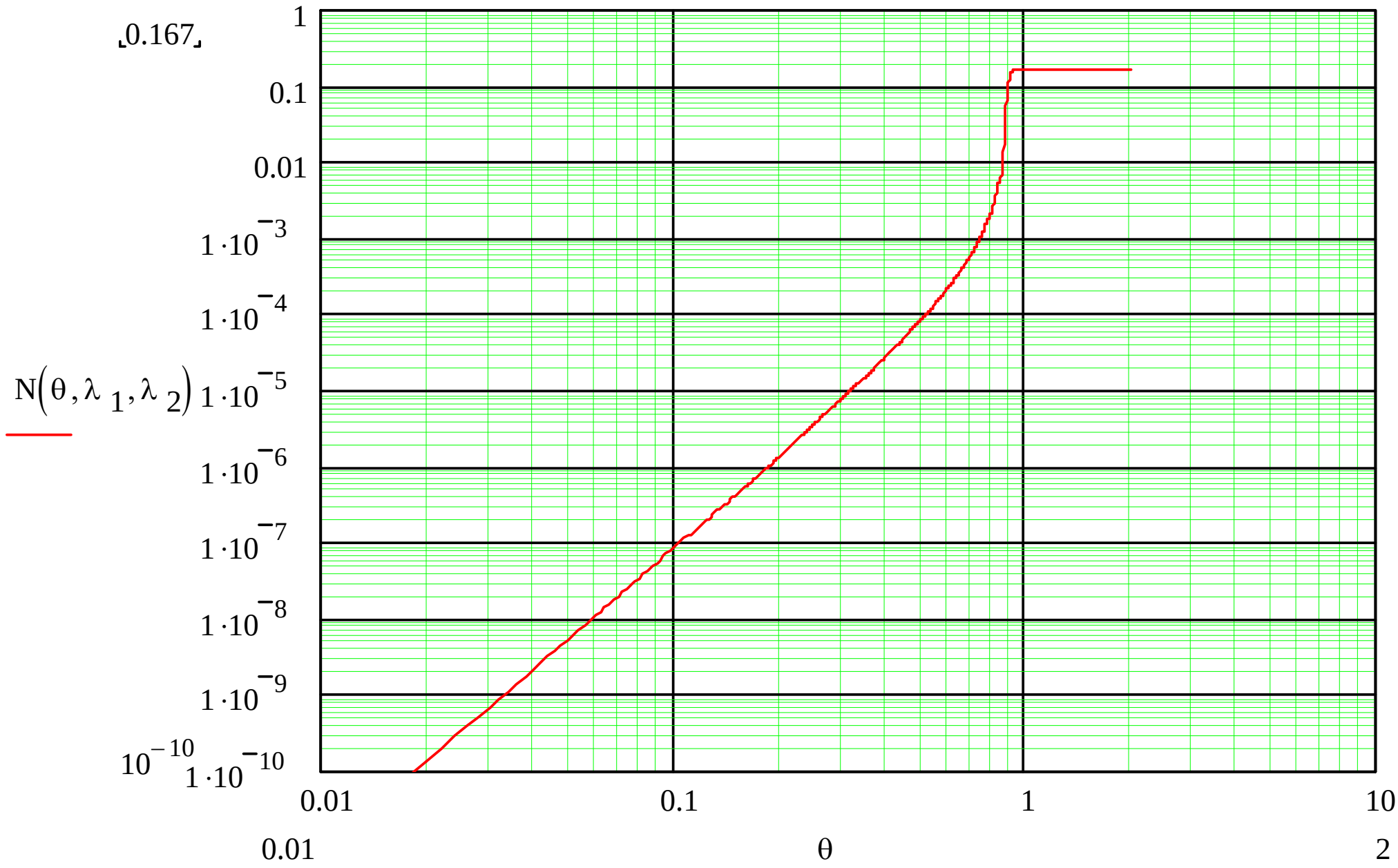
$$\alpha := \frac{1}{137} \quad l := 10^{-2}\ \text{mm} \quad \lambda_1 := 0.4\,10^{-3}\,\text{mm} \quad \lambda_2 := 0.65\,10^{-3}\ \text{mm} \quad k(\lambda,\theta) := \frac{l}{\beta \cdot \lambda} \cdot \left(1 - \beta \cdot n_m \cdot \cos(\theta)\right)$$

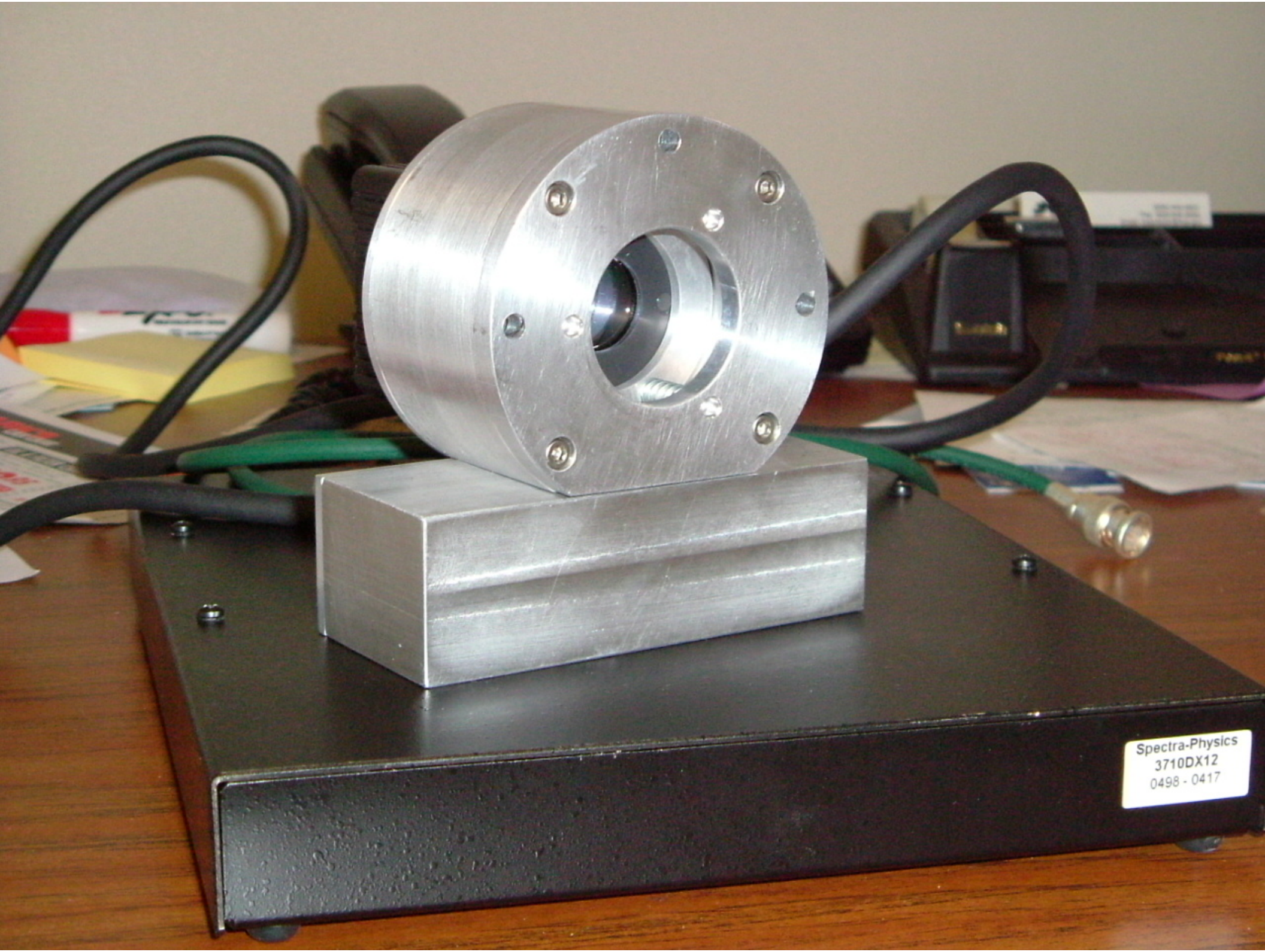
$$\theta_c := \operatorname{acos}\left(\frac{1}{n_m}\right)$$

$$N(\theta, \lambda_1, \lambda_2) := 2\pi \cdot \alpha \cdot l^2 \cdot \int_0^{\theta} \int_{\lambda_1}^{\lambda_2} \frac{1}{\lambda^3} \left(\frac{\sin(\pi \cdot k(\lambda,\theta))}{\pi \cdot k(\lambda,\theta)}\right)^2 d\lambda \cdot \left(\sin(\theta)\right)^3 d\theta$$

$$\beta = 0.99952 \quad \theta_c = 0.891$$

Total number of Cherenkov photons per centimeter over the visible spectrum one can estimate as: ***n_C ~ 490·sin²(θ_c)***. For mica ***θ_c =51.03 deg***. So ***number of photons per electron in 10 μm mica should be ~ 0.3 photons/electron***. Considering the angular distribution for 1 nC charge the total flux of Cherenkov photons should be: ***~ 0.9·10³*** photons ***i.e. ~ few photons per pixel*** considering ICCD magnification and $\Delta\lambda/\lambda$ light filtering.


0.167
1
0.1
0.01
1·10^−3
1·10^−4
N(θ, λ1, λ2)
1·10^−5
1·10^−6
1·10^−7
1·10^−8
1·10^−9
10^−10
1·10^−10
0.01
0.1
1
10
0.01
θ
2


Spectra-Physics
3710DX12
0498 - 0417