3/22/90

MI - Transition Crossing

ESME Simulation

Alex BOGACZ

- revolution frequency dispersion η

$$\rightarrow \quad \eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

$$\frac{1}{\gamma_t^2} = \left(\frac{1}{\gamma_t^0}\right)^2 + \overbrace{4 h r_p R \frac{1}{\beta^2 \gamma^7} \frac{1}{2a^2} \lambda(\theta)}^{\text{Umstätter effect}} - \underbrace{2 j(t) \frac{1}{\gamma^3}}_{\gamma_t - \text{jump}}$$

$$+ \underbrace{\alpha_1 \frac{\Delta p}{p}}_{\text{Johnsen effect}}$$

- Longitudinal Space-Charge Force

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_s}\right) = \frac{eV}{2\pi}\left(\sin\phi - \sin\phi_s\right) + e\,E_\theta(\theta)R$$

$$\underbrace{E_\theta(\theta) = \frac{g_0\, e h^2}{4\pi\epsilon_0 R^2}\frac{d}{d\theta}\lambda(\theta)}_{\text{longitudinal space charge}}$$

- $E-\theta$ oscillations - Equations of motion

→ nonadiabatic time

$$T = \frac{R}{c}\left\{\frac{2\pi}{h}\left(\frac{mc^2}{eV}\right)^2\frac{\gamma^4}{\beta}\frac{1}{\sin\phi_s\,|\cos\phi_s|}\right\}^{1/3}$$

→ dimensionless time $\tau = t/T$

$$\begin{cases} \dfrac{d}{d\tau}\theta = \Big[\, \overbrace{\tau - \mu\, j(\tau) + \epsilon\, \lambda(\theta,\tau) + \tfrac{1}{2}\alpha_1 \mu \gamma^3 \tfrac{\Delta p}{p}}^{0} \,\Big]\, p \\[2ex] \dfrac{d}{d\tau} p = \dfrac{\sin(\phi_s + \theta) - \sin\phi_s}{|\cos\phi_s|} + \xi \dfrac{\partial}{\partial\theta}\lambda(\theta,\tau) \end{cases}$$

$$\mu = \left\{ \frac{4\pi h}{\beta^2\gamma^4} \frac{mc^2}{eV} \frac{|\cos\phi_s|}{\sin^2\phi_s} \right\}^{1/3}$$

$$\epsilon = 4 h r_p R \frac{1}{2a^2} \left\{ \frac{h\pi}{2\beta^8\gamma^{16}} \frac{mc^2}{eV} \frac{|\cos\phi_s|}{\sin^2\phi_s} \right\}^{1/3}$$

$$\xi = \frac{2\pi h^2 g_0}{\gamma^2 |\cos\phi_s|} \frac{r_p}{R} \frac{mc^2}{eV}$$

small θ approximation

$$\frac{\sin(\phi_s + \theta) - \sin\phi_s}{|\cos\phi_s|} \longrightarrow \theta\, \mathrm{sgn}(\cos\phi_s)$$

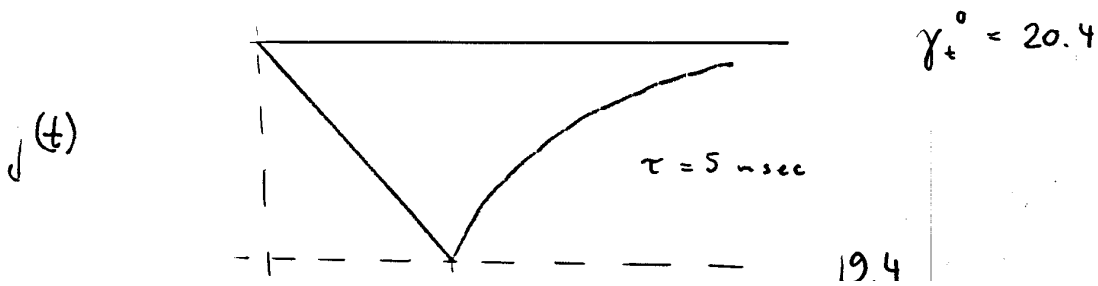
- γ_t jump – compensation of the Umstätter and Johnsen effects

$$j(\tau) = \frac{\epsilon}{\mu}\,\lambda(\theta,\tau)\Big|_{max} + \frac{1}{2}\alpha_1\,\gamma_t^3\,\frac{\Delta p}{p}\Big|_{max}$$

$\delta = \frac{\Delta p}{p}$

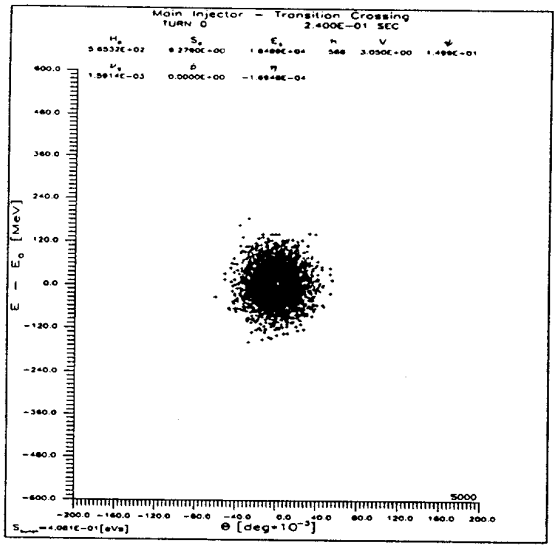
T	μ	ε λ(θ)	$\mu\gamma^3\alpha_1\delta$	j	$\frac{d\lambda}{d\theta}\frac{\xi}{\theta}$	$\frac{d\gamma}{dt}$
			MI			
2.3×10^{-3}	5.8	5.6×10^{-1}	4.9	0.94	2.1×10^{-2}	125.
			MR			
3.7×10^{-3}	3.7	4.3×10^{-1}	4.2	1.24	4.2×10^{-2}	68.
			BOOSTER			
2.3×10^{-4}	13.	7.6×10^{-1}	1.6	0.17	2.6×10^{-2}	250.

MI γ_t – jump



$\alpha_1 = 0$

1



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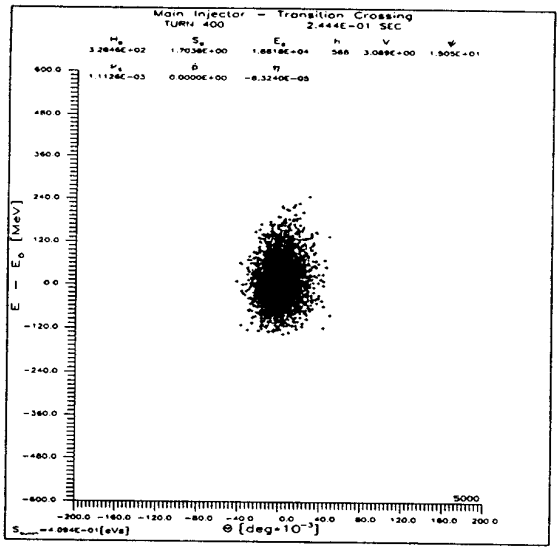

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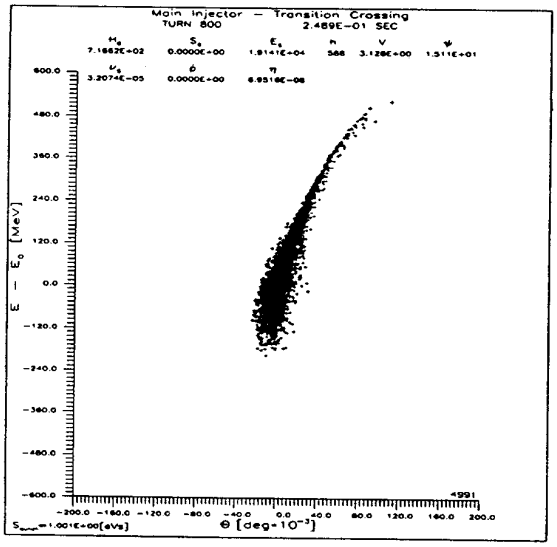

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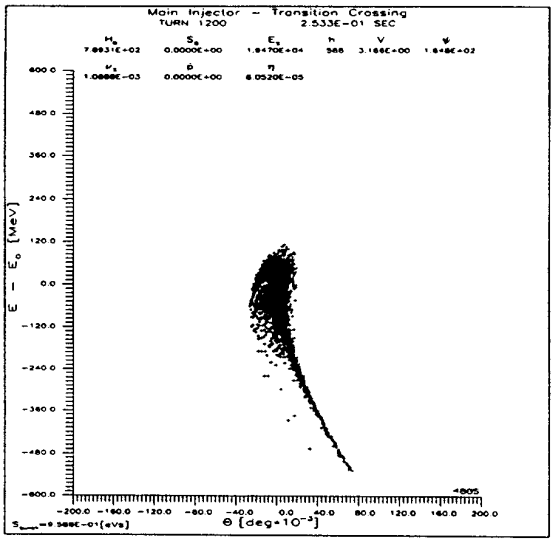

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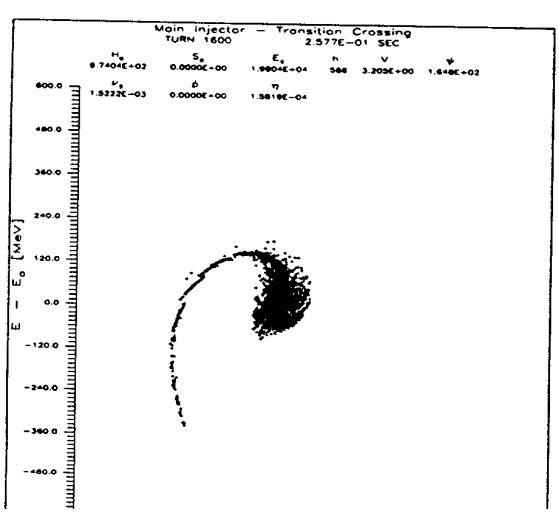

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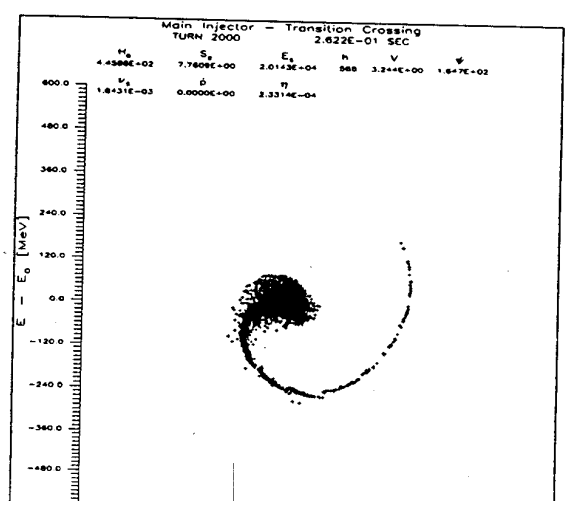
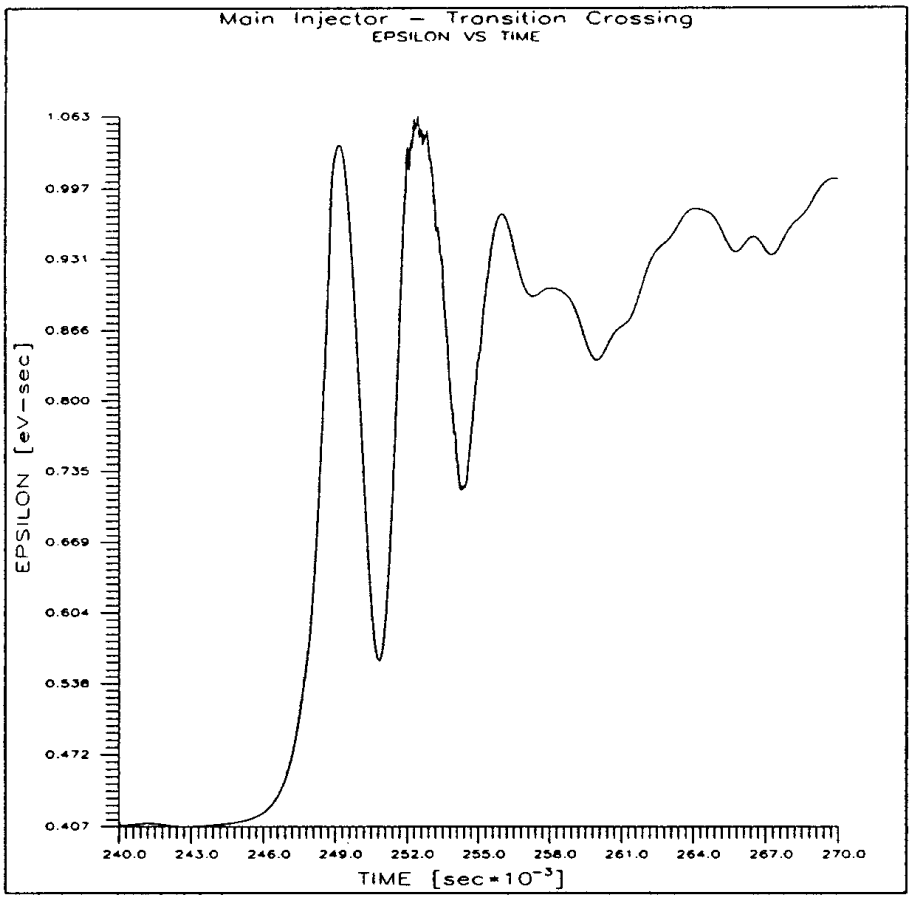


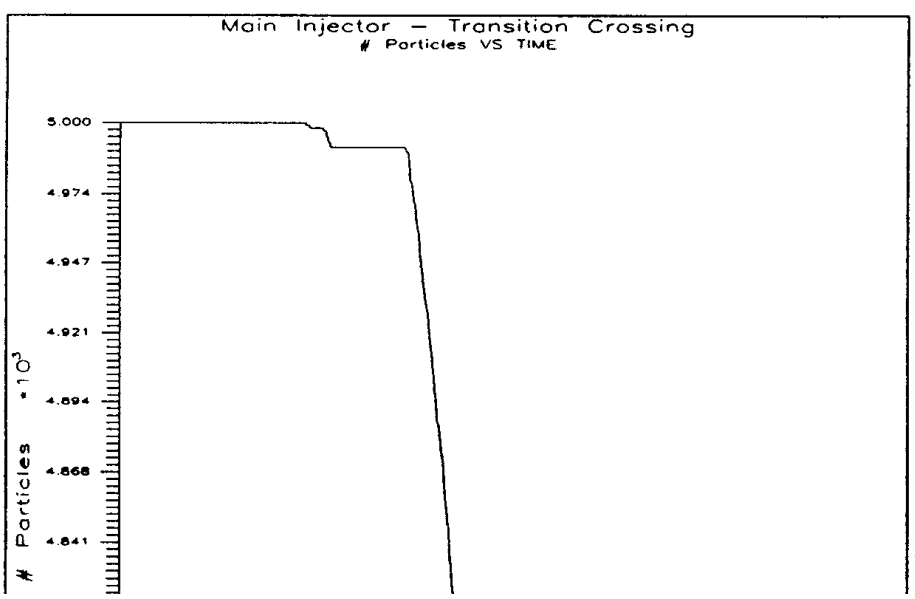
longitudinal emittance



2.5

blow-up

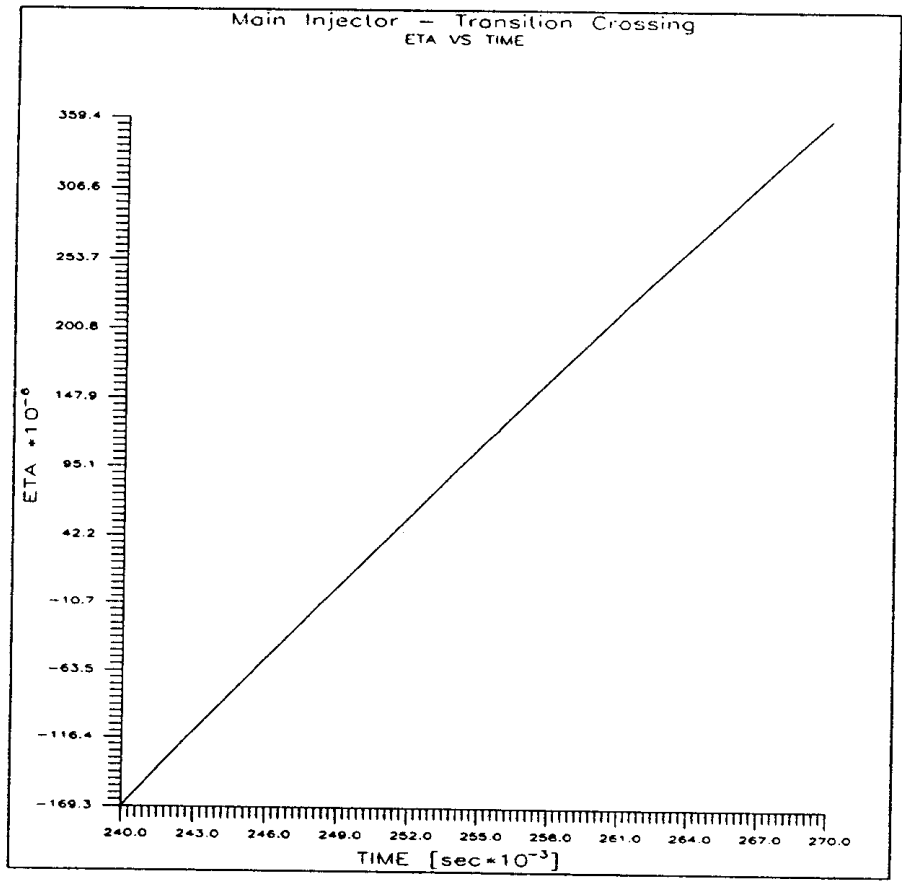
# of particles



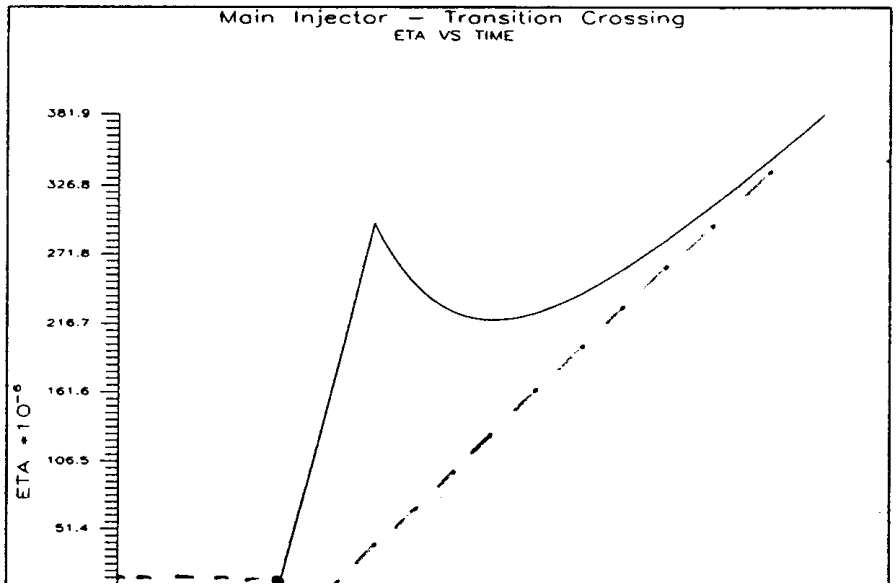
5.5%

lost

η vs time

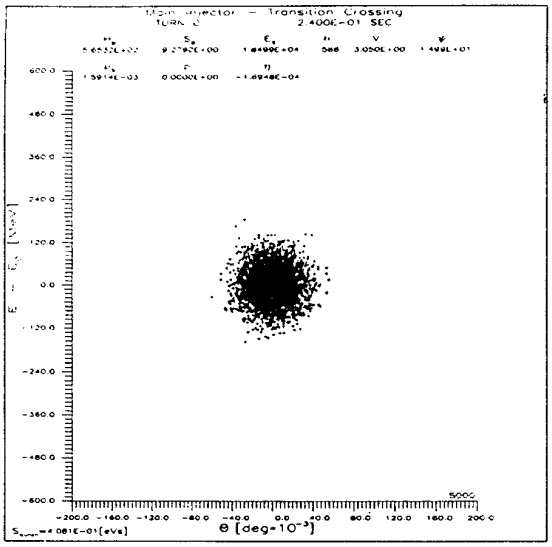


γ_t - jump



$\alpha_1 = 0$ γ_t jump

1



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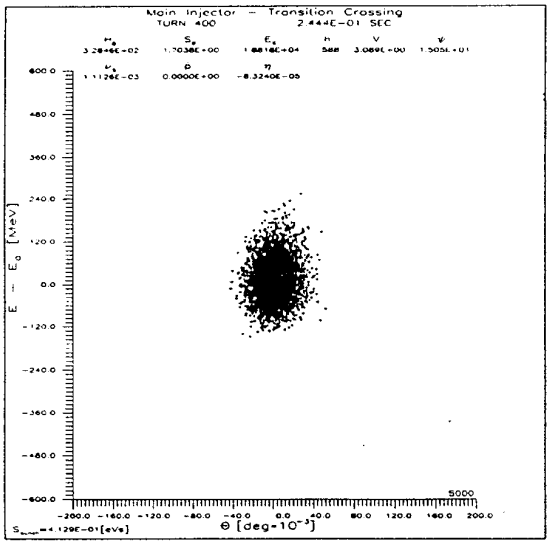

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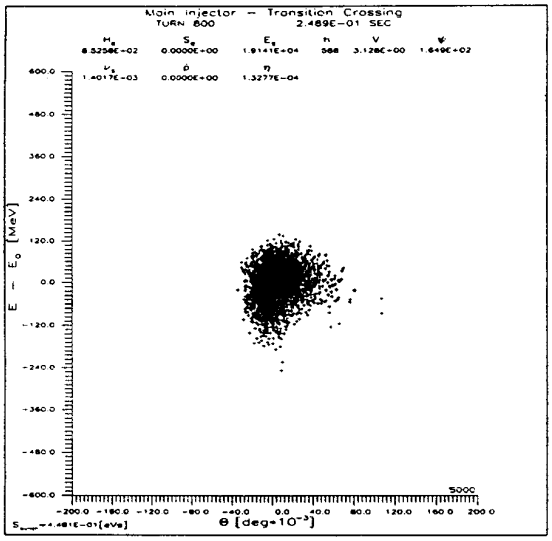

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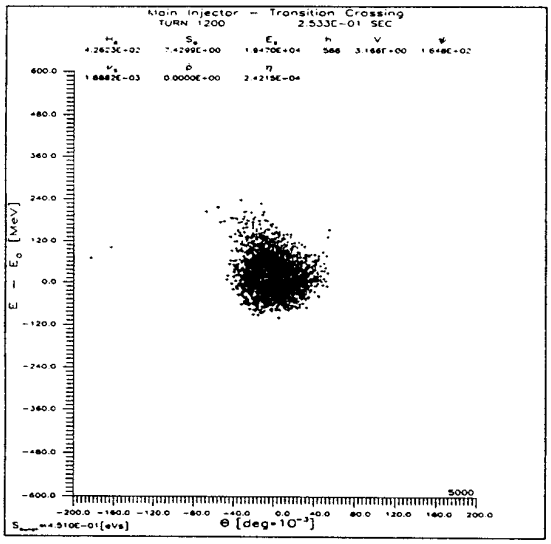

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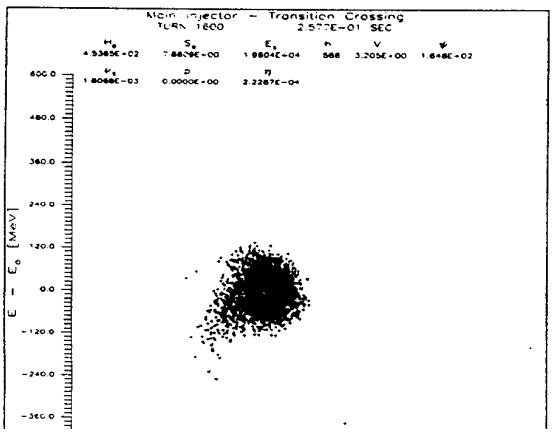

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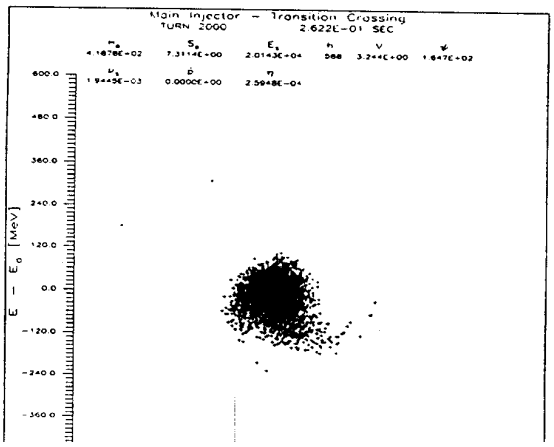
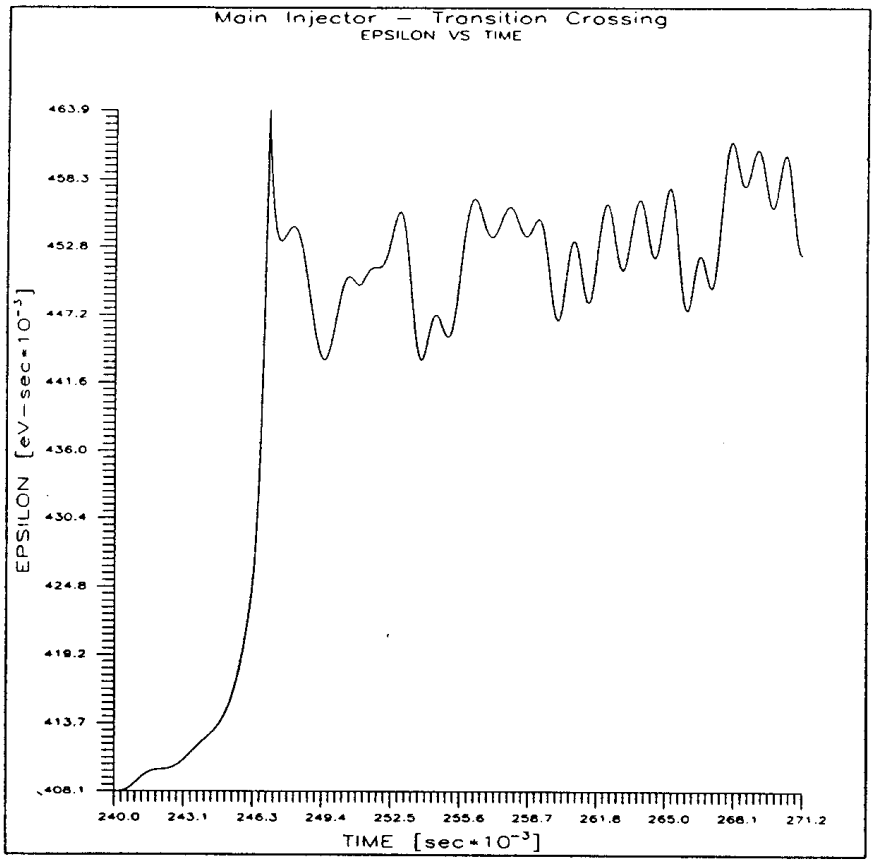


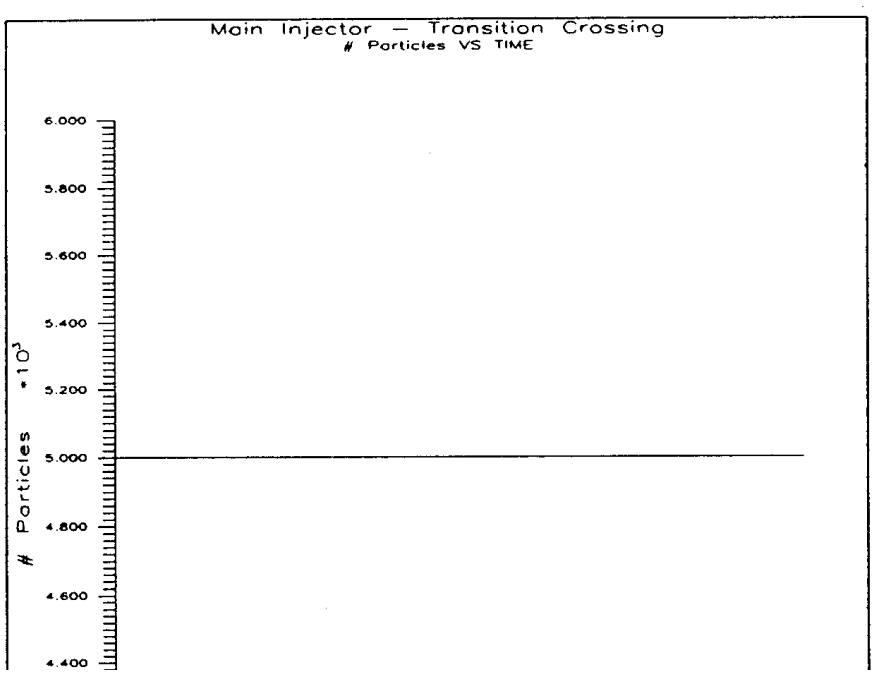
longitudina emittance



10 %

blow up

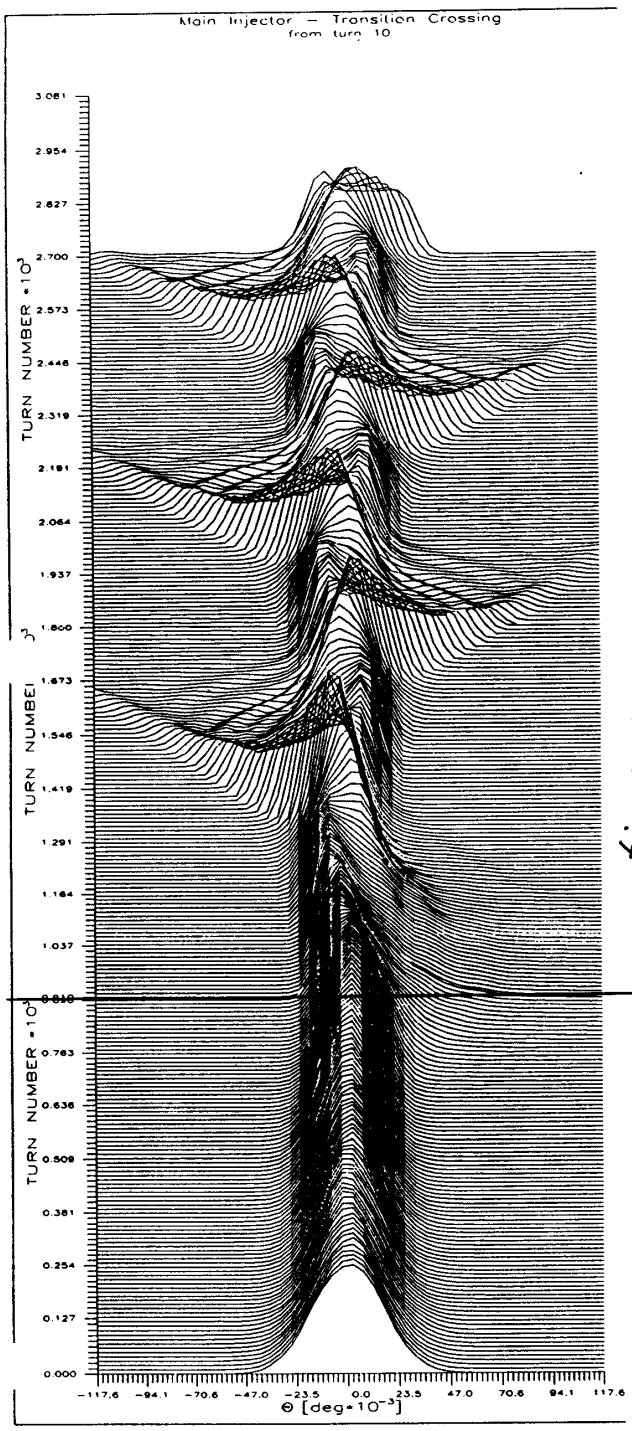
# of particles



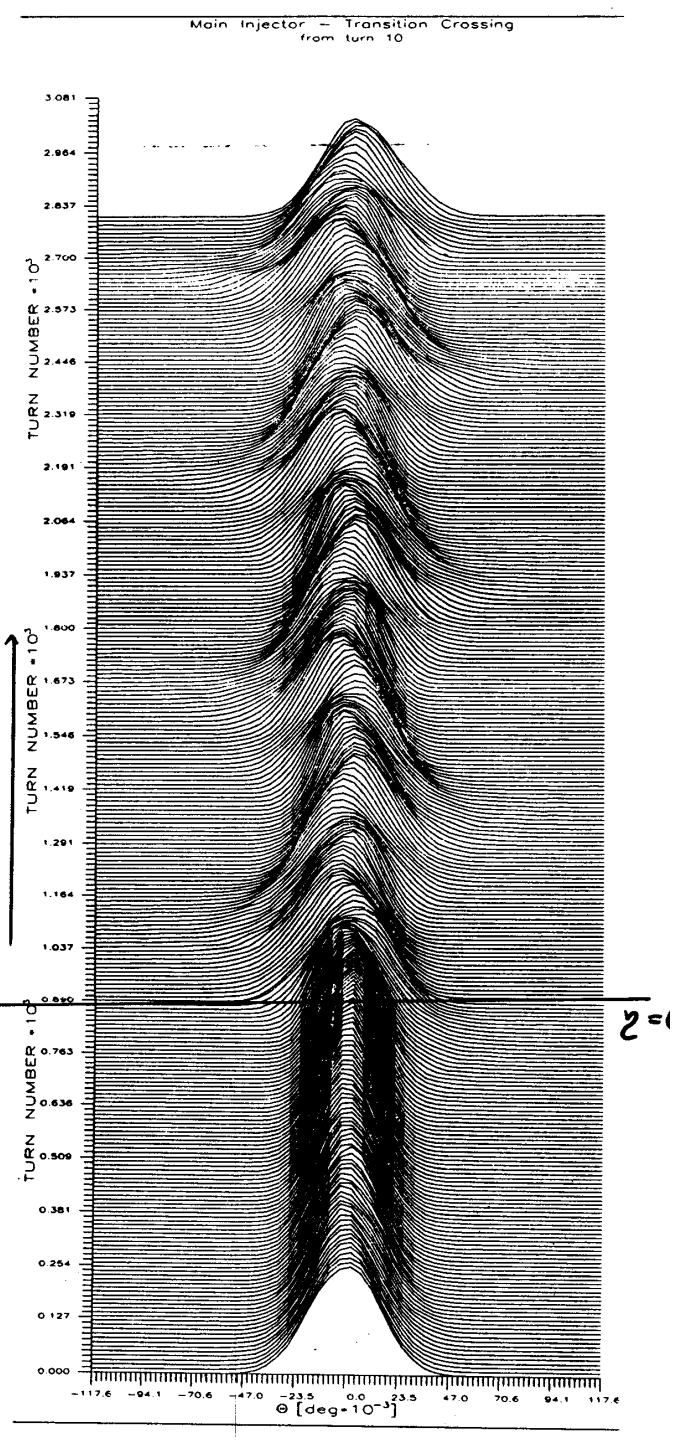
no particle lost

$\alpha_1 = 0$

no γ_t jump



γ_t jump

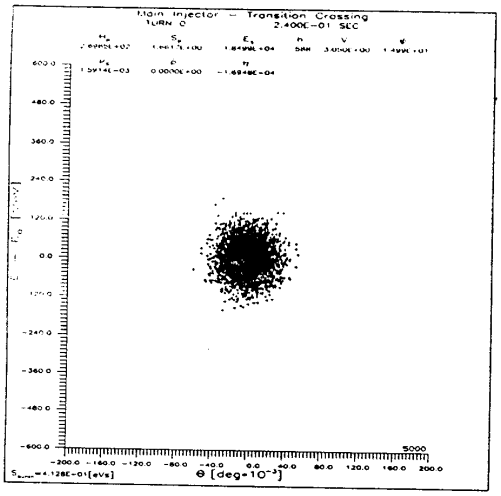


time

$\eta = 1$

$\alpha_1 = 5 \times 10^{-3}$

1



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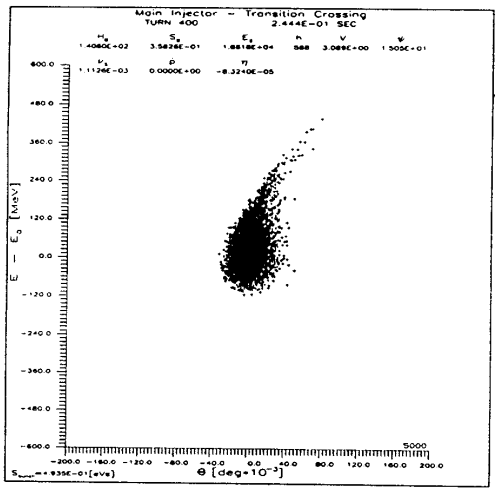

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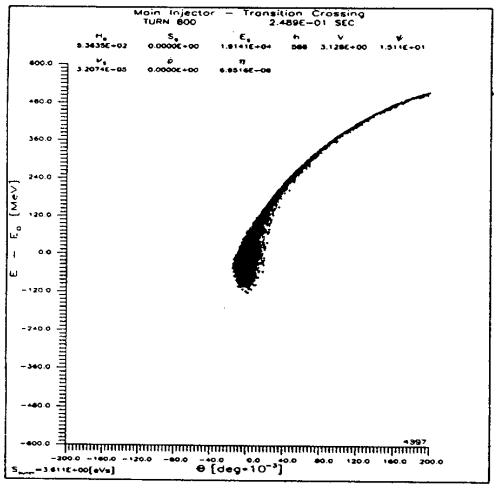

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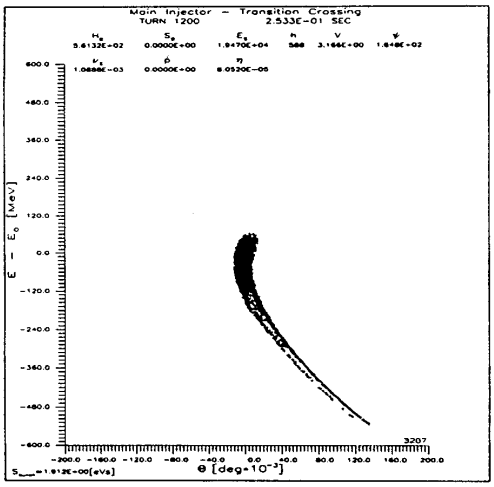

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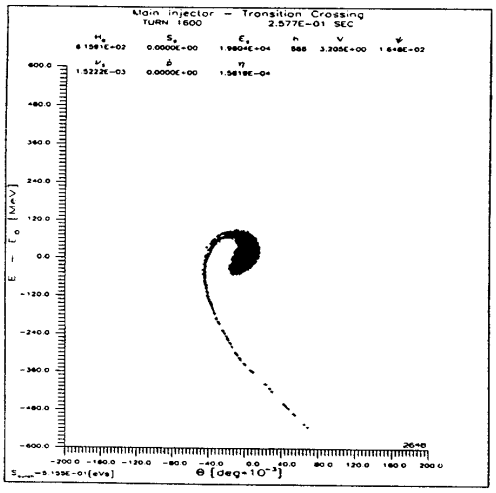

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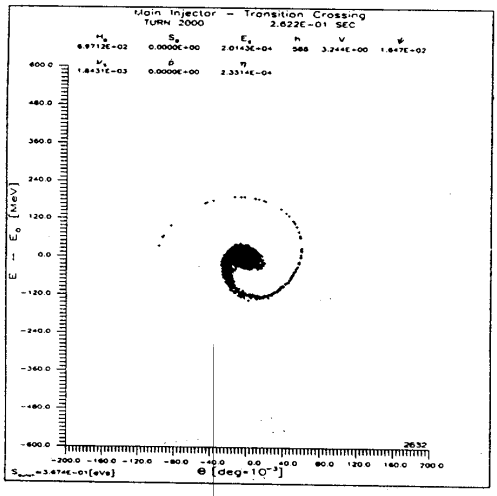
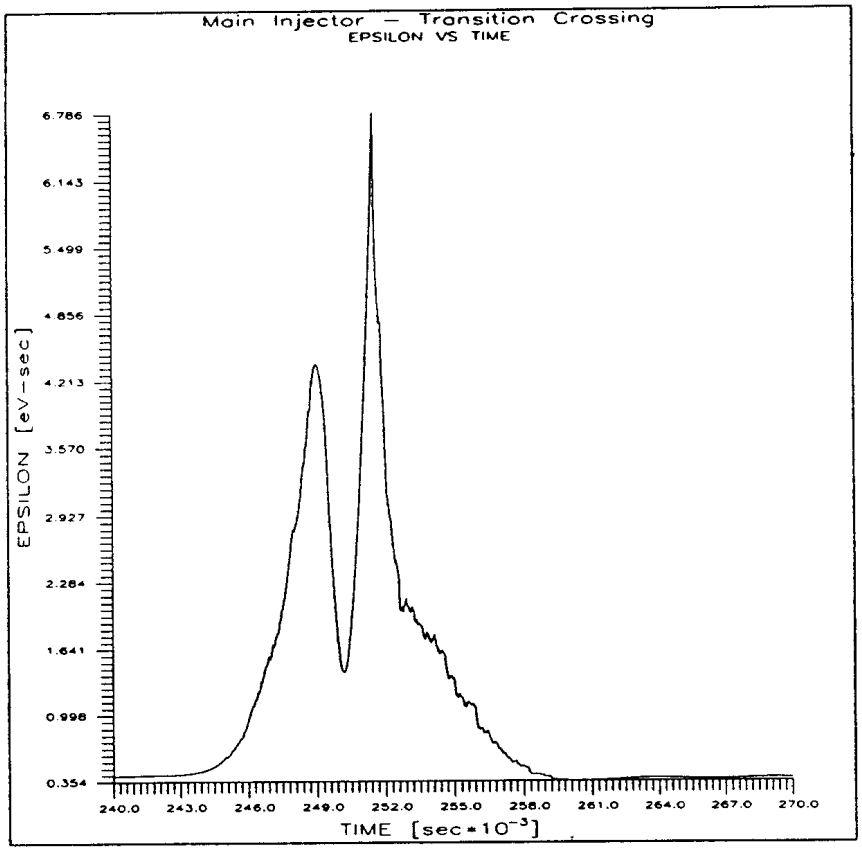
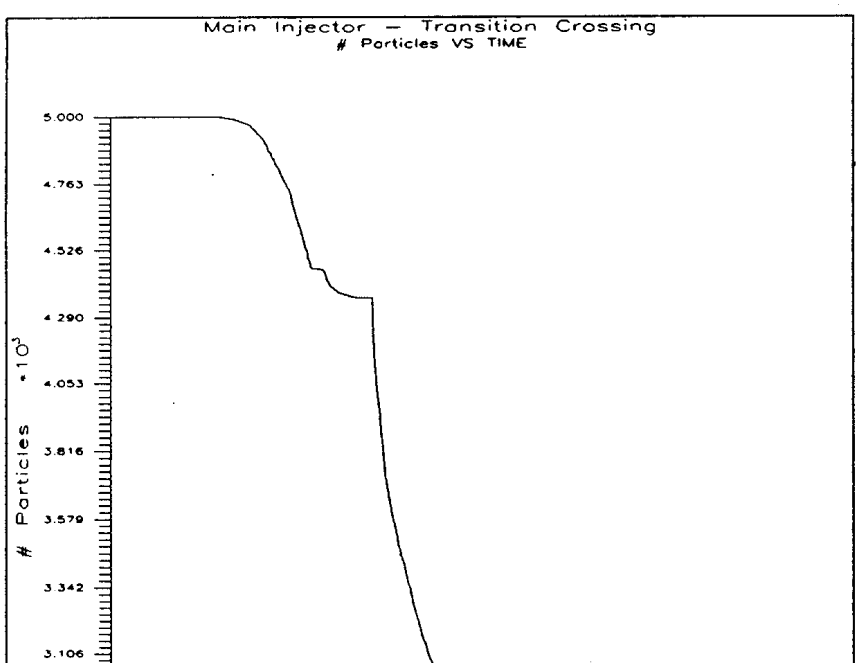


longitudinal emittance



compacted beam

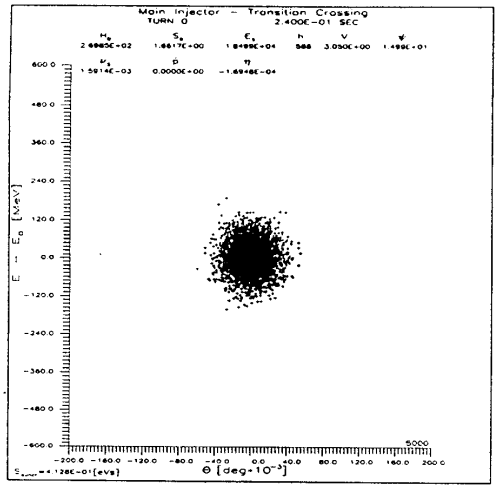
# of particles



50% beam loss

$\alpha_1 = 5\times10^{-3}$ γ_t jump

1



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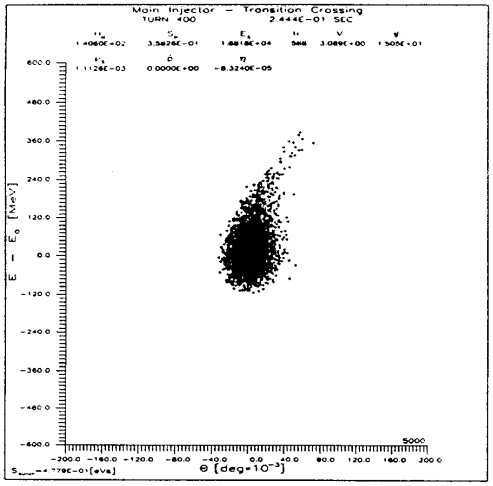

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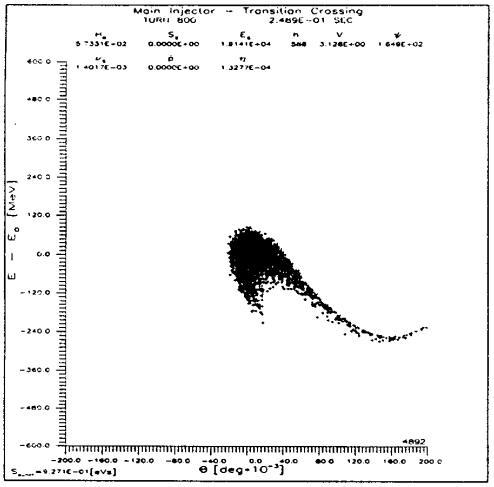

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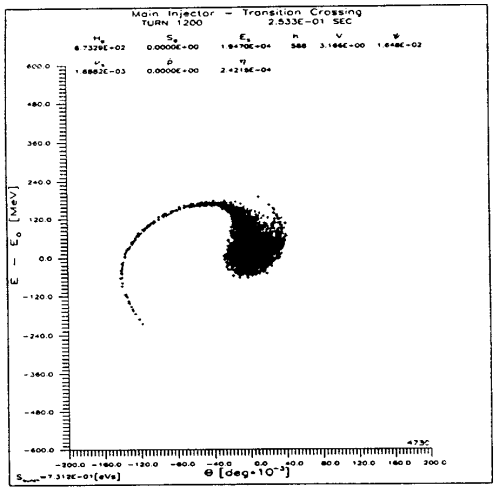

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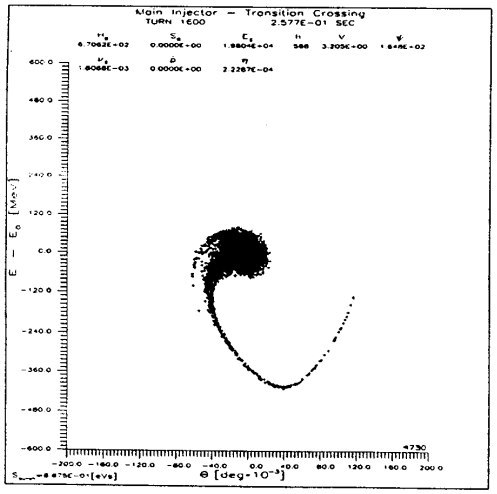

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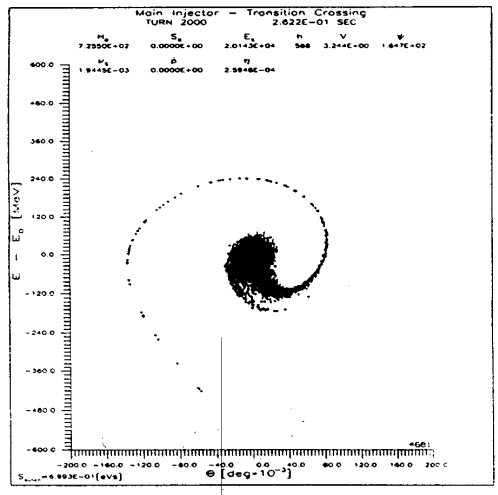
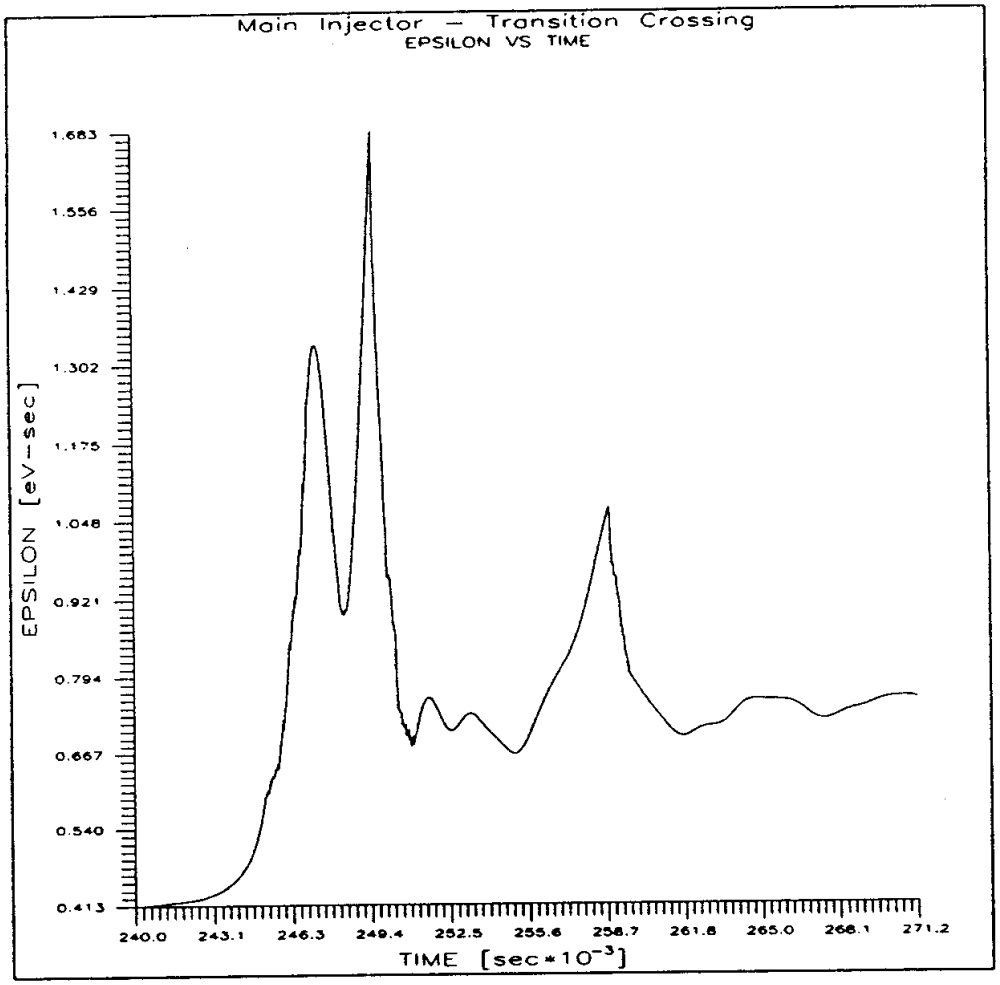
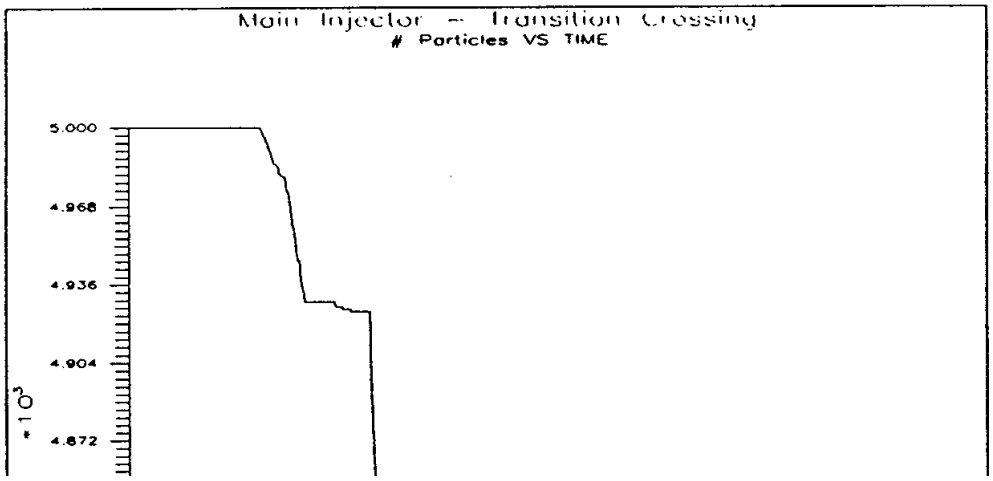


Longitudinal emittance



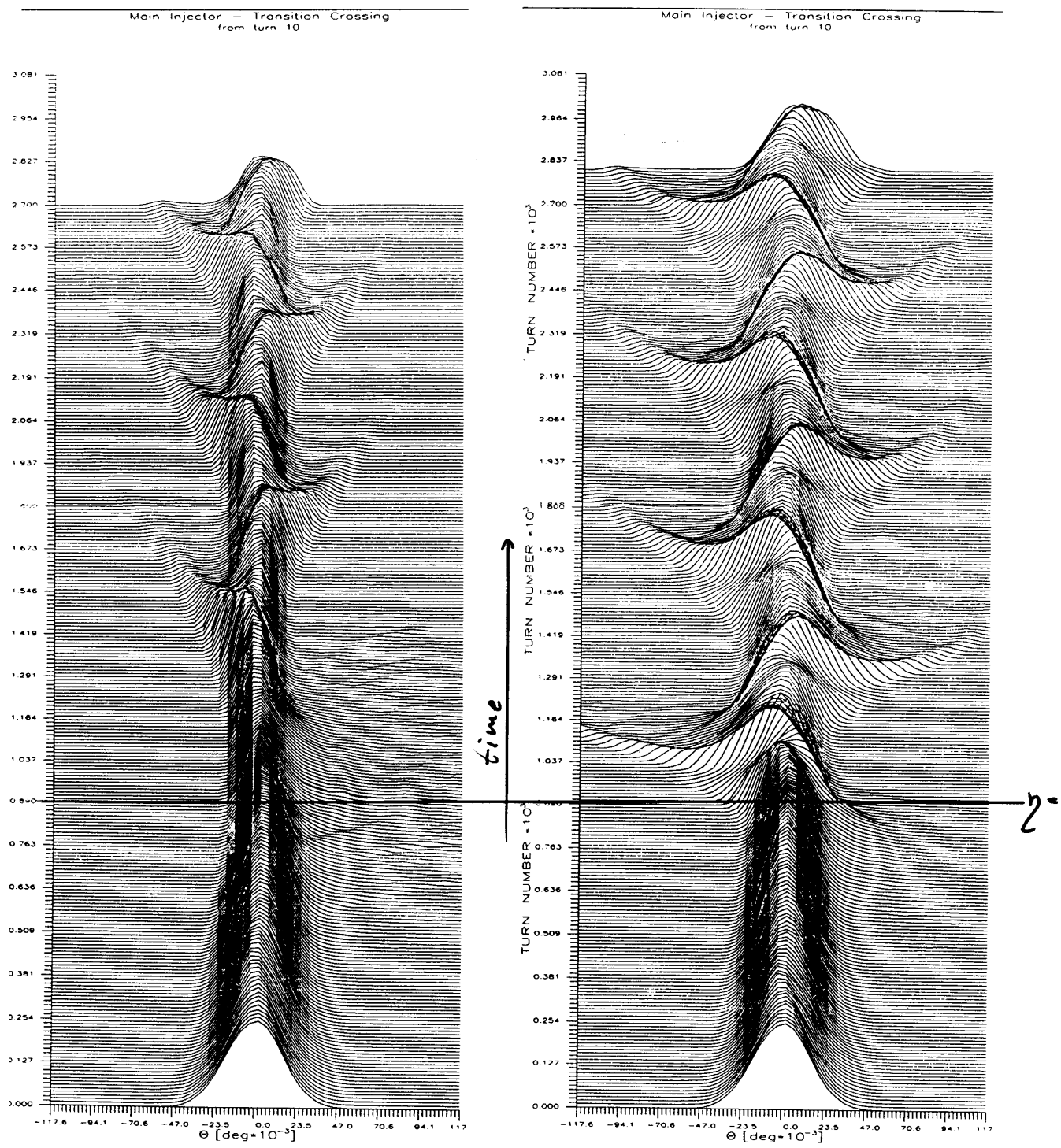
75 % blow up

# of particles



6.5 %

$\alpha_1 = 5 \times 10^{-3}$

no γ_t jump

γ_t jump