The Definition of α_1: Circumference, Johnsen, or ESME?

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October, 1990

For reference purposes, this note describes how the three definitions of α_1 and the one definition of α_P, all of which are common in the literature, are related. The "circumference" definition of α_1 comes from expanding the difference in the circumference of the closed orbit, $\Delta C = C - C_0$, as a polynomial in the off momentum parameter, $\delta = (p - p_0) / p_0$,

$$\frac{\Delta C}{C_0} = \alpha_0 + \alpha_{c1}\delta^2 + \cdots$$

The definition introduced by Johnsen is very similar

$$\frac{\Delta C}{C_0} = \alpha_0\delta + \alpha_0\alpha_{J1}\delta^2 + \cdots$$

The variation of the transition energy γ_T with δ is directly described by α_P, defined through

$$\alpha_P(\delta) = \frac{1}{\gamma_T^2(\delta)} = \frac{p}{C}\frac{dC}{dp} = \alpha_0 + \alpha_{E1}\delta + \cdots$$

introduces the ESME definition of α_1. Note that equation 3 represents the <u>local</u> derivative at some momentum p. WARNING - some lattice design codes return $\alpha_P(\delta)$, but some return

$$\frac{p_0}{C_0}\frac{dC}{dp} = \alpha_0 + \alpha_{C1}\delta + \cdots$$

when they are used with a constant momentum offset. Comparing equations 1 and 2 gives

$$\alpha_{J1} = \frac{\alpha_{C1}}{\alpha_0}$$

Performing the differentiation in equation 3 and expanding gives

$$\alpha_{E1} = \alpha_0 + \alpha_{C1} - \alpha_0^2$$

The reader is NOT implored to adopt one or another of the definitions introduced here, but rather is asked to be careful to specify which definition he or she id using. He or she IS implored not to invent any more definitions for α_1.
