MI-0000
Main Injector Dipole Magnet: Lorentz Forces

Jean-François Ostiguy
AD/Accelerator Physics

October 9, 1990

1 Introduction

In theory, the calculation of the Lorentz force acting on a body is a simple matter. In practice, due to numerical difficulties and limitations of the postprocessing software, things tend to be slightly more complicated. Three methods can be employed:

- The net force can be obtained by integrating the force due to the magnetic field acting each differential element

$$\mathbf{F} = \int \left[\mathbf{J} \times \mathbf{B} + \frac{1}{\mu_0}(\mathbf{M} \cdot \nabla)\mathbf{H} \right] \, dV \tag{1}$$

  For conducting media such as copper, $\mathbf{M} = 0$ and the only difficulty arises from the shape of the integration region. In contrast, the situation is extremely complex in the case of a ferromagnetic material because the permeability depends on the state of stress of the body.

- For materials whose constitutive relation is linear, it is possible to express the integrand in equation (1) as the divergence of a tensor (the Maxwell stress tensor) and to transform the resulting expression into a surface integral.

$$F_i = \int \partial_j T_{ij} \, dS \tag{2}$$

  with

$$T_{ij} \equiv H_i B_j - \frac{1}{2}\delta_{ij} H_k B_k \tag{3}$$

  or, in vector form

$$\mathbf{F} = \int \left[(\hat{\mathbf{n}} \cdot \mathbf{B})\mathbf{H} - \frac{\mathbf{B} \cdot \mathbf{H}}{2}\hat{\mathbf{n}} \right] \, dS \tag{4}$$

- Finally the force can also be calculated by using the principle of virtual work. In essence, one needs to calculate the total energy

$$W = \int \left[\int \mathbf{H} \cdot \mathbf{dB} \right] dV \tag{5}$$

over all space, move the body by a small amount $\Delta \mathbf{s}$ and recalculate the energy. The component of the force acting in the direction of the displacement $\mathbf{s}$ is then given by

$$\mathbf{F} \cdot \hat{\mathbf{s}} = -\frac{\Delta W}{\Delta s} \tag{6}$$

Using the finite element codes at our disposal, all these methods are available. In practice, we must live with the limitations of the postprocessing software which, hopefully, will disappear in the future.

2 Body Forces

The code PE2D integrates

$$\mathbf{F} = \int \mathbf{J} \times \mathbf{B} \, dV \tag{7}$$

accurately and has been used to determine the forces acting on each one of the four conductors in the body of the magnet. The results apply to the conductors situated in the first quadrant ($0 < x < \infty, 0 < y < \infty$). Conductor no 1 is the innermost conductor.

BODY FORCES I=9417 Current in Amperes. Field in Tesla. Force in Newton/m				
	conductors		conductor+core	
Conductor	f_x	f_y	f_x	f_y
1	+953	−354	+2460	+1560
2	+350	−391	+1810	+963
3	−216	−396	+1080	+664
4	−816	−368	+3040	+515

3 End Section Forces

The forces acting on the conductor in the end regions are much more difficult to evaluate than the body forces . Although TOSCA, our 3D code, does allow the user to integrate $\mathbf{J} \times \mathbf{B}$, the integration can only be performed over an entire coil. Because of restrictions imposed on the surface on which the Maxwell stress tensor can be integrated and the very primitive support for the virtual work option, we have opted for a simple estimation based on the assumption that each of the four conductors is a filament exposed to a field equal to the one at the center of the actual conductor. In component form, one has

$$f_x = I_y B_z - I_z B_y \tag{8}$$
$$f_y = I_z B_x - I_x B_z \tag{9}$$
$$f_z = I_x B_y - I_y B_x \tag{10}$$

where I_x, I_y and I_z are the components of the current vector and f_x, f_y and f_z represent the forces per unit of conductor length. The calculations were performed for two differents lines. Line "A" is parallel to the longitudinal axis. Line "B" bissects the circular arcs sections of each racetrack winding. Both lines pass through the middle of the conductor thickness. The situation is schematized in figure 2. It should be noted that the special end section pole profile has not been modeled in detail (see figure 1). To the extent that we are only interested in rough estimates of the forces, this should not affect the validity of the results.

END SECTION FORCES
CONDUCTORS ONLY
Line "A"
Current in Amperes. Field in Tesla. Force in Newton/m

Conductor	I_x	I_y	I_z	B_x	B_y	B_z	f_x	f_y	f_z
1	−9417	0.0	0.0	0.0	−0.171	−0.0326	0.0	−306.5	+1608.0
2	−9417	0.0	0.0	0.0	−0.099	−0.0347	0.0	−326.3	+930.7
3	−9417	0.0	0.0	0.0	−0.031	−0.0346	0.0	−325.9	+295.3
4	−9417	0.0	0.0	0.0	+0.040	−0.0324	0.0	−305.3	−380.8

END SECTION FORCES
CONDUCTORS ONLY
Line "B"
Current in Amperes. Field in Tesla. Force in Newton/m

Conductor	I_x	I_y	I_z	B_x	B_y	B_z	f_x	f_y	f_z
1	−6659	0.0	+6659	−0.0212	−0.1824	−0.0246	+1213.7	−305.3	+1213.7
2	−6659	0.0	+6659	−0.0219	−0.1208	−0.0244	+804.2	−308.2	+804.2
3	−6659	0.0	+6659	−0.0223	−0.0601	−0.0222	+400.1	−282.4	+400.1
4	−6659	0.0	+6659	−0.0185	−0.0093	−0.0185	−62.3	−235.9	−62.3

END SECTION FORCES CONDUCTORS + CORE Line "A" Current in Amperes. Field in Tesla. Force in Newton/m									
Conductor	I_x	I_y	I_z	B_x	B_y	B_z	f_x	f_y	f_z
1	−9417	0.0	0.0	0.0	−0.138	+0.278	0.0	+2611	+1300
2	−9417	0.0	0.0	0.0	−0.145	+0.200	0.0	+1887	+1364
3	−9417	0.0	0.0	0.0	−0.103	+0.122	0.0	+1151	+974
4	−9417	0.0	0.0	0.0	−0.030	+0.072	0.0	+673	+286

END SECTION FORCES CONDUCTORS + CORE Line "B" Current in Amperes. Field in Tesla. Force in Newton/m									
Conductor	I_x	I_y	I_z	B_x	B_y	B_z	f_x	f_y	f_z
1	−6659	0.0	+6659	+0.285	−0.275	+0.1231	+1836.0	+2719.2	+1836.0
2	−6659	0.0	+6659	+0.891	−0.195	+0.0688	+1296.2	+1052.2	+1296.2
3	−6659	0.0	+6659	+0.032	−0.117	+0.0305	+777.1	+422.6	+777.1
4	−6659	0.0	+6659	+0.015	−0.032	+0.0120	+214.3	+180.2	+214.3

4 Conclusion and Comments

The calculations show that each coil is subject to forces which tend to expand it "outward" in the horizontal plane. Due to the influence of the core, the coils are also subjected to forces that tend to increase the vertical distance between the top and bottom coils. These results are very different from those obtained in the case where the core is absent. In that case, the forces acting between the top and lower coils packs are attractive. The effect of the core can be understood by imagining image currents parallel to the coil currents and circulating behind every iron boundary. In the vertical direction, the force due to these image currents is sufficient to overcome the natural attractive force between the top and bottom coils.

The results presented in this note are expressed in MKS units. Conversion to the more familiar English units can be performed by using the conversion factors

$$\begin{aligned} 1 \text{ Newton} &\simeq 0.2248\,\text{lb} \\ 1 \text{ meter} &\simeq 39.37\,\text{in} \end{aligned} \tag{11}$$

One can get a feel for the magnitude of the Lorentz forces by comparing them with the force due to gravity which is approximately 1.28 lb/in or 224 N/m for each conductor. Roughly speaking, the maximum force represents about ten times the force due to gravity.

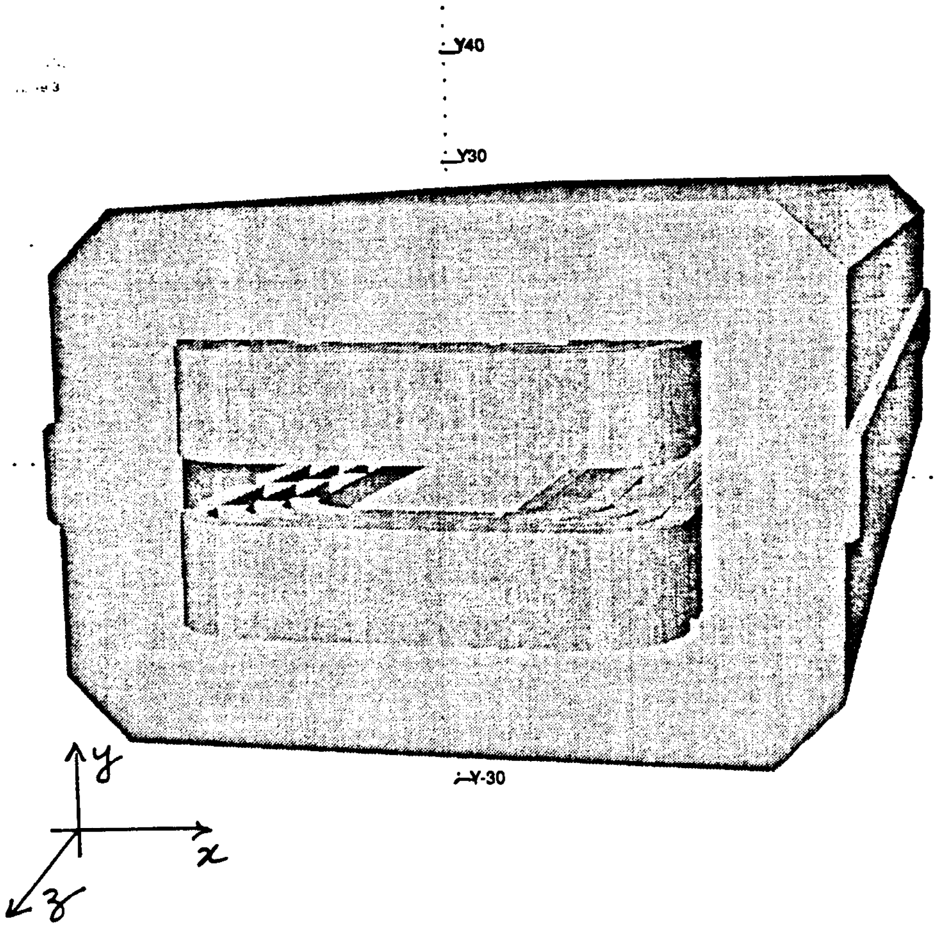

Figure 1: A perspective view of the Main Injector magnet model.

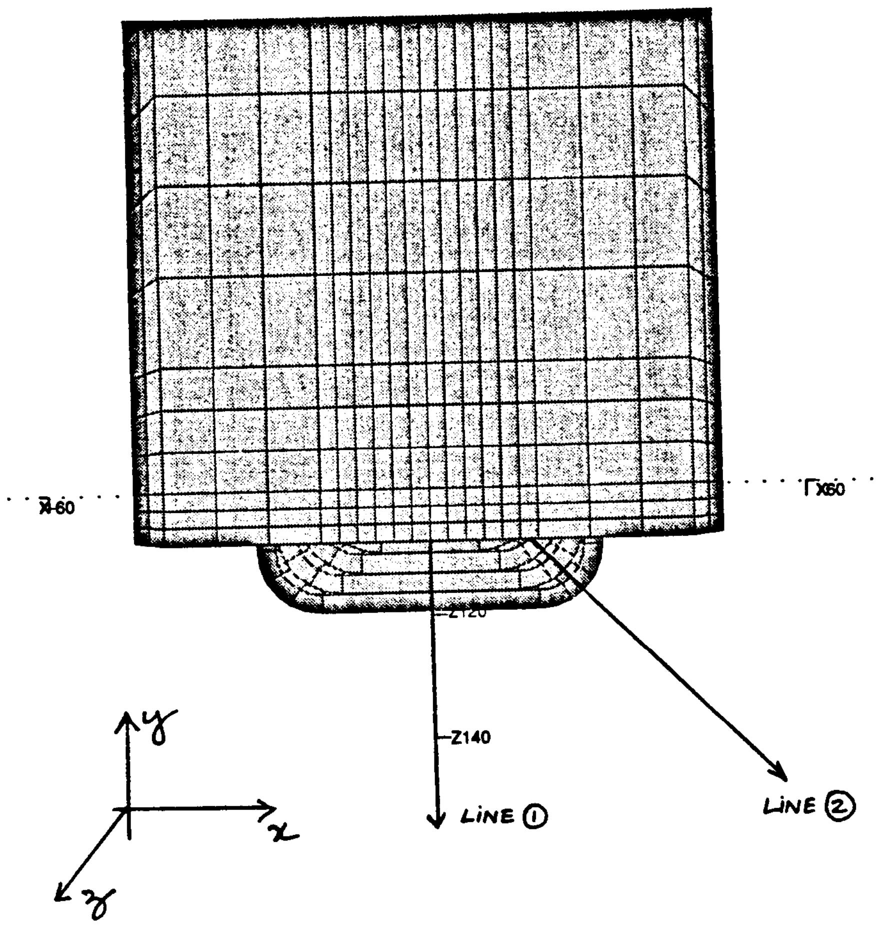

Figure 2: Lines used to estimate the forces in the end section regions. Line "B" bissects the circular arc of the conductors. Both lines vertical offset is equal to half the conductor thickness.

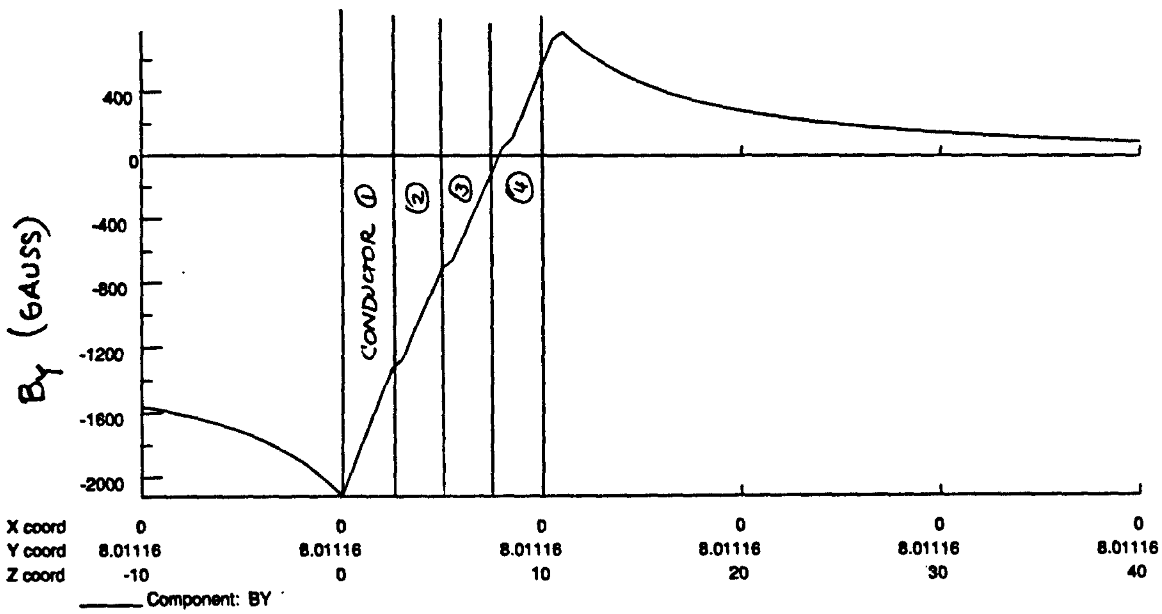

Figure 3: B_y vs position along line "A" in the case where the field is due to the conductors only.

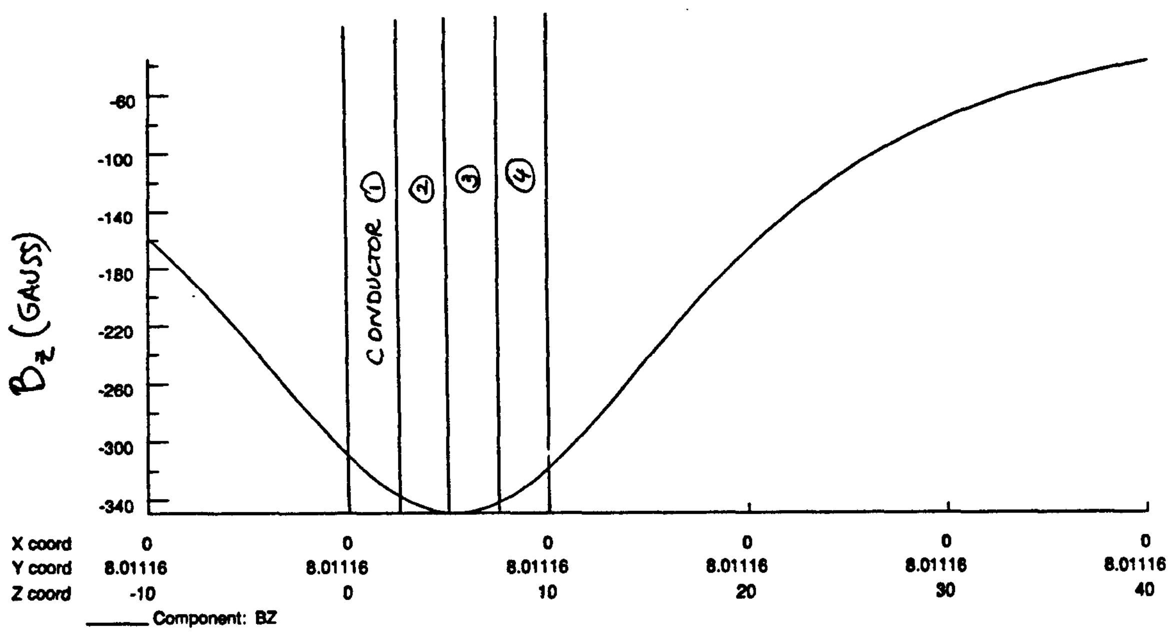

Figure 4: B_z vs position along line "A" in the case where the field is due to the conductors only.

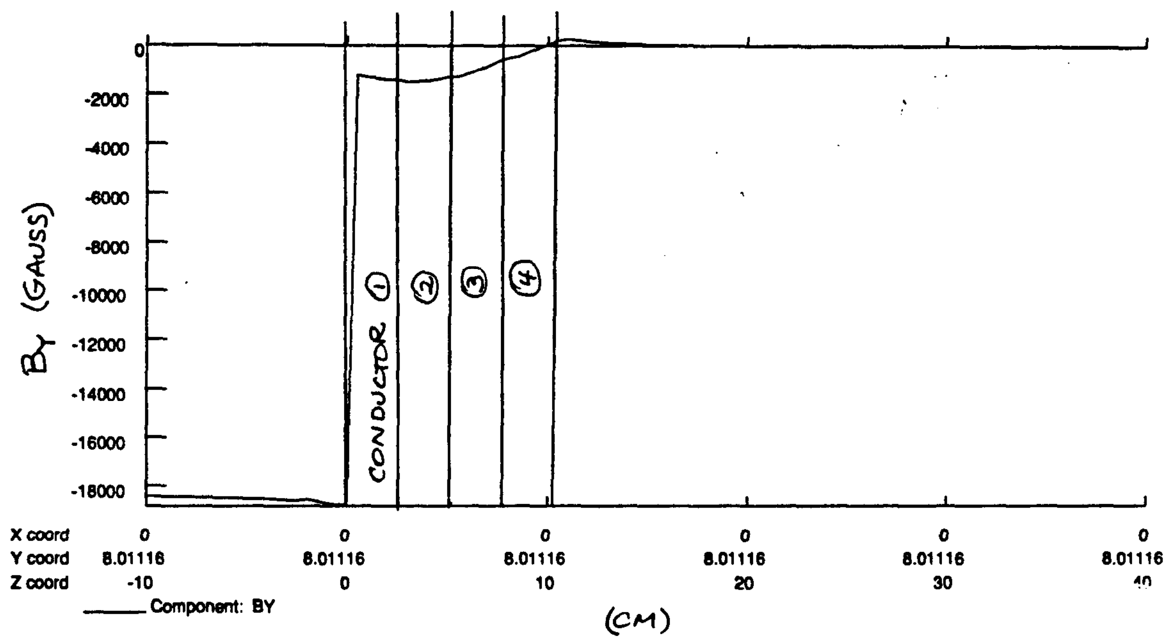

Figure 5: B_y vs position along line "A" in the case where the core is present.

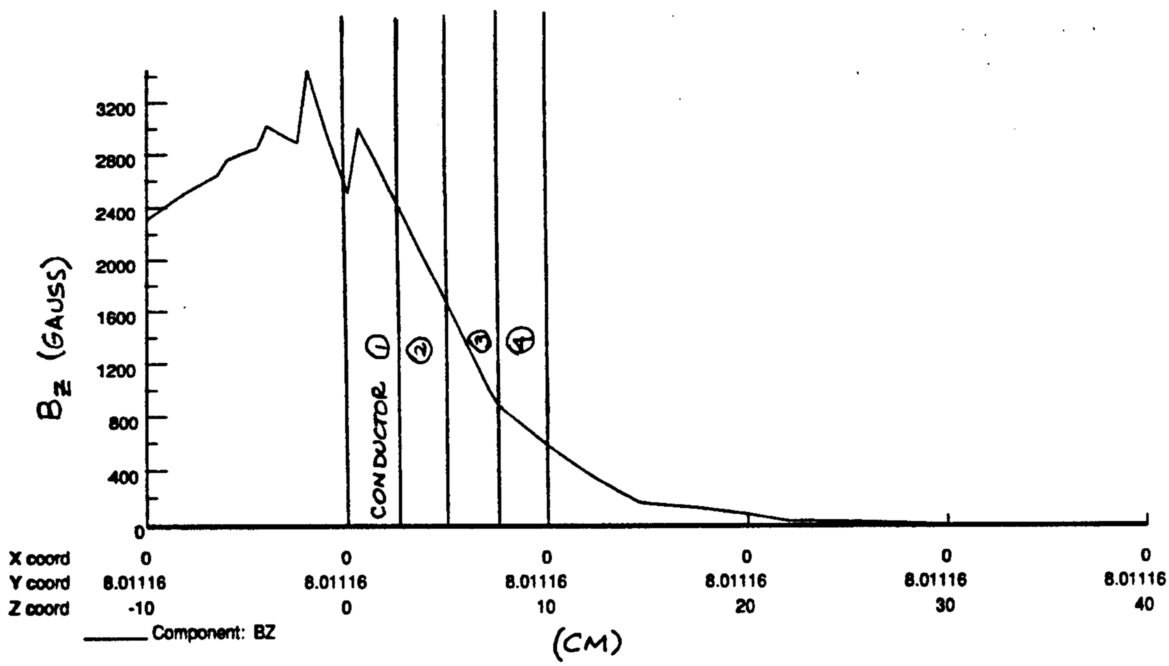

Figure 6: B_z vs position along line "A" in the case where the core is present.

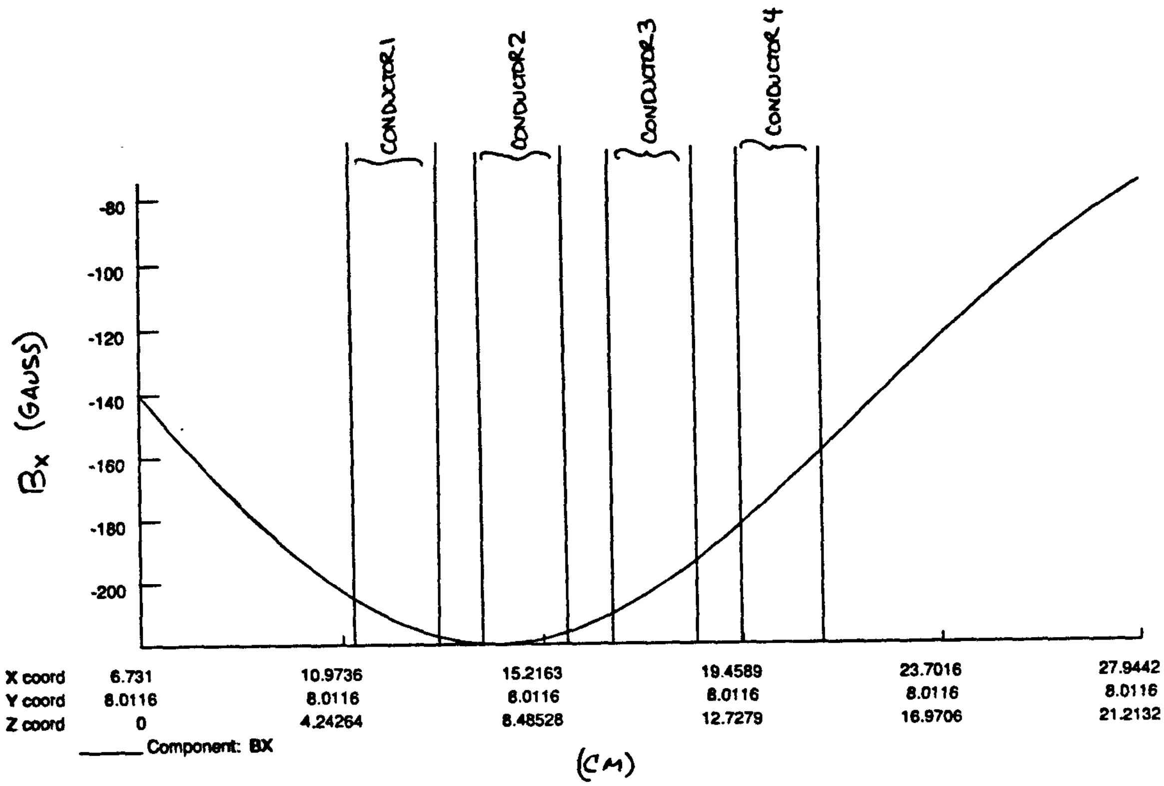

Figure 7: B_x vs position along line "B" in the case where the field is due to the conductors only.

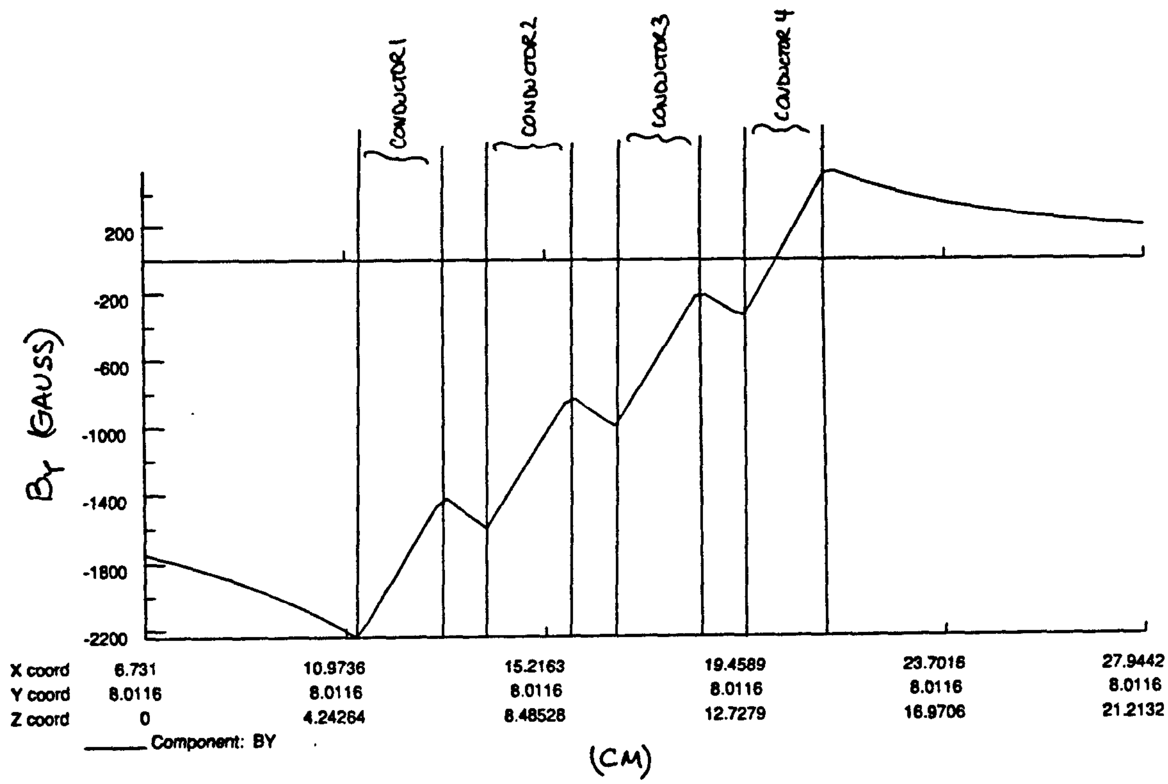

Figure 8: B_y vs position along line "B" in the case where the field is due to the conductors only.

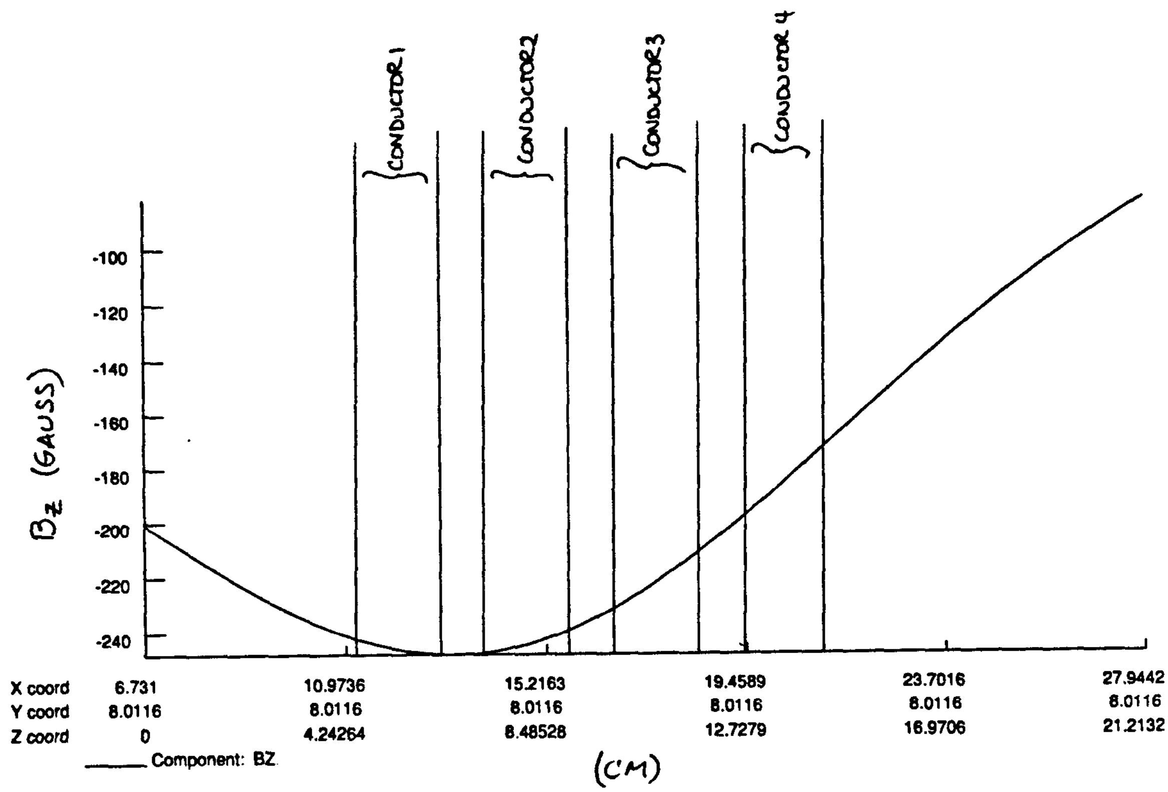

Figure 9: B_z vs position along line "B" in the case where the field is due to the conductors only.

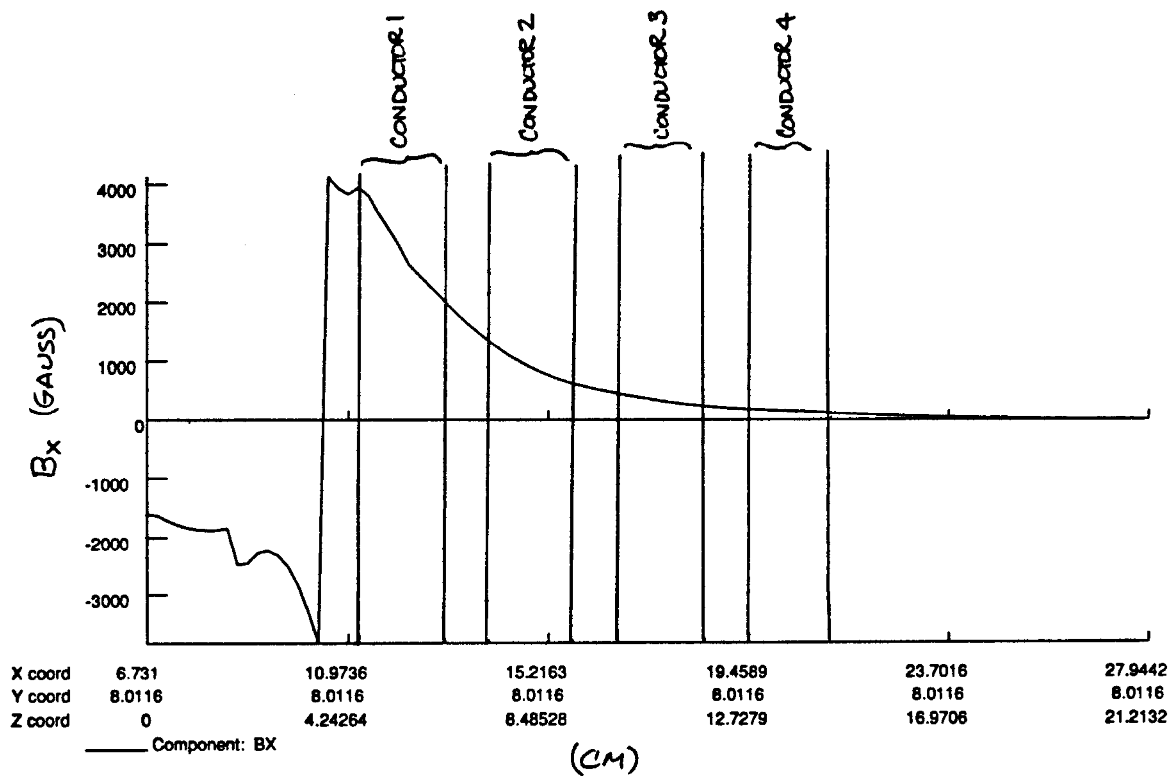

Figure 10: B_x vs position along line "B" in the case where the core is present.

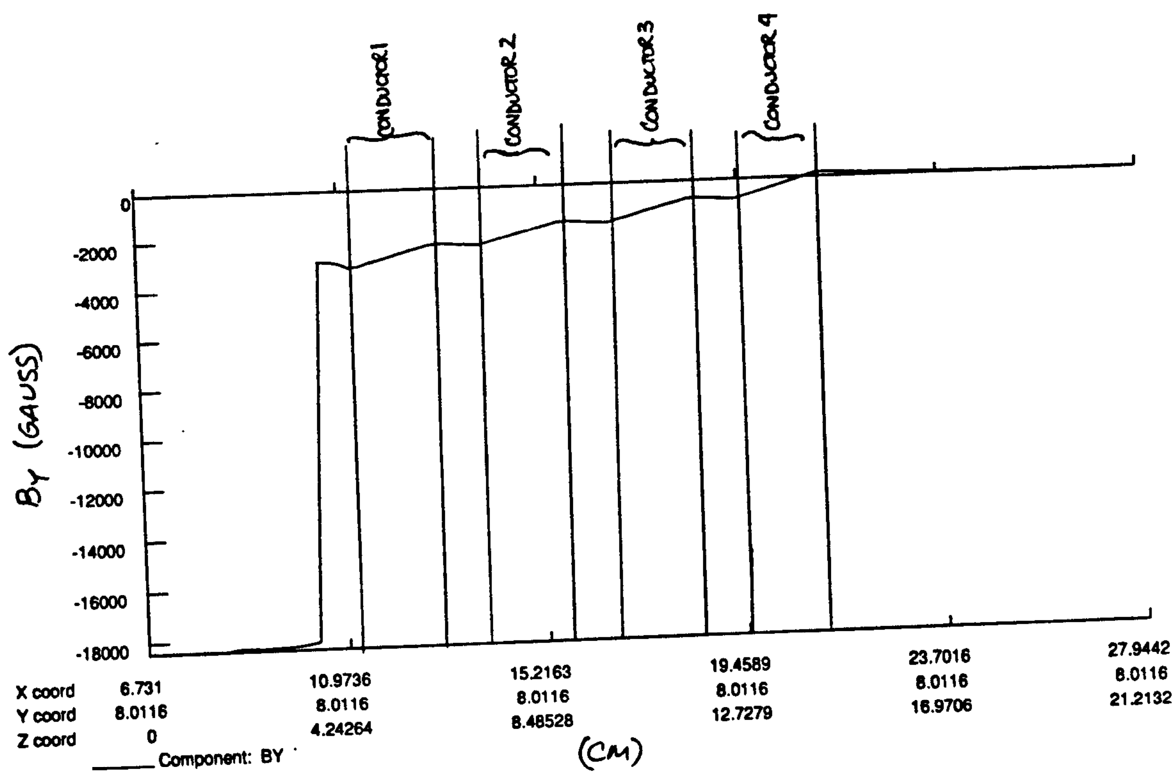

Figure 11: B_y vs position along line "B" in the case where the core is present.