

MI-0048

α_1 DISMAY

JAMES A. MAC LACHLAN

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James A. MacLachlan

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Note MI-0038^[1] calls attention to the difference between expansion of the momentum compaction

$$\alpha_p = (p/r)(\partial R/\partial p) \quad (1)$$

and $\Delta R/R$ in powers of the fractional momentum offset

$$\delta = (p - p_0)/p . \quad (2)$$

The expansion of $\Delta R/R$ is

$$\frac{\Delta R}{R} = \alpha_0 \delta + \alpha_1 \delta^2 + \alpha_3 \delta^3 + \dots \quad (3)$$

or some times

$$\frac{\Delta R}{R} = \alpha_0 \delta (1 + \alpha_1^{(J)} \delta + \dots) , \quad (4)$$

where $\alpha_0 = \gamma_T^{-2}$ and $\alpha_1^{(J)}$ is the "Johnsen parameter" called by him α_2 in his original paper.^[2] MI-0038 labels α_1 as used in eq. 3 the "circumference" definition of α_1 .

So far, so good. The principal point, however, of MI-0038 was to explain the relation of the ALPHA1 parameter used in the program ESME to α_1 and $\alpha_1^{(J)}$. The statement is made that ALPHA1 is the expansion coefficient for α_p , which is calculated from eq. 1 to be

$$\alpha_p = \frac{p}{R} \frac{\partial R}{\partial p} \approx \alpha_0 + (\alpha_0 + 2\alpha_1 - \alpha_0^2) \delta . \quad (5)$$

Unfortunately, this is an error carried over from the ESME user's guide.^[3] In fact,

$\text{ALPHA1} \equiv \alpha_1 \quad (\text{circumference})$
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There are several notes relating to the difference equations which are quite clear that it is the path length that is being expanded.^{[4],[5],[6]} It has also been reasonably clear in the code itself. To reduce the likelihood of future errors, the phase slip per turn has been rewritten in the code almost directly in the form

$$S_i = \frac{\Omega_s}{\Omega_i} = \frac{\beta_s R_i}{\beta_i R_s} , \quad (6)$$

where the Ω 's are orbital angular velocities of the i -th and the synchronous particles and where R_i and R_s are expanded separately about the reference trajectory $R_o = R(p_o)$ in powers of $\delta = (p - p_o)/p$:

$$R(p) = R_o(1 + \alpha_o\delta + \alpha_1\delta^2 + \alpha_3\delta^3 + \dots) . \quad (7)$$

A note updating the ESME user's guide is forthcoming. The purpose of this is to correct the misrepresentation in MI-0038 and apologize to colleagues who may have drawn unwarranted conclusions on the importance of the Johnsen effect in MI or Main Ring.

References

- [1] J. MacLachlan, K. Y. Ng, S. Peggs, "The Definition of α_1 : Circumference, Johnsen, or ESME?", Fermilab Main Injector internal note MI-0038 (October 1990)
- [2] K. Johnsen, in "Proc. of the CERN Symposium on High Energy Accelerators" vol. 1 (1956), p106
- [3] S. Stahl and J. MacLachlan, "User's Guide to ESME v. 7.1", Fermilab internal note TM-1650 (26 February 90)
- [4] J. A. MacLachlan, "Fundamentals of Particle Tracking for the Longitudinal Projection of Beam Phasespace in Synchrotrons", Fermilab note FN-481 (15 April 88)
- [5] J. A. MacLachlan, "Difference Equations for Longitudinal Motion in a Synchrotron", Fermilab note FN-529 (15 December 89)
- [6] J. A. MacLachlan, "Differential Equations for Longitudinal Motion in a Synchrotron", Fermilab note FN-532 (25 January 90)