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# Chromaticity Compensation Sextupole Strengths

S. Alex BOGACZ

- sensitivity matrix $\mathbb{M}$ (2×2 matrix)

- chromaticity vector $\underline{\chi} = \begin{pmatrix} \chi_x \\ \chi_y \end{pmatrix}$ (hor. & vert)

$$\underline{\chi} = \underline{\chi}^0 + S_e \, \underline{e} + \mathbb{M} \begin{pmatrix} S_f \\ S_d \end{pmatrix}$$

($\underline{\chi}^0$ ↑ natural chrom.)

- sextupole strength : $\Delta x' = -\underbrace{(g \, \Delta L)}_{S\,[m^{-2}]} x^2$

$$g = (B\rho)\, S = \frac{p\left[\frac{eV}{c}\right]}{c\left[\frac{m}{s}\right]} S\,[m^{-2}] = [\text{Tesla } m^{-1}]$$

- chromaticity compensation (external sextupoles)

$$\underline{g} = \frac{p}{c} \mathbb{M}^{-1} \left[ \underline{x} - \underline{x}^{0} - \underbrace{\frac{2\pi}{N_{bend}} \left( b_2^{sat} + b_2^{edd} + b_2^{rem} \right) \underline{e}} \right]$$

$$S_e = \Theta_{dip} \cdot b_2^{tot}$$

- sensitivity coefficients for mi_16 (mad8)

$$\mathbb{M} = \begin{pmatrix} 8.93 & 0.971 \\ -1.96 & -4.76 \end{pmatrix} \times 10^{2}$$

$$\underline{e} = \begin{pmatrix} 1.59 \\ -1.49 \end{pmatrix} \times 10^{3}$$

- zero chromaticity compensation

$$\begin{cases} S_f = 4.73 \times 10^{-2} - 1.51 \times S_e \\ S_d = -8.86 \times 10^{-2} - 2.50 \times S_e \end{cases}$$

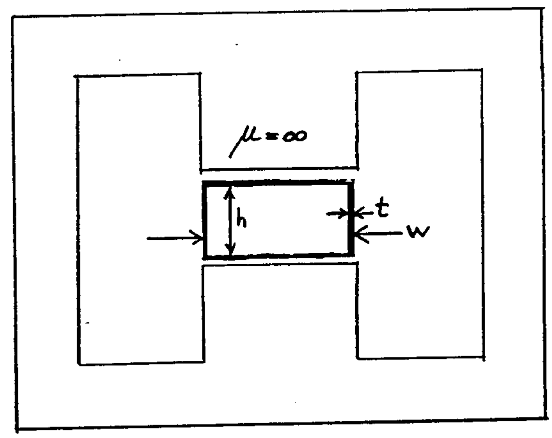
- Eddy current sextupole component (JFO)

**ASSUMPTIONS:**

$$
\begin{aligned}
B_0 &= 0.205 && \text{T} \\
\dot{B}_0 &= 1.725 && \text{T/s} \\
\sigma &= 2.13 \times 10^6 && \Omega^{-1} - \text{m}^{-1} \\
t &= 0.0016 && \text{m} \\
g &= 0.0508 && \text{m} \\
h &= 0.0450 && \text{m}
\end{aligned}
$$

| width $w$ | dipole | sextupole | decapole | 14-pole |
|---|---|---|---|---|
| 0.0300 | −5.827 | +0.9132 | −1.0774 | +0.7525 |
| 0.0325 | −6.427 | +1.1456 | −1.1890 | +0.6798 |
| 0.0350 | −7.053 | +1.3795 | −1.2672 | +0.5815 |
| 0.0400 | −8.384 | +1.8497 | −1.3270 | +0.3581 |
| 0.0450 | −9.821 | +2.2978 | −1.2810 | +0.1573 |
| 0.0600 | −14.766 | +3.3589 | −0.8657 | −0.1221 |
| 0.0700 | −18.573 | +3.8108 | −0.5814 | −0.1313 |
| 0.0800 | −22.771 | +4.1024 | −0.3609 | −0.1006 |
| 0.0900 | −27.345 | +4.2995 | −0.2265 | −0.0675 |
| → 0.1000 | −32.294 | +4.4138 | −0.1359 | −0.0425 |
| 0.1100 | −37.604 | +4.4823 | −0.0804 | −0.0257 |
| 0.1200 | −43.275 | +4.5225 | −0.0471 | −0.0153 |
| 0.1300 | −49.303 | +4.5452 | −0.0274 | −0.0089 |
| 0.1400 | −55.689 | +4.5574 | −0.0158 | −0.0052 |
| 0.1500 | −62.431 | +4.5630 | −0.0091 | −0.0030 |

**NOTE: ALL DIMENSIONS IN MKS UNITS. MULTIPOLES IN NORMALIZED UNITS $\times 10^4$ @ 2.54 cm = 1 inch.**

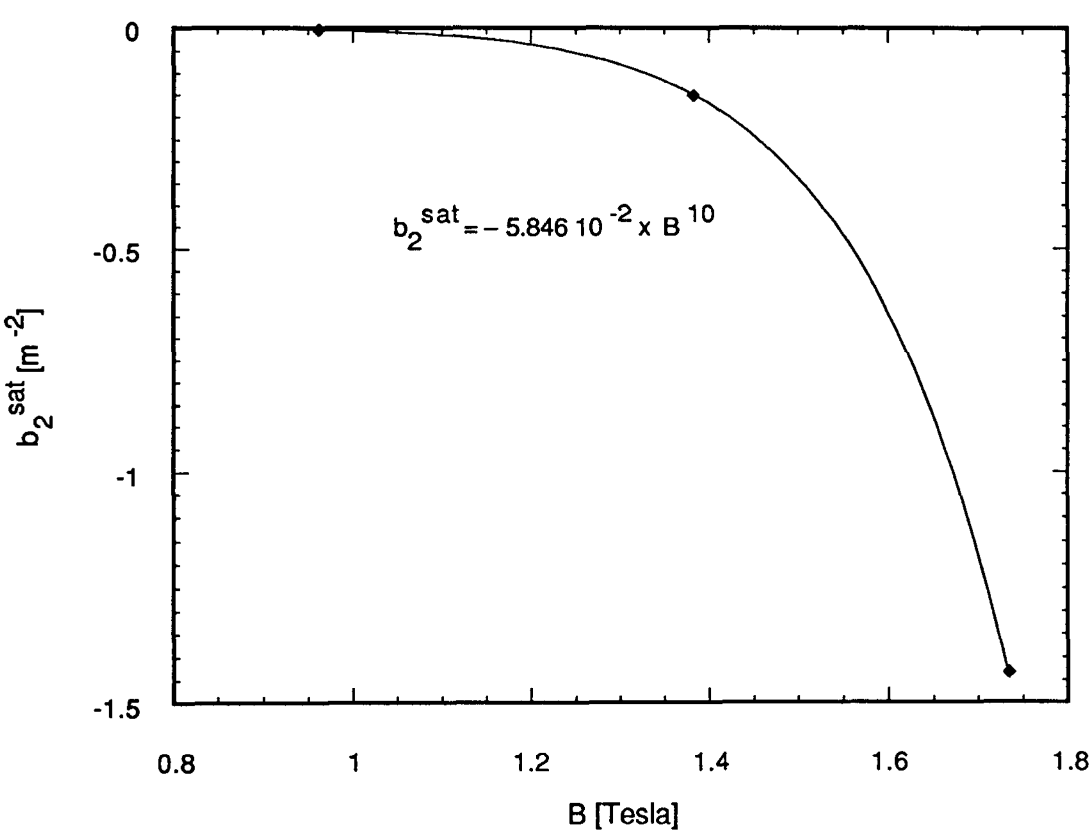



- linear approximation

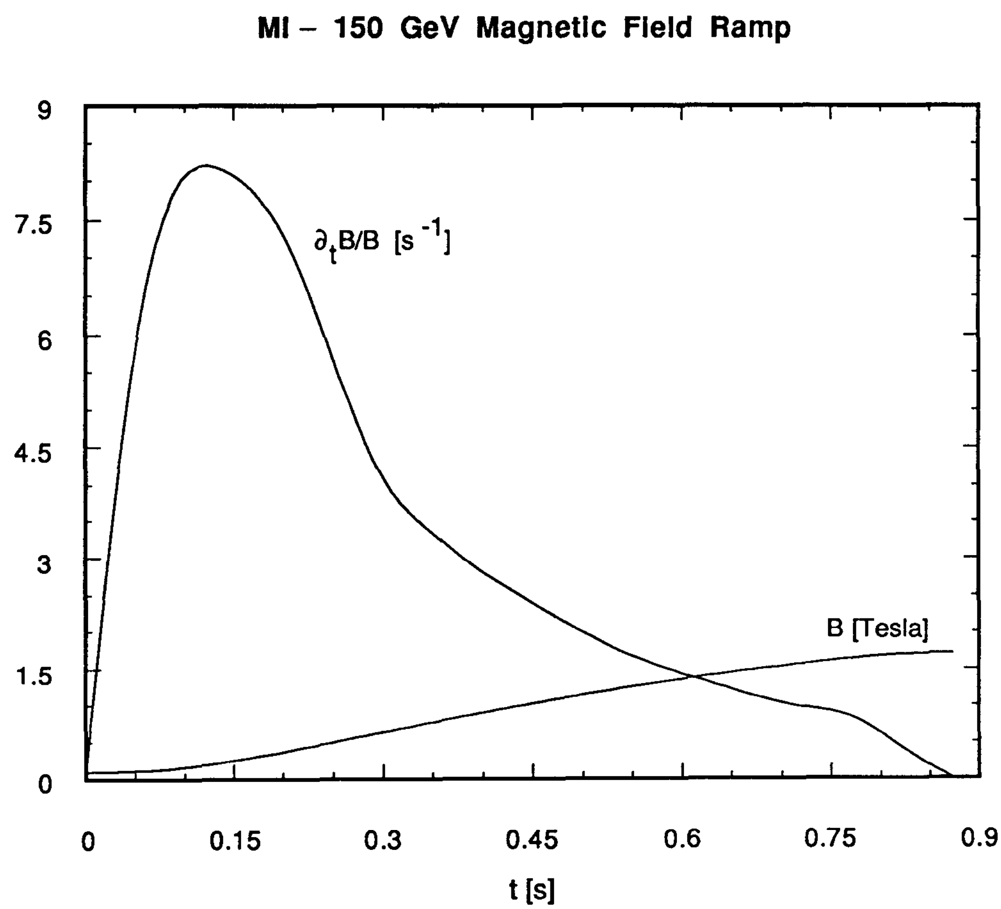
$$b_2^{edd} = 8.128 \times 10^{-2} \; \frac{\dot{B}_0}{B_0} \, [s^{-1}] = [m^{-2}]$$

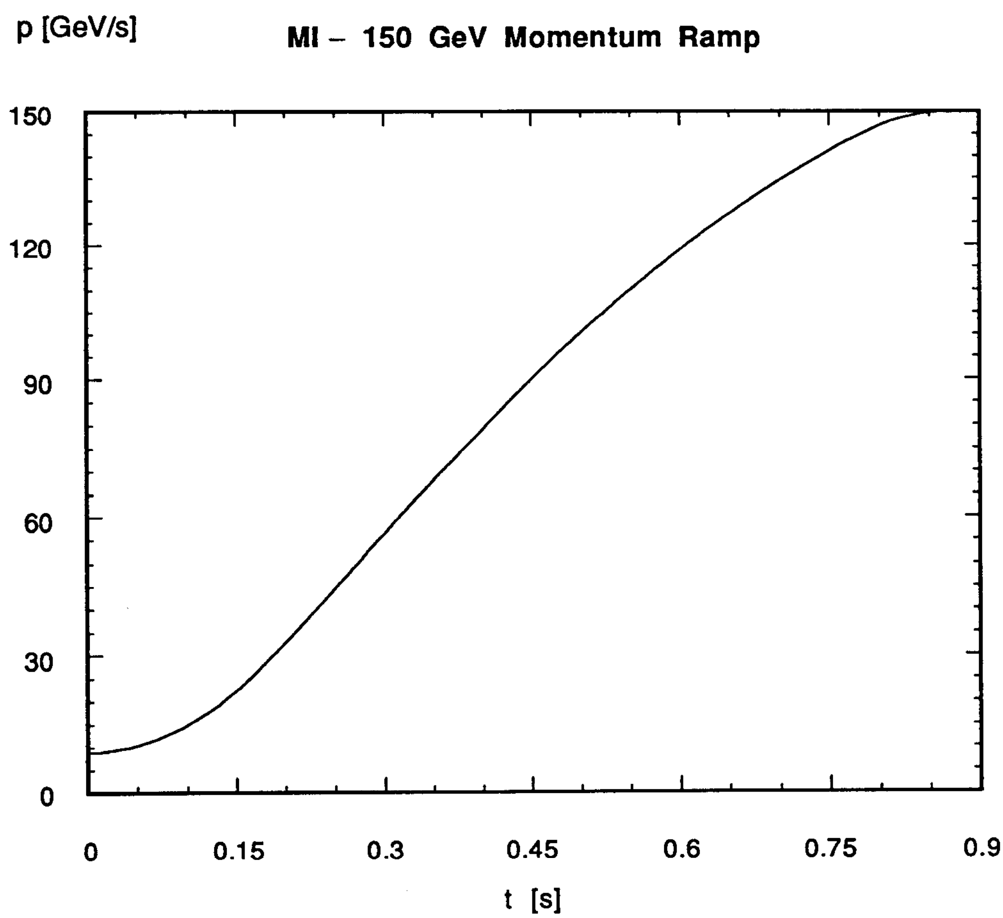
| MAIN INJECTOR DIPOLE<br>NORMAL MULTIPOLES AT $r = r_0 = 2.54$ cm<br>ARMCO steel laminations<br>LINEAR EXTRAPOLATION<br>Dipole Field in Tesla<br>Higher Harmonics in relative units $\times 10^4$ | | | | |
|---|---|---|---|---|
| | $I = 502$ A<br>$J = 19.4525$ A/cm$^2$ | $I = 4900$ A<br>$J = 189.875$ A/cm$^2$ | $I = 7100$ A<br>$J = 275.125$ A/cm$^2$ | $I = 9417$ A<br>$J = 364.909$ A/cm$^2$ |
| pole | | | | |
| 2 | +0.09925 | +0.96187 | +1.38247 | +1.73467 |
| 6 | +0.22003 | −0.02544 | −0.97478 | −9.24396 |
| 10 | +0.18395 | +0.15194 | −0.04830 | −1.29193 |
| 14 | +0.04032 | +0.00523 | −0.01839 | −0.15969 |

**MI – Sextupole Saturation in a Dipole Magnet**



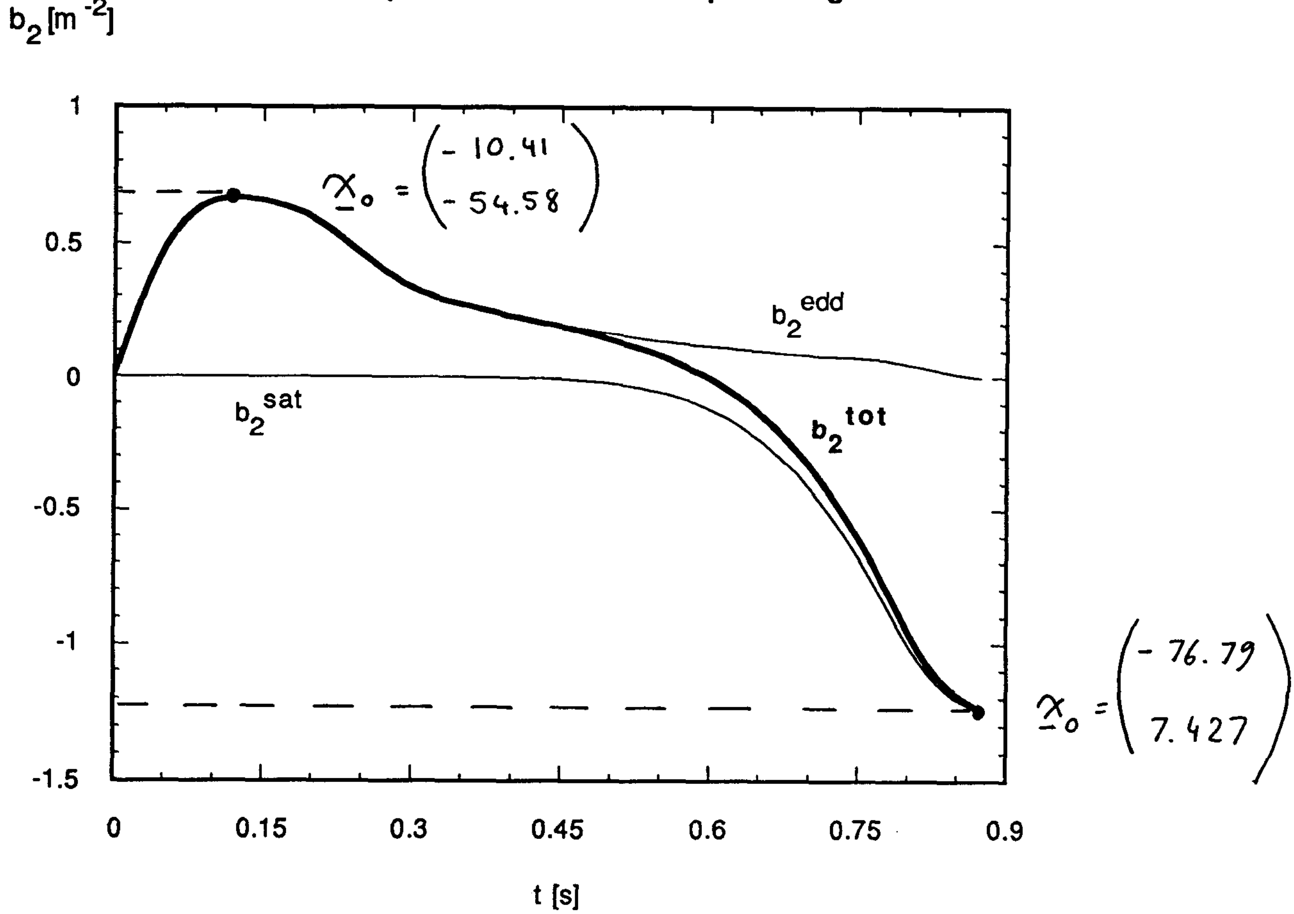

data from JFO


MI – 150 GeV Magnetic Field Ramp
∂tB/B [s -1]
B [Tesla]
9
7.5
6
4.5
3
1.5
0
0
0.15
0.3
0.45
0.6
0.75
0.9
t [s]



p [GeV/s]
MI – 150 GeV Momentum Ramp
150
120
90
60
30
0
0
0.15
0.3
0.45
0.6
0.75
0.9
t [s]

**MI – Sextupole Content of a Dipole Magnet**




- natural chromaticities (hor. & vert) at extreme values of the sextupole components in a dipole magnet (mad 8)