Measurement of the Momentum Dependance of Orbit Length in the Main Ring

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ABSTRACT

The second order dependence of orbit length on momentum offset from the central orbit is inferred from the transition time for radially offset beam. The transition time is found by minimization of the shape oscillation of the bunches after transition; because the minimum is not distinct and the available momentum aperture is limited, the result is scarcely better than order of magnitude. We find

$$\alpha_1 = \frac{1}{2}\frac{d^2}{d\delta^2}\frac{\Delta R}{R}\bigg|_{\delta=0} = 2.13\cdot 10^{-3} \pm 30\% \qquad (\delta = \Delta p/p) \ .$$

1. Introduction

The behavior of beam bunches as they are accelerated through the transition energy is a matter of practical interest for obtaining beam of the highest intensity or brightness with the least loss. Furthermore, understanding which of several effects peculiar to energies near transition dominate the growth of bunch area may be useful in designing better accelerators.

The principal parameters governing the beam dynamics in the energy-phase plane for a beam near transition energy are listed in Table I with calculated and measured values for the Main Ring where known. The parameter α_1, which gives the lowest order nonlinear dependence of path length on momentum,

$$\frac{\Delta R}{R} = \alpha_0\frac{\Delta p}{p} + \alpha_1\left(\frac{\Delta p}{p}\right)^2 + \cdots \ ,$$

has not been previously reported. The values for γ_T and the associated momentum p_T, $\dot{p}_T$, *etc.* are derived from the ramp parameters displayed on page M3 and the observed transition time for centered beam. The values are given in the table because they are needed to determine α_1, but they are not intended to be best effort

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measurements. There are good ways to get γ_T and perhaps better measurements have been made.

At transition the first order dependance of circulation period on momentum vanishes; *i.e.*,

$$\eta = \alpha_o - \frac{1}{\gamma^2} = \frac{1}{\gamma_{T0}^2} - \frac{1}{\gamma^2} \equiv 0 \ .$$

The next order in η is $\Delta\eta = \eta_1 \Delta p/p$ where

$$\eta_1 = \alpha_1 - \alpha_o^2 + \alpha_o \gamma_s^{-2} + \frac{3}{2}\beta_s^2 \gamma_s^{-2}$$

and β_s and γ_s are the Einstein-Lorentz kinematic parameters v/c and $E/m_o c^2$ for the synchronous particle.[1] Here α_1 and the last term in parentheses are the principal terms; indeed at transition

$$\eta_1 = \alpha_1 + \frac{3}{2}\beta_T^2 \alpha_o \ .$$

Likewise, other parameters, some frequently taken as constants, have a strong momentum dependance near transition. For example, the transition energy itself depends on momentum:

$$(\gamma_T(\Delta p) - \gamma_{T0})/\gamma_{T0} = (\alpha_o/2 - \alpha_1/\alpha_o - 1/2)\Delta p/p_T \ ,$$

where p_T is $(\gamma_{T0}^2 - 1)^{1/2} m_o c$. The range of γ_T resulting from the momentum distribution within a bunch results in a distribution of transition times. The interval either side of the nominal transition time during which there are particles both above and below transition is usually called the Johnsen time[2] or nonlinear time:

$$t_J = \frac{\gamma_{T0}}{\dot{\gamma}}(3/2 + \alpha_1/\alpha_o - \alpha_o/2)\frac{\Delta p}{p} \ .$$

For large emittance bunches the rf phase is wrong for a significant fraction of the particles over a non-negligible time; the result is a disruption of the bunch shape and eventual growth of the effective emittance independent of the beam current. One can see that if $\alpha_1 \approx -3\alpha_o/2$, the nonlinear time is zero; for this special value the entire bunch passes transition at the same time.

We are aware of no previous attempt to measure α_1 for the Main Ring. Sho Ohnuma speculated (1982) that $\alpha_1 \approx 0.14\alpha_o$ with Main Ring dipole systematic sextupole error included and the natural chromaticity cancelled.[3] Dejan Trbojevič calculated γ_T for various off-momentum orbits in the range $\Delta p/p = \pm 0.001$ using SYNCH; the results are plotted in Fig. 1. They may be used to calculate $\alpha_1 = 0.572\alpha_o$ using the above expression for $\gamma_T(\Delta p)$. The lattice contained sextupoles to correct natural chromaticity but no eddy current or bending magnet contribution. However, Bill Ng has compared SYNCH values for $\gamma_T(p)$ to analytic calculations for a simple FODO lattice; he reports disagreement.[4] The SYNCH result is probably nearly correct; it does not have a high confidence level, however.

2. Experimental Program

If one can determine the transition crossing times for beams of different momenta with sufficient accuracy, the expression for t_J can be used to calculate α_1. The basic idea is to introduce a radial position offset during the period including the transition time t_T. Figure 2 shows a 200 Hz fast time plot of ROFF, RFSUM, and PHIS. The transition time is marked by the jump in PHIS. The t_T value is the setting for the PHIS jump for which the bunchwidth shows minimum oscillation after the jump. The corresponding momentum offset is found by differencing the closed orbits with and without the radial offset. To reasonable accuracy this difference is proportional to the dispersion X_p; the proportionality constant is the relative momentum offset $\Delta p/p$. One would want in general to check this procedure at the energy of interest by measuring the rf frequency. However, near transition that is not a sensitive test. Below it will be seen that the typical error in $\Delta p/p$ at other energies is so small compared to the error in the t_T that it is reasonable to neglect the momentum calibration.

The measurement is crude because the determination of the minimum in the shape oscillation is a qualitative judgement on a complicated beam current signal. There appears to be more than one significant effect on the bunches so that the oscillation can change its qualitative character without clear change in amplitude. A significant contributor to the fuzzyness of the signal is a momentum spread of O(0.1 %) within the bunch itself so that changes in t_T corresponding to smaller momentum steps are washed out. Because the Main Ring in its current incarnation has momentum aperture of only about 0.3 % full width, the range of the controlled variable is very limited.

The basic idea of measuring α_1 from t_T has been described by Boussard.[5]

$$t_T = -\left(\frac{3}{2} + \frac{\alpha_1}{\alpha_0}\right)\frac{\gamma_T}{\dot{\gamma}}\frac{\Delta p}{p} = -\left(\frac{3}{2}\alpha_0 + \alpha_1\right)\frac{\gamma_T^3}{\dot{\gamma}}\frac{\Delta p}{p}$$

where now $\Delta p/p$ is the momentum offset on the radially displaced orbit and $\dot{\gamma}$ is taken as a constant during the nonlinear time. As mentioned, the theme is to adjust the transition switch timing looking for a setting that minimizes the shape oscillations after that time.

3. Measurements

The values of t_T and relative momentum offset are given in Table II and plotted in Fig. 3. The errors on the momentum are representative of experience at other momenta and the errors in transition time are a representation of the result of a couple of remeasures plus the observation of the breadth of the minimum being sought. The transition time for zero radial offset is used along with the ramp parameters from page M3 to measure γ_T and $\dot{\gamma}$ which are needed to derive α_1 from the transition times. This is not intended to be a best measurement of γ_T; there are more refined techniques available. It is possible that γ_T is not exactly the nominal value. Both nominal and observed values are reported in Table I. The observations are not precise; it may take hard work or a good idea to get significantly refined data. Reducing the momentum spread of the bunches by collimating in the 8 GeV line is a possible improvement on the reported procedures. However, this requires dedicated study time to set up the collimation, match the new bunch shape, and make the observations. The study being reported was conducted on two and one half hours of borrowed 29 cycles during Tevatron studies.

3. Analysis

The line in Fig. 2 is a linear fit with a slope

$$\frac{\Delta t_T}{\Delta(\Delta p/p)} = -0.4863 \text{ ms/per mil} \quad .$$

The expression for the nonlinear time gives

$$\frac{\Delta t_T}{\Delta(\Delta p/p)} = -(\frac{3}{2}\gamma_T^{-2} + \alpha_1)\frac{\gamma_T^3}{\dot{\gamma}} \quad .$$

The only unknown quantity is α_1; for γ_T and $\dot{\gamma}$ one can take either the nominal values in Table I or the values observed in this study. It happens that for the precision of this study the result is the same with either:

$$\begin{aligned} \alpha_1 &= 0.796\alpha_o = 2.13 \cdot 10^{-3} \pm 30\% (\text{measured } \gamma_T) \\ &= 0.800\alpha_o = 2.28 \cdot 10^{-3} \qquad (\text{nominal } \gamma_T) \quad . \end{aligned}$$

These values agree within error to the value $\alpha_1 = 0.76\alpha_o$ reported informally (Kourbanis, MacLachlan) in the days immediately following the study. The slight difference here arises from a least squares fit rather than an "eyeball" fit to the transition time *vs.* momentum. The SYNCH result of $\alpha_1 = 0.572\alpha_o$ also fits barely within our generous error estimate.

Acknowledgements

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References

1. J. Wei, "Longitudinal Dynamics of the Non-Adiabatic Regime on Alternating Gradient Synchrotrons", thesis, SUNY Stony Brook (May 1990) p10

2. K. Johnsen, *Proc. CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (CERN, Geneva, 1956) Vol. 1, p106

3. S. Ohnuma, priv. comm. (a Sho Note) (27 July 1982)

4. K. Y. Ng, priv. comm.

5. D. Boussard, "Study of the beam loss at transition", SPS Improvement Report no. 106, CERN internal note (22 September 1977)

Table I: Main Ring Parameters Near Transition

	Nominal	Measured	Calculated	Comments
R [m]	1000			meas. prob. exists
$\gamma_T = \alpha_0^{-\frac{1}{2}}$	18.75	19.32		good meas. may exist
p_T [GeV/c]	17.5676	18.1		from t_T and p. M3
$\dot{p}_T$ [GeV/c/s]	83.3244	85.7		
$\ddot{p}_T$ [GeV/c/s^2]	400.000			
h	1113			
$V(t_T)$ [MV]	1.746	2.15		RFSUM
t_T [s]	0.208311	0.229		after T2
α_1		$2.13 \cdot 10^{-3}$	$1.63 \cdot 10^{-3}$	calc. SYNCH
			$0.40 \cdot 10^{-3}$	calc. analyt.

Table II: Transition time *vs.* Momentum Offset

ROFF	$\frac{\Delta p}{p}$[°/₀₀] ±0.2	t_T [s] ±0.004
1.0	0.80	0.3787
0.0	0.00	0.3790
-2.0	-1.52	0.3795
-3.0	-1.92	0.3802

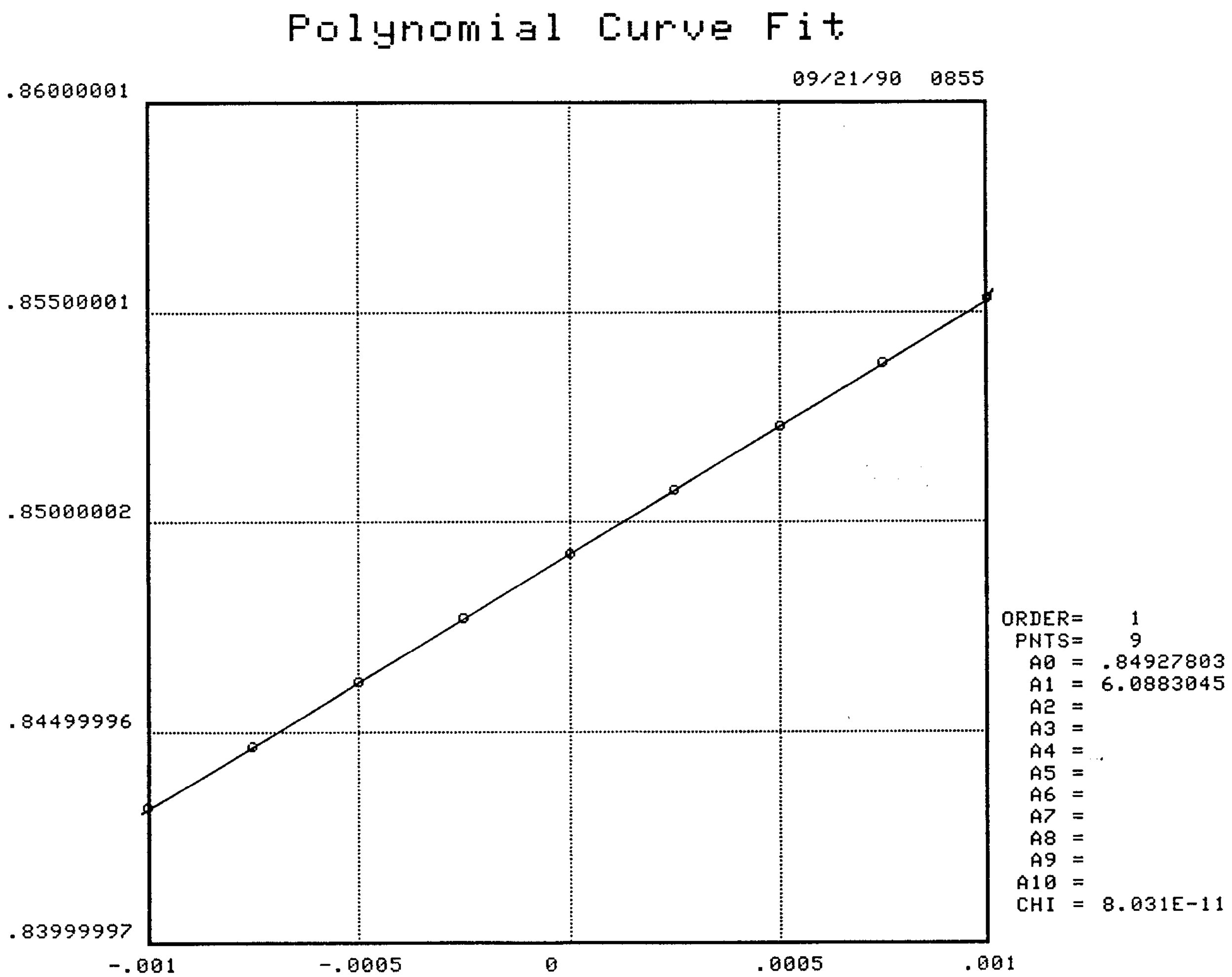

Figure 1: Fit of SYNCH Values for γ_T *vs.* $\frac{\Delta p}{p}$

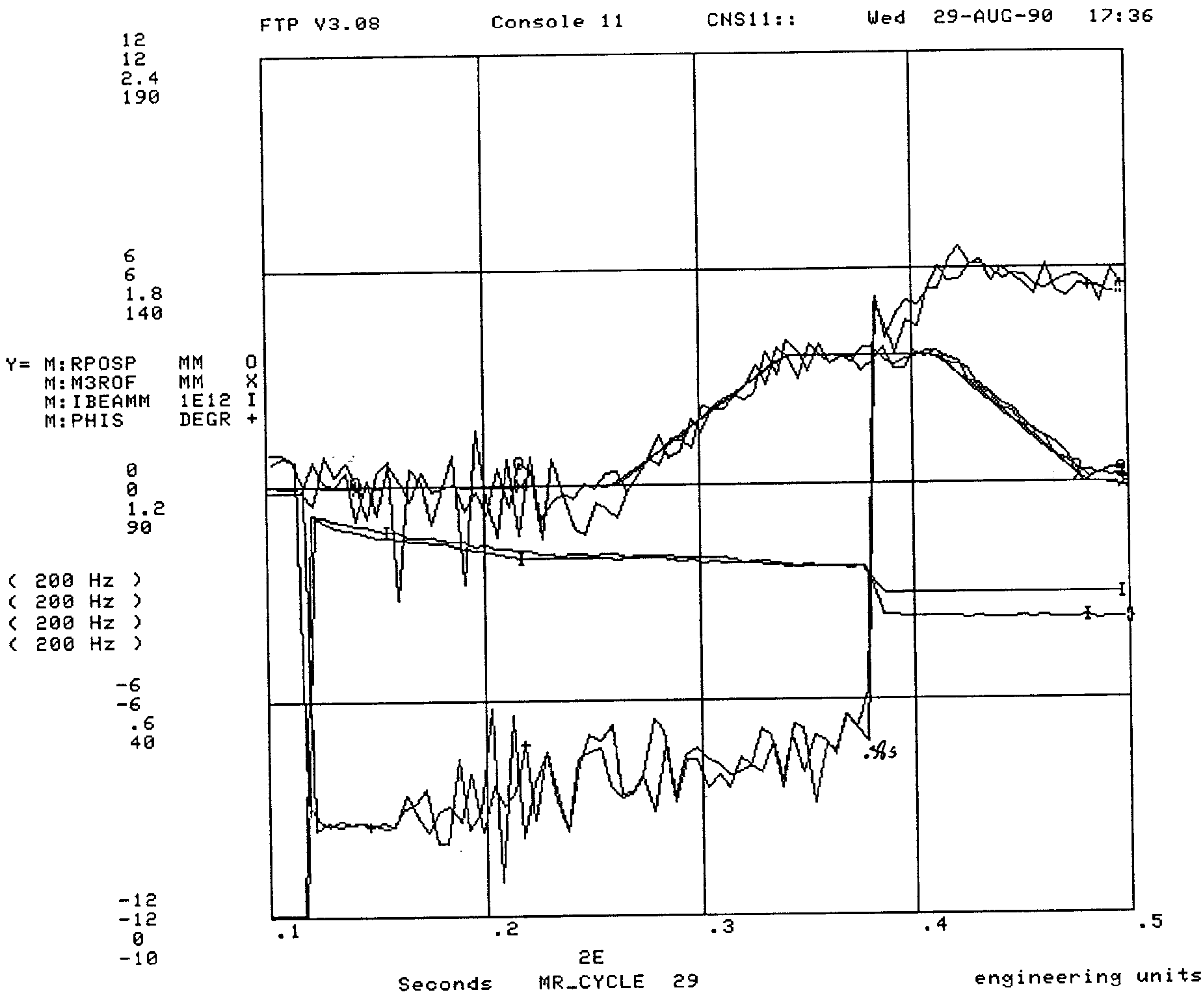

Figure 2: Radial position offset (ROFF = 1): M3ROF, RPOSP, IBEAM, and PHIS *vs.* time in seconds

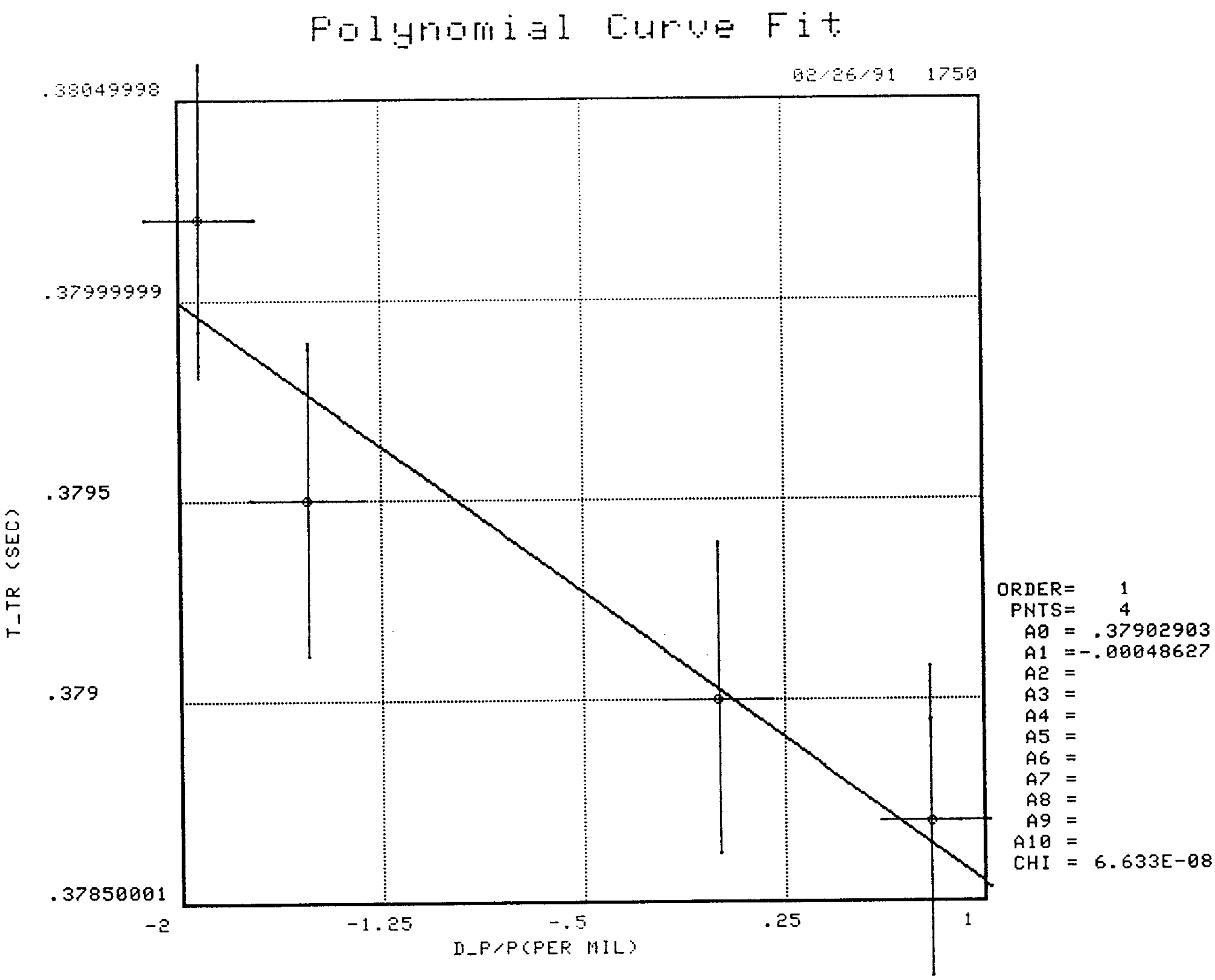

Figure 3: Measured transition time *vs.* $\frac{\Delta p}{p}$ with linear fit