BEAM LOADING EFFECTS FOR A PARTIALLY FILLED MI RING

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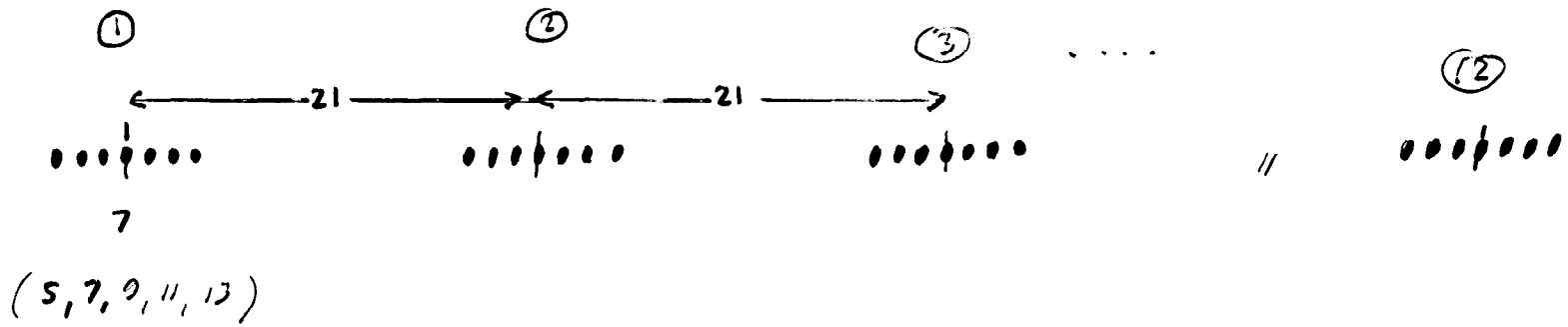
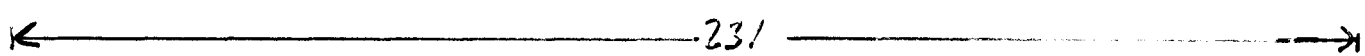
- Analytic expression for the beam loading force acting on the n-th bunch in case of a general resonant impedance (FN-611)

- MR cavity impedance measurement (fundamental + higher parasitic modes) (Ping)





$588 = 21 \times 28$

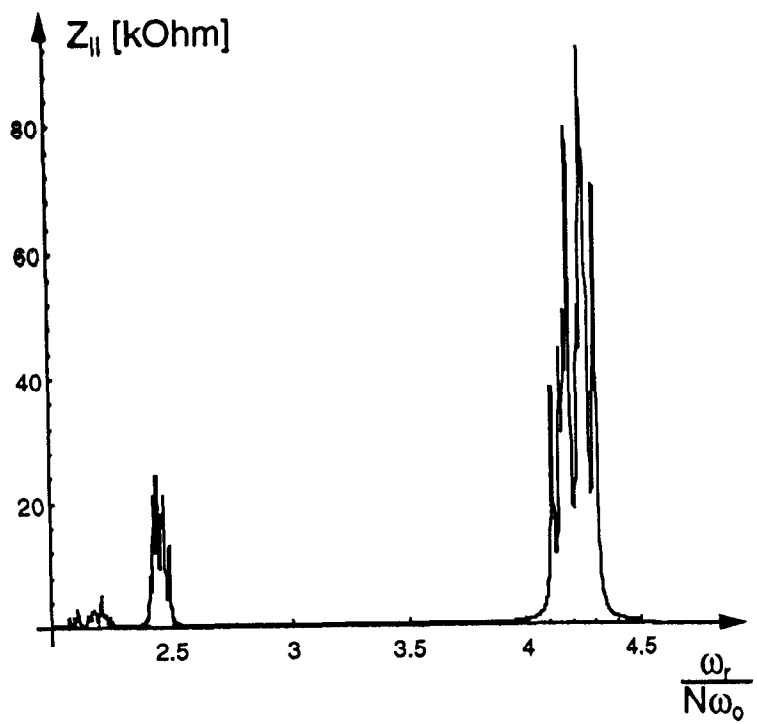
- two sampling scales – partial symmetry preserved

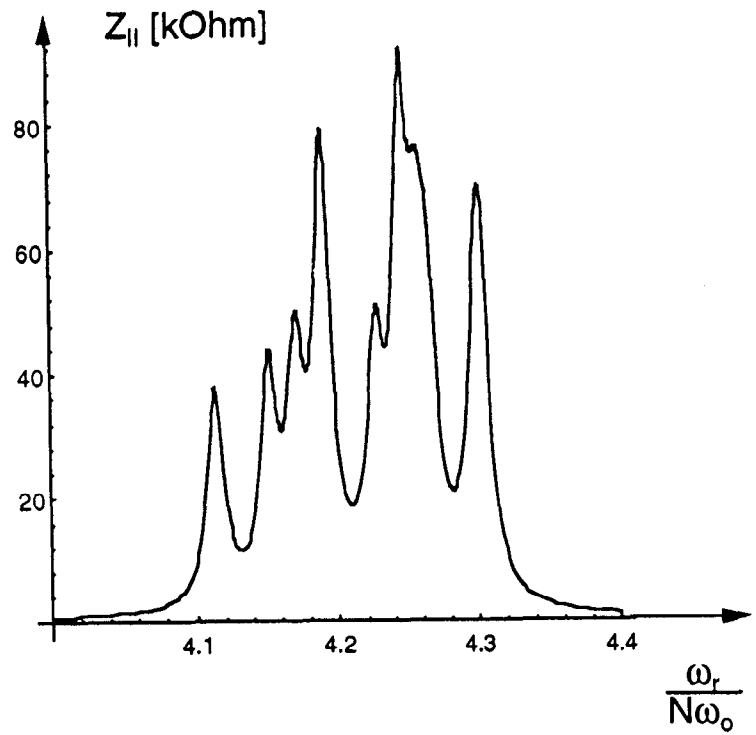
→ $N = 588$ $M = 7$

→ $n = 28$ $m = 12$

Main Ring Cavities (17) - Parasitic Modes

Q = 300





- superposition of sharp Lorentzian peaks

$\frac{1}{20}$ $\quad$ $1/5$ $\quad$ 1

$R/Q = 1400$

$Q \sim 300$

$R = 4.2 \times 10^5$ Ohm

	some of MR cavity resonant modes				
cavity number	mode 1(Mhz)	signal(Db)	mode 2(Mhz)	signal(Db)	mode 3(Mhz)
1	115.5		130.1		226
2	116.6		130.8		222
3	N/A		132		220
4	**117.1**		130.8		228
5	112		129.3		N/A
6	118		127.7		224
7			**EMPTY CAVITY**		
8	119.1		132		227.8
9	**117.3**		N/A		225
10	111.3		129.1		222
11	**117.3**		N/A		218
12	114.6		129.3		N/A
13	**117.3**		131.1		221
14	110		N/A		225.6
15	112		128.5		N/A
16	115.7		130.7		225
17	115.3		130.4		N/A
18	114.1		129.3		N/A
	↑ 16		↑ 14		↑ 12

- fundamental mode

53 MHz)

$Q \approx 2000$ $\quad$ $R/Q = 1.4 \times 10^3$

$R = 2.8 \times 10^6$ Ohm

COUPLED MULTI-BUNCH MOTION

We assume a storage ring of a harmonic number N populated by a train of M consecutive bunches, where $M \leq N$.

$$\frac{\partial^2}{\partial t^2} y_n(t) + \omega^2 y_n(t) = A \sum_{k=-\infty}^{\infty} \sum_{m=0}^{M-1} W'\left(-(k + \tfrac{m-n}{N})T_0 + \tfrac{1}{c}\left[y_n(t) - y_m\left(t - (k + \tfrac{m-n}{N})T_0\right)\right]\right) ,$$

$$A = \frac{N_0 r_0 \eta \omega_0}{2\pi\gamma} .$$

$$\frac{\partial^2}{\partial t^2} y_n(t) + \omega^2 y_n(t) = A \sum_{k=-\infty}^{\infty} \sum_{m=0}^{M-1} W'\left(-(k + \tfrac{m-n}{N})T_0\right) +$$

$$+ A \frac{1}{c} \sum_{k=-\infty}^{\infty} \sum_{m=0}^{M-1} W''\left(-(k + \tfrac{m-n}{N})T_0\right) \left[y_n(t) - y_m\left(t - (k + \tfrac{m-n}{N})T_0\right)\right] .$$

$$* \quad f_n = A \sum_{k=-\infty}^{\infty} \sum_{m=0}^{M-1} W'\left(-\left(k + \frac{m-n}{N}\right)T_0\right).$$

$$\omega_n^2 = \omega^2 - \Delta\omega_n^2$$

$$\Delta\omega_n^2 = A \frac{1}{c} \sum_{k=-\infty}^{\infty} \sum_{m=0}^{M-1} W''\left(-\left(k + \frac{m-n}{N}\right)T_0\right)$$

$$\frac{\partial^2}{\partial t^2} y_n(t) + \left(\omega^2 - \Delta\omega_n^2\right) y_n(t) = f_n +$$

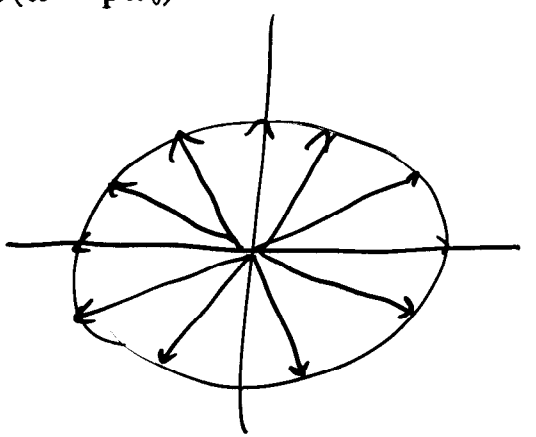
$$- A \frac{1}{c} \sum_{k=-\infty}^{\infty} \sum_{m=0}^{M-1} W''\left(-\left(k + \frac{m-n}{N}\right)T_0\right) y_m\left(t - \left(k + \frac{m-n}{N}\right)T_0\right),$$

$$\begin{cases} w(t) = c\,\dfrac{R}{Q}\,\epsilon^{-t\frac{\omega_r}{2Q}}\,\sin\omega_r t \\ \updownarrow \\ Z(\omega) = \dfrac{R}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)} \end{cases}$$

BEAM LOADING – FULL VS PARTIALLY FILLED RING

$$W'(t) = \frac{c}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{i\omega t}\, Z_{\parallel}(\omega) \quad .$$

$$f_n = \frac{Ac}{2\pi}\sum_{k=-\infty}^{\infty}\sum_{m=0}^{M-1}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega(k+\frac{m-n}{N})T_0}\, Z_{\parallel}(\omega) \ , \qquad \sum_{k=-\infty}^{\infty} e^{-i\omega k T_0} = \omega_0 \sum_{p=-\infty}^{\infty}\delta(\omega - p\omega_0)$$



$$f_n = \omega_0\,\frac{Ac}{2\pi}\sum_{p=-\infty}^{\infty}\sum_{m=0}^{M-1} Z_{\parallel}(p\omega_0)\ e^{2\pi i p\frac{n-m}{N}} \ ,$$

$$(f_n)_{full} = \omega_0\,\frac{Ac}{2\pi}\,N\sum_{q=-\infty}^{\infty} Z_{\parallel}(Nq\omega_0) \ ,$$

BEAM LOADING

$$f_n = \omega_0 \frac{Ac}{2\pi} \sum_{p=-\infty}^{\infty} e^{2\pi i (p\omega_0)\frac{n}{N\omega_0}} \quad Z_{\parallel}(p\omega_0) \; \frac{1 - e^{-2\pi i (p\omega_0)\frac{M}{N\omega_0}}}{1 - e^{-2\pi i (p\omega_0)\frac{1}{N\omega_0}}} = \sum_{p=-\infty}^{\infty} F(p\omega_0)$$

$$\sum_{p=-\infty}^{\infty} F(p\omega_0) = \sum_{q=-\infty}^{\infty} \frac{1}{\omega_0} \int_{-\infty}^{\infty} d\omega \; e^{2\pi i q \frac{\omega}{\omega_0}} \; F(\omega) \; ,$$

$$f_n = iAc \sum_{q=-\infty}^{\infty} \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \; Z_{\parallel}(\omega) \; \frac{e^{2\pi i (q + \frac{n}{N}) \frac{\omega}{\omega_0}} - e^{2\pi i (q - \frac{M-n}{N}) \frac{\omega}{\omega_0}}}{1 - e^{-2\pi i \frac{\omega}{N\omega_0}}} \; .$$

$$I^{+}(k) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \; Z_{\parallel}(\omega) \; \frac{e^{2\pi i k \frac{\omega}{N\omega_0}}}{1 - e^{-2\pi i \frac{\omega}{N\omega_0}}} \quad , \qquad k \geq 0$$

$$I^{-}(k) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \; Z_{\parallel}(\omega) \; \frac{e^{-2\pi i k \frac{\omega}{N\omega_0}}}{1 - e^{-2\pi i \frac{\omega}{N\omega_0}}} \quad , \qquad k \geq 0$$

$$f_n = iAc \left\{ I^{+}(n) - I^{-}(k) + \sum_{q=1}^{\infty} \left[I^{+}(Nq + n) + I^{-}(Nq - n) + \right.\right.$$

$$\left.\left. - I^{+}(Nq - (M - n)) - I^{-}(Nq + (M - n)) \right] \right\} \quad ,$$

$$Z_{\parallel}(\omega) = \frac{R}{1 + iQ \left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega} \right)} \quad ,$$

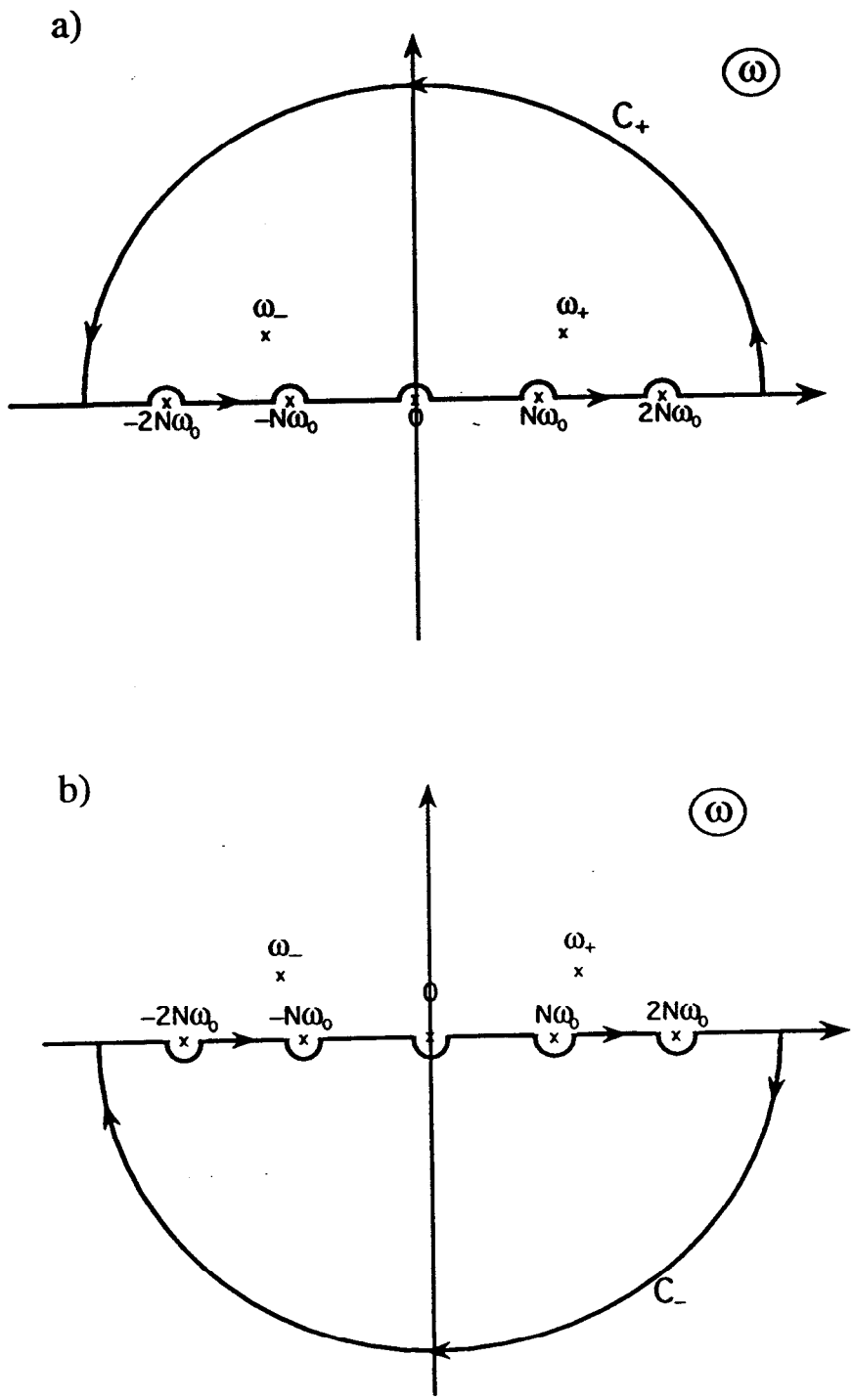


Figure 5

$$* \qquad Z_{\parallel}(\omega) = -iR\,\frac{\omega_r}{Q}\,\frac{\omega}{(\omega - \omega_+)\,(\omega - \omega_-)} \quad ,$$

$$\omega_{\pm} = \omega_r(\pm 1 + i\delta), \quad \omega_- = -\,\omega_+^{*}\,, \quad \delta = \frac{1}{2Q} \quad .$$

$$I^{+}(k) = \frac{1}{2}\,\omega_0\,\frac{1}{2\pi i}\,N \sum_{p=-\infty}^{\infty} Z_{\parallel}(Np\omega_0) - i\,\frac{R}{2Q}\left[\frac{\omega_+\,e^{2\pi i k \frac{\omega_+}{N\omega_0}}}{1 - e^{-2\pi i \frac{\omega_+}{N\omega_0}}} - \frac{\omega_-\,e^{2\pi i k \frac{\omega_-}{N\omega_0}}}{1 - e^{-2\pi i \frac{\omega_-}{N\omega_0}}}\right] ,$$

$$I^{-}(k) = -\frac{1}{2}\,\omega_0\,\frac{1}{2\pi i}\,N \sum_{p=-\infty}^{\infty} Z_{\parallel}(Np\omega_0) \quad ,$$

$$\sum_{q=1}^{\infty} e^{2\pi i q \frac{\omega_\pm}{\omega_0}} = \frac{e^{2\pi i k \frac{\omega_\pm}{\omega_0}}}{1 - e^{2\pi i \frac{\omega_\pm}{\omega_0}}} ,$$

- $$f_n = \omega_0 \frac{Ac}{2\pi} N \sum_{p=-\infty}^{\infty} Z_{\parallel}(Np\omega_0) +$$

$$+ Ac \frac{R}{2Q} \left[\frac{\omega_+ e^{2\pi i n \frac{\omega_+}{N\omega_0}} \left(1 - e^{2\pi i (N-M) \frac{\omega_+}{N\omega_0}}\right)}{\left(1 - e^{2\pi i \frac{\omega_+}{N\omega_0}}\right)\left(1 - e^{2\pi i \frac{\omega_+}{\omega_0}}\right)} - \frac{\omega_- e^{2\pi i n \frac{\omega_-}{N\omega_0}} \left(1 - e^{2\pi i (N-M) \frac{\omega_-}{N\omega_0}}\right)}{\left(1 - e^{2\pi i \frac{\omega_-}{N\omega_0}}\right)\left(1 - e^{2\pi i \frac{\omega_-}{\omega_0}}\right)} \right] .$$

- $$(f_n)_{full} = \frac{AcR\omega_0}{\pi} \left(\frac{\pi\omega_+}{N\omega_0}\right) N \delta^2 \frac{\left(\frac{\pi\omega_+}{N\omega_0}\right) - \sin\left(\frac{\pi\omega_+}{N\omega_0}\right)\cos\left(\frac{\pi\omega_+}{N\omega_0}\right)}{\sin^2\left(\frac{\pi\omega_+}{N\omega_0}\right) + \left(\frac{\pi\omega_+}{N\omega_0}\right)^2 \delta^2} .$$

FINALLY

$$* \quad f_n = \frac{AcR\omega_o}{\pi} \left\{ \left(\frac{\pi\omega_I}{N\omega_o}\right) N \ \delta^2 \ \frac{\left(\frac{\pi\omega_I}{N\omega_o}\right) - \sin\left(\frac{\pi\omega_I}{N\omega_o}\right)\cos\left(\frac{\pi\omega_I}{N\omega_o}\right)}{\sin^2\left(\frac{\pi\omega_I}{N\omega_o}\right) + \left(\frac{\pi\omega_I}{N\omega_o}\right)^2 \delta^2} + \right.$$

$$\left. - \frac{(N - M)\left(\frac{\pi\omega_I}{N\omega_o}\right)^2 \delta^2 + \sin\left(\frac{2\pi\omega_I}{N\omega_o}\right)\sin\left(\frac{2\pi\omega_I}{N\omega_o} M\right)\cos\left(\frac{2\pi\omega_I}{N\omega_o}(2n - M - 1)\right)}{\sin^2\left(\frac{\pi\omega_I}{N\omega_o}\right) + \left(\frac{\pi\omega_I}{N\omega_o}\right)^2 \delta^2} \right\}$$

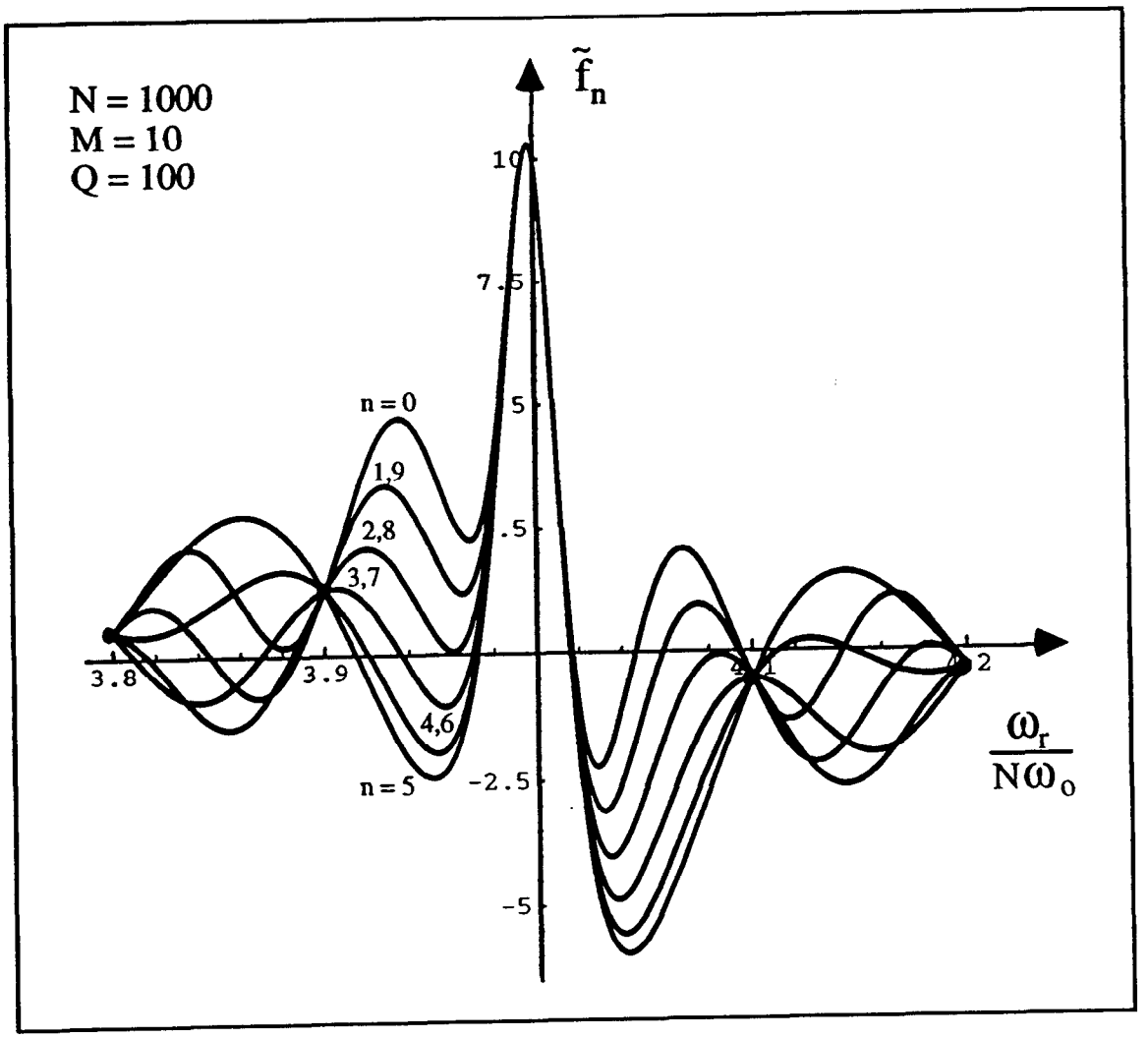


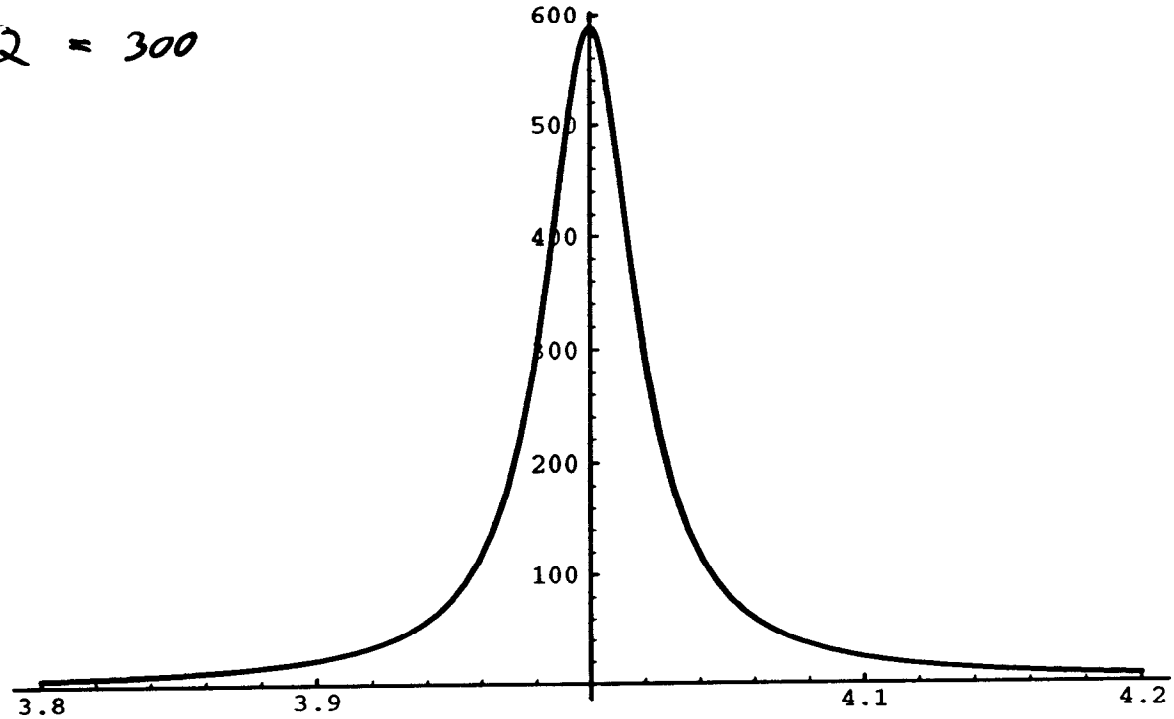
Figure 1

$$\omega_r = \frac{N\omega_0}{M}$$

$$\delta\omega \sim \frac{1}{Q^2}$$

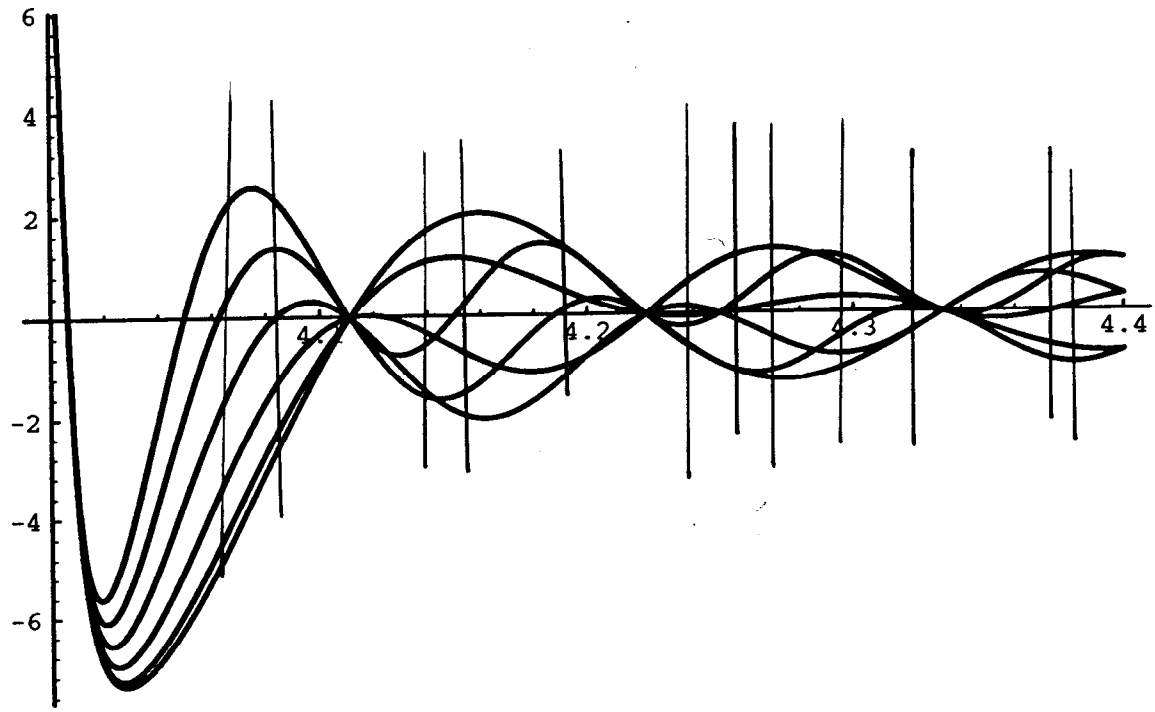
$\tilde{f}_{full}$

$Q = 300$



$\frac{\omega_r}{N\omega_0}$

9 • $\tilde{f}_n$ $Q = 300$



$\frac{\omega_r}{N\omega_0}$