MI-0114
Eddy Currents in the Main Injector Dipole Magnet Coils

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Introduction

The purpose of this note is to address some concerns expressed by the review panel (in particular Klaus Halbach) at the March 1994 review of the FMI.

The Main Injector has been designed to be a faster ramping machine than the existing Main Ring (240 GeV/s vs 120 GeV/s). Furthermore, stronger dipole magnets are used since the MI has a smaller circumference (28/55) than the Main Ring. In order to minimize cost and accommodate these requirements, the MI dipole magnet inductance has been reduced by doubling the conductor area and halving the number of turns. A cross-section of the main injector dipole magnet is shown in Figure 1.

A side-effect of the increased copper area and higher ramping rate is to enhance eddy currents. The following questions were therefore raised by the review panel :

- Can the non-uniform spatial distribution current density resulting from the presence of eddy currents have a deleterious effect on field quality ?
- Does the current redistribution cause a significant increase in the effective resistance of the coil ?
- Does the time-dependent inductance associated with this current distribution need to be accounted for in the power supply design ?

Field Quality

Although in some dipole magnet designs conductor position is used to control field quality, this is not the case for the main injector magnets: there is no conductor in the midplane; field quality depends exclusively on the pole profile. Figure 1 compares the calculated field homogeneity in the dipole magnet

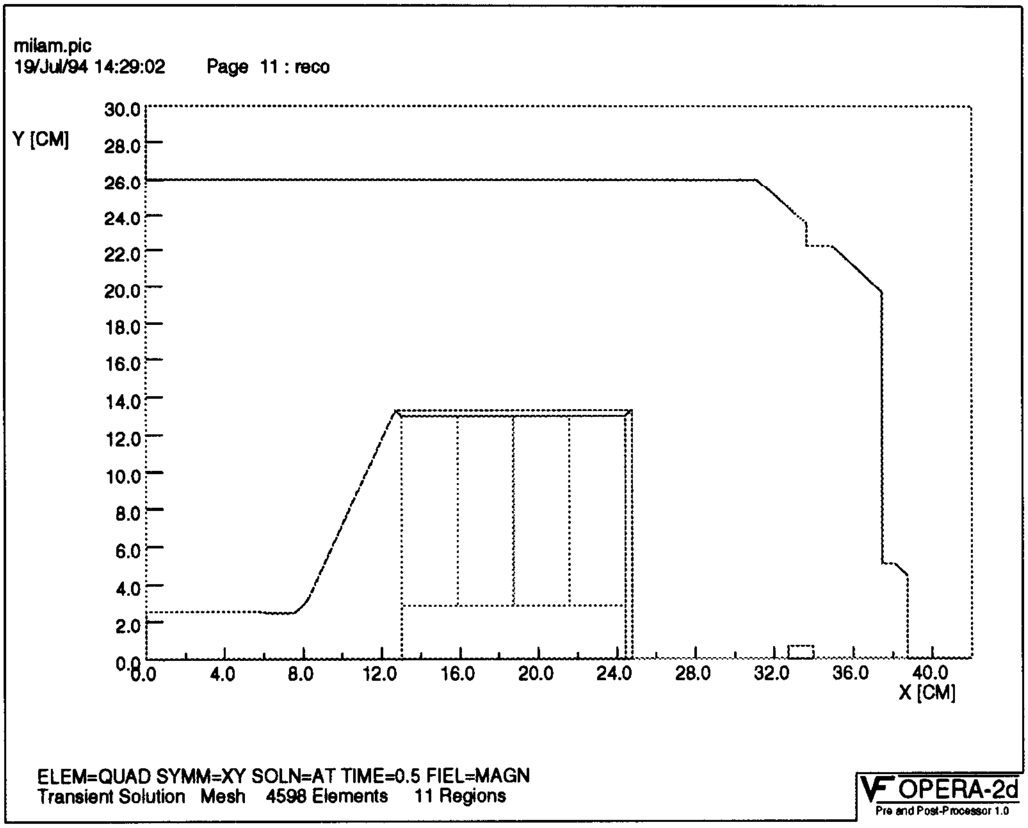

Figure 1: *1/4 of the the main injector dipole cross-section. There are 4 turns per coil.*

midplane for the case where current density is the same in all 4 conductors to the case where the current density is (arbitrarily) redistributed in conductors 1 2 3 and 4 (from left to right) in the proportions 1.5:1.5:0.5:0.5. A constant relative permeability of 300 (a value typical of the average permeability in the magnet near maximum excitation) is assumed. No significant change in the field homogeneity is observed at the 10^{-4} level, even for such a dramatically uneven current distribution.

For completeness, a numerical calculation of the current density distribution in the conductor cross-section was performed using the transient solver module of the standard finite element code PE2D. The code solves the diffusion equation

$$\nabla^2 A_z - \sigma\mu\mu_0 \frac{\partial A_z}{\partial t} = \mu_0 J_s \tag{1}$$

where A_z is the magnetic vector potential and J_s represents current supplied by an external source. The total current in each conductor I is prescribed

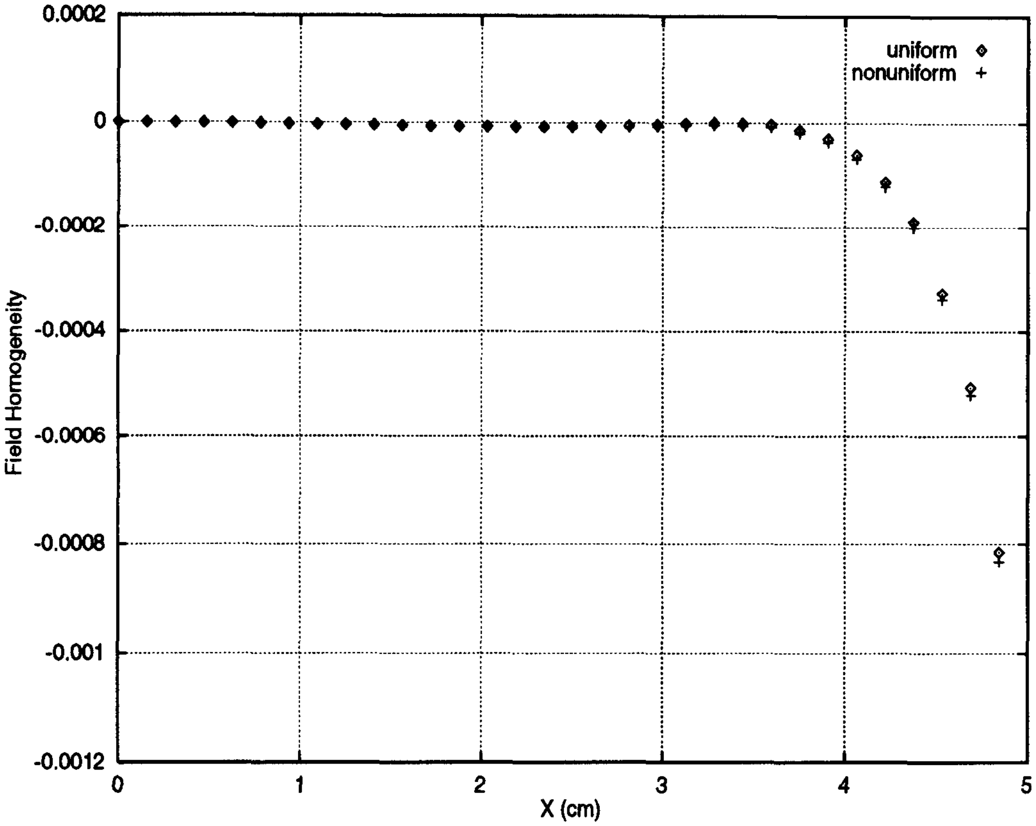

Figure 2: *Field homogeneity in the dipole magnet midplane assuming a constant permeability of 300. For the "non-uniform" case, the current in each conductor has been distributed (arbitrarily) in the proportions 1.5:1.5:0.5:0.5 (from left to right). No significant change in field homogeneity is observed.*

and the following constraint is enforced for each conductor:

$$I = \int J_s + \sigma \frac{\partial A_z}{\partial t} dxdy \tag{2}$$

i.e. the net circulating eddy current is zero in each conductor. From a circuit standpoint, we are assuming that the magnet is fed by a high impedance current source. In practice, of course, the power supply delivers a specified voltage. To the extent that the load impedance is known, a voltage ramp can be specified so as to produce the required current. In the first approximation, the magnet impedance is purely inductive and

$$I(t) = \frac{1}{L} \int V(t)dt \tag{3}$$

Thus if $V(t)$ is linear, $I(t)$ is parabolic.

Figure 2 represents contours of the relative deviation from uniformity of the current distribution and the corresponding field homogeneity plot at $t = 0.30$ s. The maximum eddy current density is on the order of 250 A/cm^2 and occurs near the bottom left edge of the innermost conductor. Not surprisingly, there is no significant field quality degradation; the effect is an increase on the order of a fraction of a part in 10^4 in the sextupolar component of the field at 2.54 cm. The effect is larger at injection, but is very small relatively to the sextupole component due to the eddy currents induced in the vacuum chamber. In this simulation, the resistivity of the coil was set to $\rho = 1.71$ $\mu\Omega$-cm the excitation assumed to be as shown in Figure (up ramp 14615 A/s i.e approximately 230 GeV/s) and the total current specified independently in each one of the four conductors. As discussed above, dI/dt varies along the ramp. The assumed excitation results in a slight overestimation of the eddy current magnitude.

Time-dependent Impedance

Due to the presence of eddy currents, the current density is not uniform and this results in an increase in power dissipation. A time-dependent resistance $R(t)$ can be calculated as follows:

$$P(t) = \int \frac{1}{\sigma} J^2(x, y, t)dxdy = I(t)^2 R(t) \tag{4}$$

where

$$I(t) = \int J(x, y, t)dxdy \tag{5}$$

is the net current. In the same manner, the current distribution causes a small change in the energy stored magnetic energy. The time-dependent inductance $L(t)$ can be extracted from the stored magnetic energy W_M

$$L(t) = \frac{2W_M(t)}{I^2(t)} \tag{6}$$

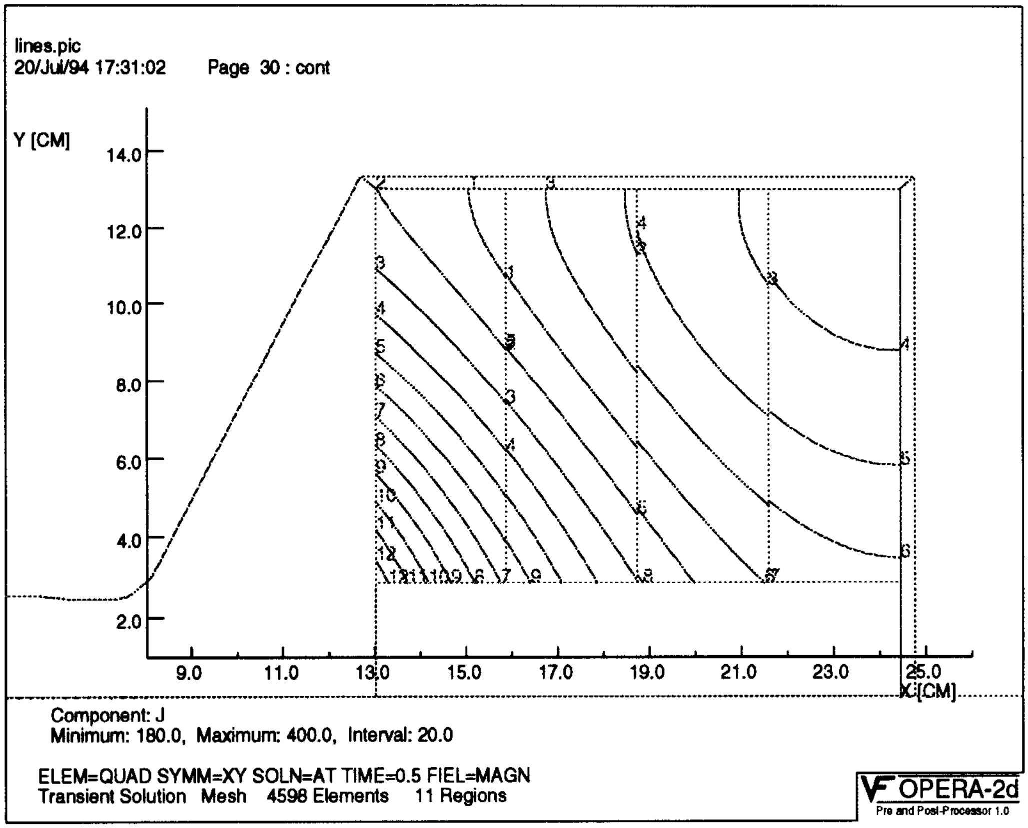

Figure 3: *Contours of equal current density at t = 0.30 s. A total of 12 contour lines are shown. The current density varies from 180 to 400 A/cm^2. The current is 4413 A, corresponding to an average current density of 152 A/cm^2. The relative permeability was arbitrarily set to 5000.*

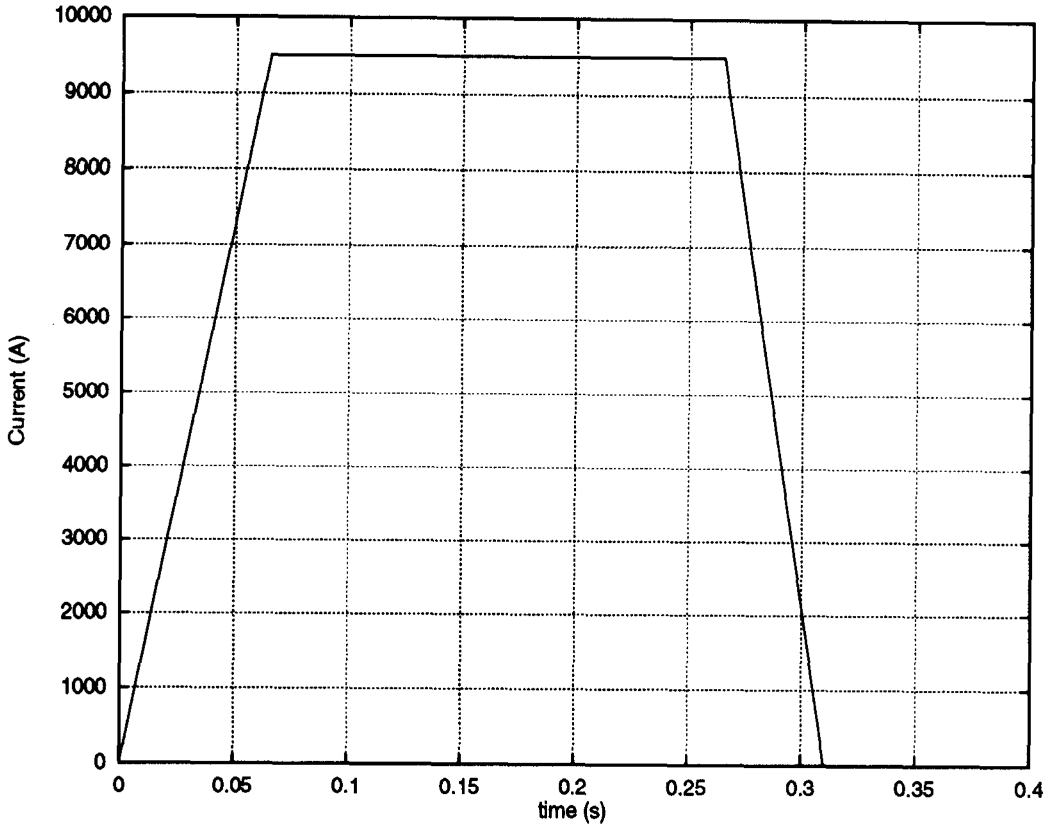

Figure 4: *Excitation used in the simulation.*

Time (s)	Resistance R ($\mu\Omega$/cm)	Inductance L (μH/cm)
static	0.942	3.84
$t = 0.1$	1.312	3.77
$t = 0.3$	0.994	3.78
$t = 0.5$	0.961	3.81
$t = 1.0$	0.942	3.84

Table 1: *Resistance and inductance at different instants along the excitation ramp, assuming the excitation curve shown in Figure 2. The values are for the entire cross-section.*

where

$$W_M(t) = \int \left[\int H(t) dB(t) \right] dxdy \tag{7}$$

is the total stored magnetic energy.

The values of R and L at different instants along the excitation curve are presented in Table 1. The inductance variations are very small. This is to be expected since roughly 95% of the total field energy is located in the gap region. The inductance is lower than the static value since, in the first approximation, the eddy currents reduce the magnetic field in the conductor region without affecting it elsewhere. The effect of eddy currents on power dissipation is a bit more important. Since in this simulation the eddy currents are the same at $t = 0.1$ s and at $t = 1.0$ s, the resistance tends to be higher at low current [1]. Both L and R are essentially equal to their static values at $t = 1$ s since the eddy current magnitude is then negligible compared with the excitation current. The time-dependence of R and L should probably be accounted for in the power supply design; however the effect is very small and no serious problems are expected.

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[1] The current ramp used in the simulation is a simplification. In reality, the ramp starts at approximately 500 A (8 GeV).