Beam Heating of Vacuum Windows

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1. Introduction & Summary:

The question is: "Will we break the windows?". The answer, in short, is no.

2. Proton Energy Deposition:

The total energy lost by a single proton incident normal to a thin window is approximately:

$$\Delta E = \int_0^{\Delta z} dz \cdot \frac{\partial E}{\partial z} \approx \Delta z \cdot \frac{\partial E}{\partial z}$$

where Δz is the window thickness and $\partial E/\partial z$ is evaluated at the incident energy. The energy lost by a heavy charged projectile results primarily from ionization of the target atoms. For ultra-relativistic protons incident on a composite target of partial densities ρ_n, electronic charges Z_n, and mass numbers A_n, $\partial E/\partial z$ is[1]:

$$\frac{\partial E}{\partial z} \approx D \sum_n \frac{\rho_n Z_n}{A_n} \left[\ell n\left(\frac{2 m_e \gamma}{I_n} \right) - 1 \right]$$

with the ionization energy $I_n \approx 16\, Z_n^{0.9}$ eV and $D = 0.3070$ MeV-cm^2/gm.

The transverse proton density can be approximately characterized by the bi-Gaussian distribution:

$$\rho(x,y;t) = \frac{N}{2\pi\sigma_x\sigma_y} \cdot e^{-x^2/2\sigma_x^2} \cdot e^{-y^2/2\sigma_y^2} \cdot \Omega(t)$$

with ρ normalized to N protons/pulse, and $\Omega(t)$ describes the temporal evolution of the density.

In the absence of dispersion, the half-widths of the beam in the transverse planes are related to the momentum $\beta\gamma$, normalized (95%) emittances ε, and betatron amplitudes $\beta_{x,y}$ by:

$$\sigma_x = \sqrt{\frac{\beta_x \varepsilon_x}{6\pi\beta\gamma}} \qquad \text{and} \qquad \sigma_y = \sqrt{\frac{\beta_y \varepsilon_y}{6\pi\beta\gamma}}$$

Furthermore, if the vacuum window is located such that $\sigma_x = \sigma_y = \sigma$, cylindrical symmetry can be invoked to simplify the proton distribution on the window to:

$$\rho(r;t) = \frac{N}{2\pi\sigma^2} \cdot e^{-r^2/2\sigma^2} \cdot \Omega(t)$$

Ignoring the small increase in beam size due to multiple scattering, the energy deposited at some point (r,φ,z) is simply proportional to the beam density at that point. The energy deposited is:

$$\hat{\rho}_E(r,\varphi,z;t) = \frac{N}{2\pi\sigma^2} \cdot \frac{\partial E}{\partial z} \cdot e^{-r^2/2\sigma^2} \cdot \Omega(t)$$

Generalizing results obtained in subsequent sections to describe asymmetric density distributions poses merely a technical complication; it does not qualitatively alter any physics issues.

3. Dispersion of Heat in the Vacuum Window:

Ignoring radiative heat loss from the window (which in fact is the dominant process at high temperatures, $\Delta E \propto T^4$), the temperature distribution evolves according to the diffusion equation:

$$\frac{\partial T}{\partial t} = \frac{\hat{\rho}_E}{\rho c_p} + \frac{1}{\rho c_p} \cdot \vec{\nabla}^2(\kappa T)$$

where c_p is the specific heat, κ the thermal conductivity, and ρ the density of the window material.

Assuming, further, that thermal conductivity κ is independent of location, and ignoring the small variation in temperature across the thickness of the window, the diffusion equation in cylindrical coordinates reduces to:

$$\frac{\partial T}{\partial t} = \frac{\hat{\rho}_E}{\rho c_p} + \frac{\kappa}{\rho c_p} \cdot \frac{\partial}{r\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

With the approximation that κ and c_p are also taken to be independent of temperature, the diffusion equation has a simple analytic solution. The symmetry of the distribution recommends that this solution be obtained via Bessel transforms. Transforms of $T(r,t)$ and its Laplacian are generated as:

$$\tau(\alpha,t) \equiv \int_0^\infty dr \cdot r \cdot J_o(\alpha r) \cdot (T(r,t) - T_o) \quad \text{and} \quad \therefore \quad -\alpha^2\tau(\alpha,t) = \int_0^\infty dr \cdot r \cdot J_o(\alpha r) \cdot \vec{\nabla}^2(T(r,t) - T_o)$$

where T_o is the temperature distribution at time t=0 (assumed constant), and also the asymptotic $r \to \infty$ distribution $T(r,t)$ for all times.

The energy deposition density transforms according to:

$$\int_0^{\infty} dr \cdot r \cdot J_0(\alpha r) \cdot \hat{\rho}_E(r,\varphi,z;t) = \frac{N}{2\pi} \cdot \frac{\partial E}{\partial z} \cdot e^{-\alpha^2\sigma^2/2} \cdot \Omega(t)$$

The second-order partial differential diffusion equation is thereby transformed into the first-order differential equation in time:

$$\frac{\partial \tau(\alpha,t)}{\partial t} = -\frac{\alpha^2 \kappa}{\rho c_p} \cdot \tau(\alpha,t) + \frac{N}{2\pi\rho c_p} \cdot \frac{\partial E}{\partial z} \cdot e^{-\alpha^2\sigma^2/2} \cdot \Omega(t)$$

From the convolution properties of Laplace transforms, the generic solution for $\tau(\alpha,t)$ is obtained:

$$\tau(\alpha,t) = \frac{N}{2\pi\rho c_p} \cdot \frac{\partial E}{\partial z} \cdot e^{-\frac{\alpha^2\sigma^2}{2}} \cdot \int_0^t dt' \cdot e^{-\frac{\alpha^2\kappa}{\rho c_p} \cdot t'} \cdot \Omega(t-t')$$

The spatial temperature distribution is generated from the corresponding inverse Bessel transform:

$$T(r,t) - T_0 = \int_0^{\infty} d\alpha \cdot \alpha \cdot J_0(\alpha r) \cdot \tau(\alpha,t)$$

$$\boxed{\Rightarrow T(r,t) = T_0 + \frac{N}{2\pi\rho c_p \sigma^2} \cdot \frac{\partial E}{\partial z} \cdot \int_0^t dt' \cdot \frac{e^{-r^2/2\sigma^2\left(1+\frac{2\kappa}{\rho c_p \sigma^2} \cdot t'\right)}}{\left(1+\frac{2\kappa}{\rho c_p \sigma^2} \cdot t'\right)} \cdot \Omega(t-t')}$$

A physically realistic model of the heating process is one in which the temporal distribution Ω is approximated by a δ-function every τ seconds, representing an impulse of N protons incident on the window every cycle. The evolution of temperature with time in this case is solved to be:

$$T_{IP}(r,t) = T_0 + \frac{N}{2\pi\rho c_p} \cdot \frac{\partial E}{\partial z} \cdot \sum_{j=0}^{\eta} \frac{e^{-r^2/2\sigma^2\left(1+\frac{2\kappa}{\rho c_p \sigma^2} \cdot (t-j\tau)\right)}}{\left(1+\frac{2\kappa}{\rho c_p \sigma^2} \cdot (t-j\tau)\right)}$$

where τ is the cycle time, η is the total number of pulses since time zero, and the time t lies in the interval $\eta\tau \leq t < (\eta+1)\tau$.

Every τ seconds an impulse of N protons produces an instantaneous temperature boost ΔT, which subsequently diffuses with time. This change ΔT each cycle is:

$$\Delta T(r, t' = j\tau) = \frac{N}{2\pi \rho c_p \sigma^2} \cdot \frac{\partial E}{\partial z} \cdot e^{-r^2/2\sigma^2}$$

The preceding treatment provides the most realistic analytic description of beam heating, and is the only justifiable approach if either the instantaneous temperature rise is 'comparable' to the melting temperature of the window material, or if the cycle time τ is 'short' relative to the rate of diffusion. Apart from these extremes, however, a model in which the δ-function approximation to Ω is replaced by its time-average is more readily interpretable, yet still contains all the essential physics.

For a continuous wave of $\hat{N} \equiv N/\tau$ protons/second the temperature distribution becomes:

$$T_{CW}(r,t) = T_o + \frac{\hat{N}}{4\pi\kappa} \cdot \frac{\partial E}{\partial z} \cdot \left\{ E_1\left(\frac{r^2}{2\sigma^2(1 + 2\kappa t / \rho c_p \sigma^2)} \right) - E_1\left(\frac{r^2}{2\sigma^2} \right) \right\}$$

where E_1 is the exponential integral function: $E_1(x) \equiv \int_x^\infty d\alpha \cdot \frac{e^{-\alpha}}{\alpha}$.

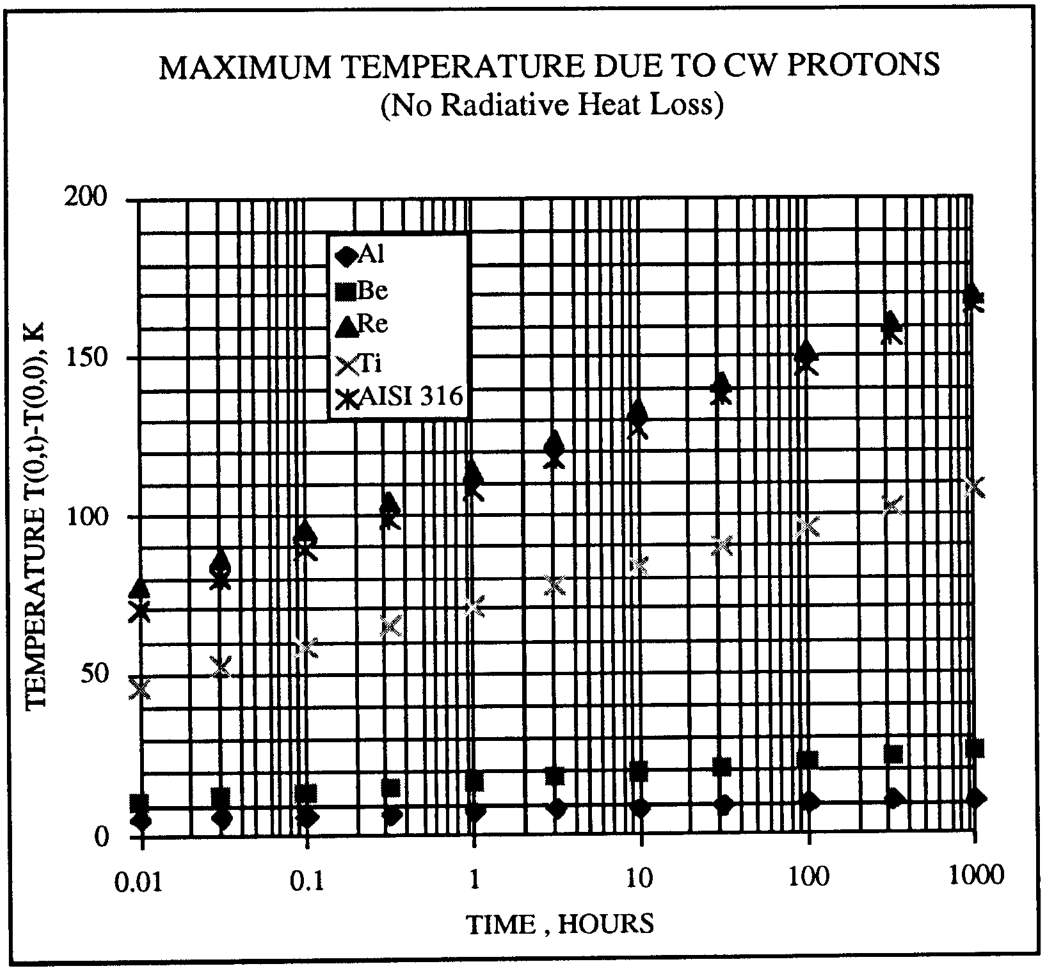
This description of the spatial variation of temperature with time is not particularly illuminating. At the hottest point in the window, however, the inverse Bessel transform simplifies considerably since $J_0(\alpha r) \equiv 1$ at $r=0$. The temperature here is found to grow logarithmically with time:

$$\boxed{T_{CW}(0,t) = T_o + \frac{\hat{N}}{4\pi\kappa} \cdot \frac{\partial E}{\partial z} \cdot \ell n \left(1 + \frac{2\kappa}{\rho c_p \sigma^2} \cdot t \right)}$$

3. Maximum Temperature Growth for Constant C_p and κ

In the Main Injector complex the most intense beam heating of vacuum windows occurs in the NuMI line, where 3×10^{13} 120 GeV/c protons are resonantly extracted in ~msec pulses with a cycle time of 1.9 seconds. The extracted beam is smallest at the target, with a 1.33 mm half-width[2]. Appropriate thermal parameters[3] are tabulated below for various window materials. Since the instantaneous maximum temperature rise ΔT <10% of the melting point for all materials considered, the continuous proton wave approximation is an acceptable model of the heating process. The maximum temperature growths corresponding to these conditions are depicted in the accompanying graph.

	M.P. K	C_p J/gm/K	κ W/cm/K	ρ^4 gm/cm^3	$\partial E/\partial z$ MeV/cm	$\Delta T(r=0)$ K
Al	933	1.236	2.110	2.70	5.03	64.9
Be	1562	3.911	0.563	1.85	3.45	20.5
Re	3460	0.196	0.717	21.1	28.8	298.9
Ti	1945	1.036	0.285	4.5	7.68	70.9
SS 316[5]	1698	0.778	0.324	8.03	13.5	93.0



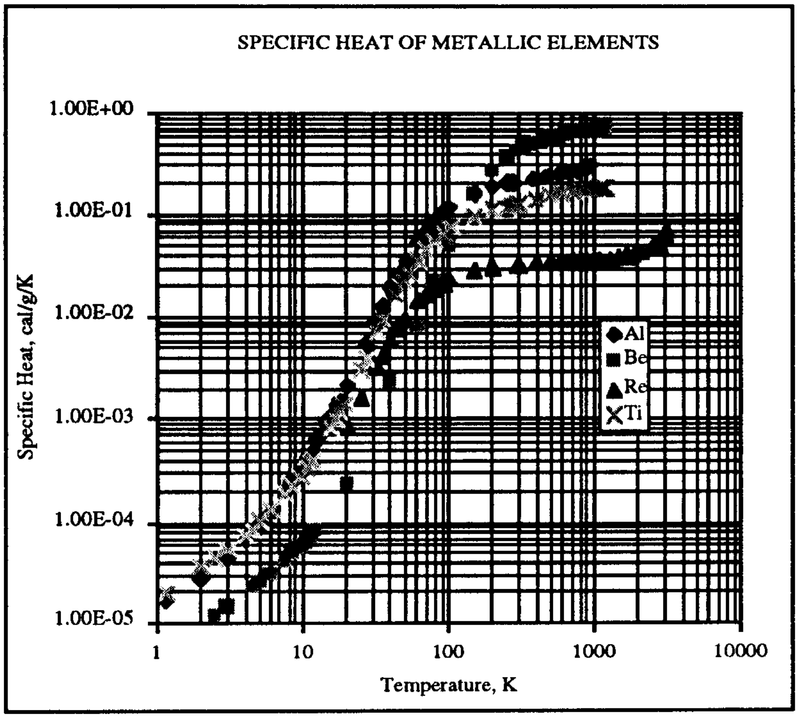
4. Summary & Discussion:

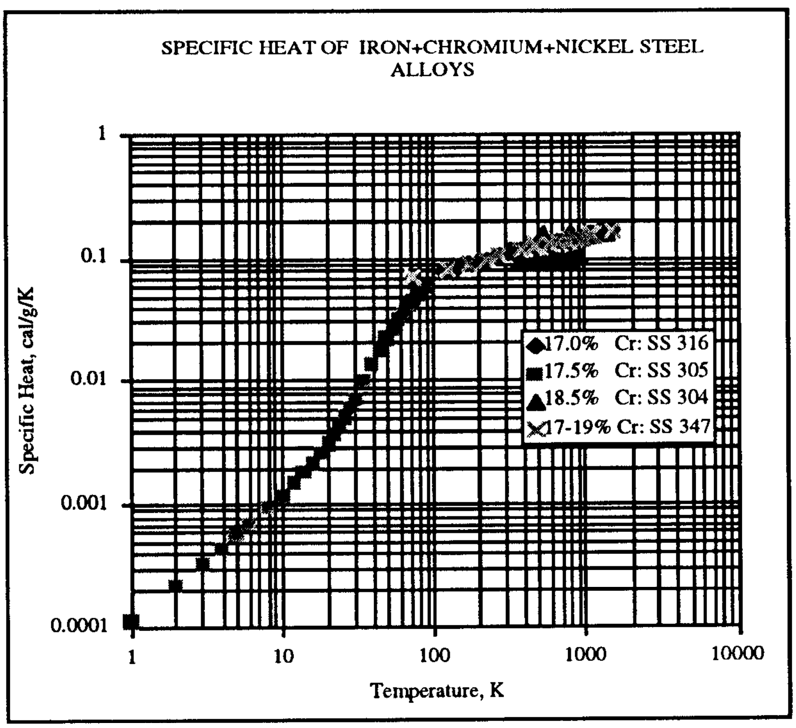
For a Gaussian-distributed proton beam incident normal to a window surface, maximum temperature was shown to grow logarithmically with time when only diffusive heat losses are considered. This is a qualitatively different result than the $\sqrt{t}$ growth rate predicted for the same beam incident at some small angle to the surface[6]. In this case the material would be expected to reach melting point in just a few minutes.

5. Conclusions:

In the absence of radiative energy losses then, yes, the calculations claim that eventually the windows will melt. However, since the time-scale for this to happen is on the order of 10^{12} years, whereas the average accelerator's lifespan is somewhat less than 10^2 years, this problem can safely be left to future generations.

Appendix I. Specific Heats of Metallic Elements & Alloys[7]



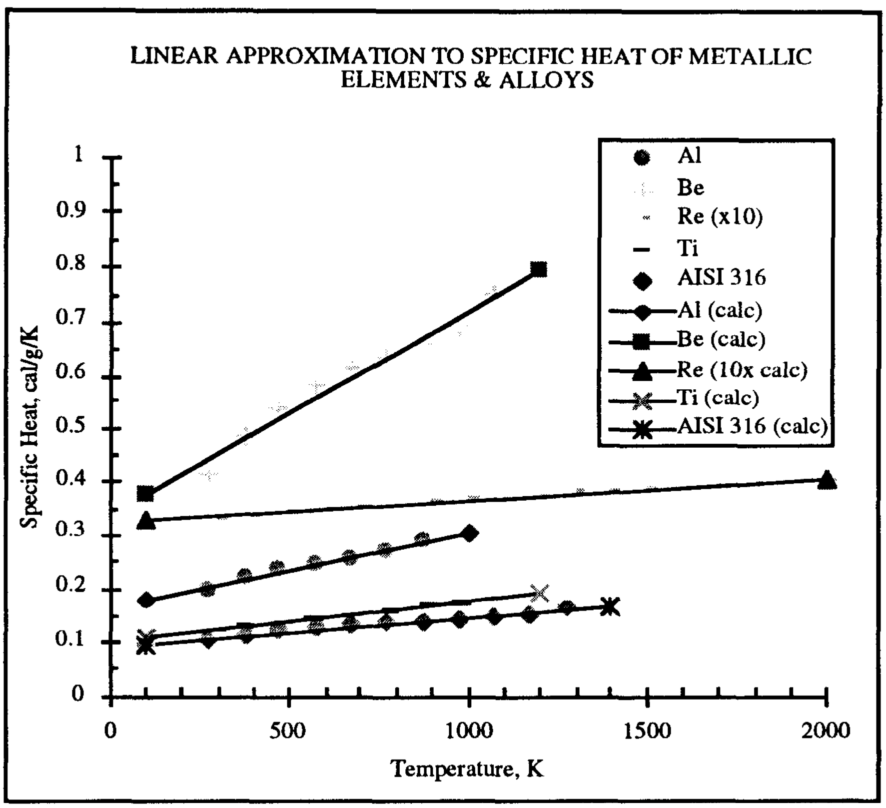


For temperatures from ≈100K up to the melting point, the specific heat of a metallic entitiy exhibits a nearly linear variation with temperature. The data from [6] have been fit using the form:

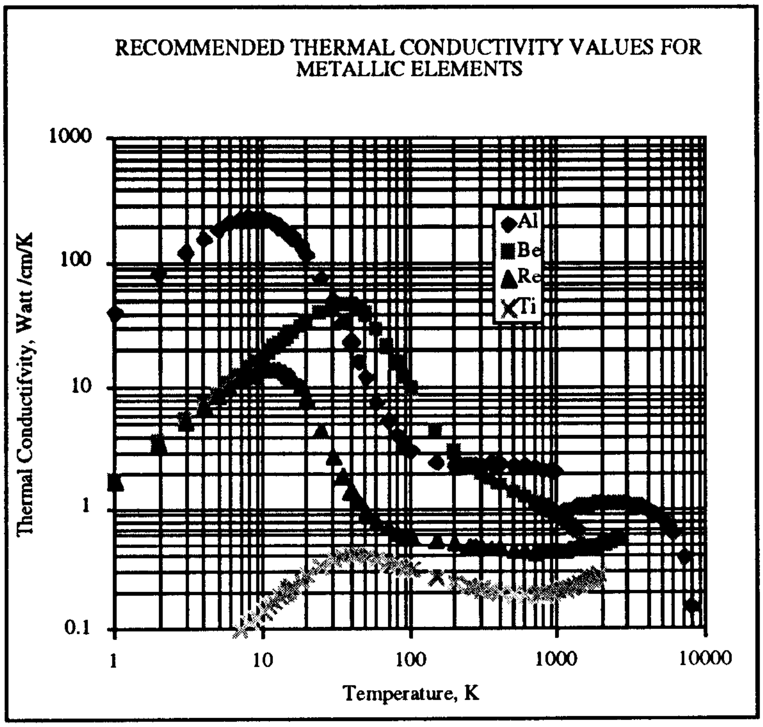
$$C_p \approx \alpha + \beta \bullet (T - 293)$$

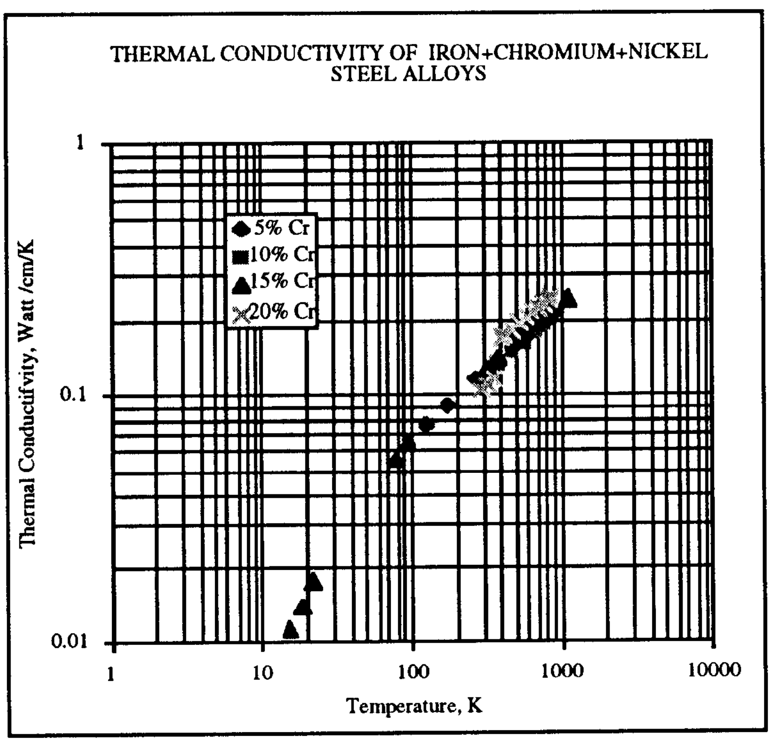
The coefficients α and β are tabulated below, and the corresponding linear fits are compared with data in the associated graph.

	α (J/gm/K)	β (J/gm/K^2)
Al	0.862	5.841×10^{-4}
Be	1.896	15.878×10^{-4}
Re	0.141	0.175×10^{-4}
Ti	0.516	3.146×10^{-4}
AISI 316	0.450	2.334×10^{-4}

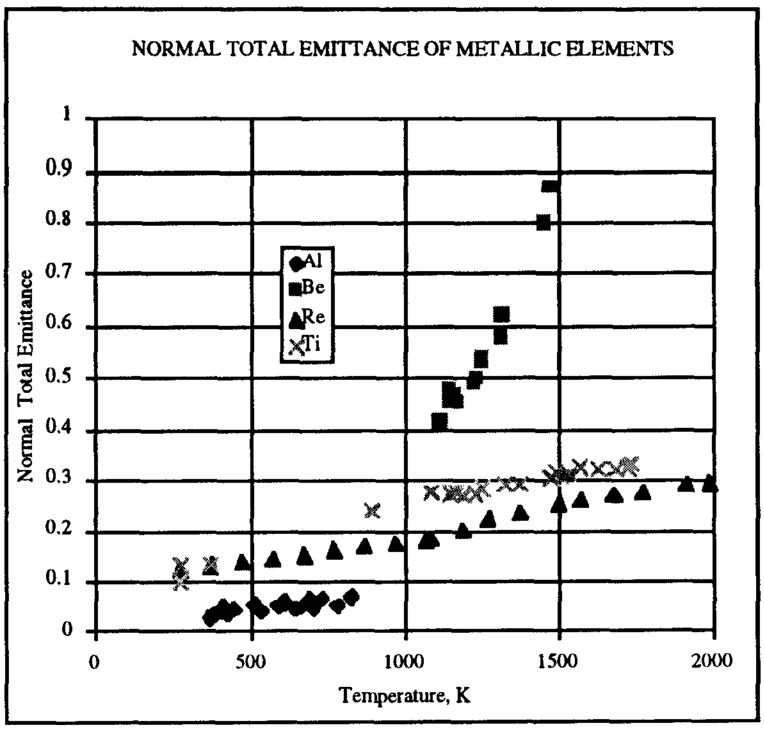


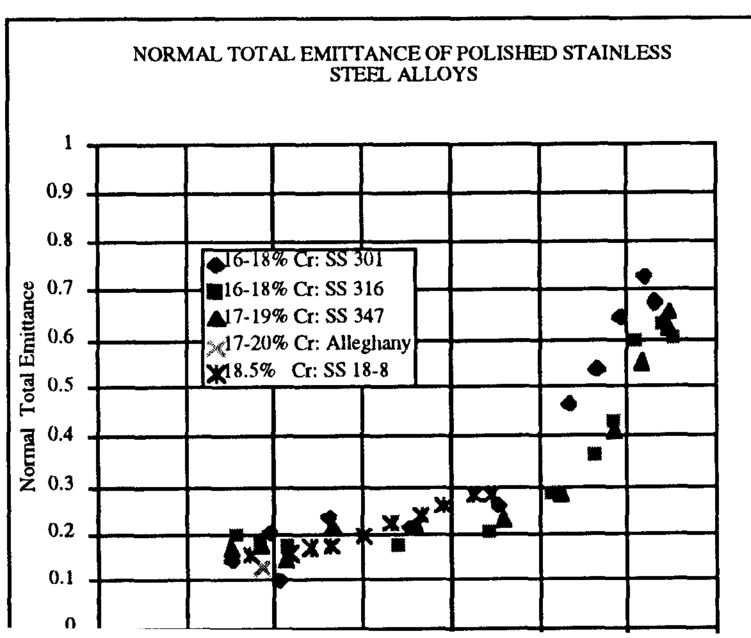
Appendix II. Thermal Conductivity of Metallic Elements & Alloys[8]





Appendix III. Emittance of Metallic Elements & Alloys[9]





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[1]Particle Data Group, Review of Particle Properties, Review of Modern Physics 52, S44, (1980).

[2]Dave Johnson, private & secret, whispered communication, and;
R. Bernstein *et al.* Neutrino Physics after the Main Injector Upgrade, CDR 1.1 (1991).

[3]Specific heats & thermal conductivities are calculated at the melting points (Appendices I & II).

[4]Y.S. Touloukian, R.K. Kirby, R.E. Taylor, P.D. Desai, Thermal Expansion- Metallic Elements and Alloys, Thermophysical Properties of Matter, vol.12, pgs: 38a-42a. (1970).

[5]AISI 316 composition: 74% Fe, 18% Cr, & 8% Ni.

[6]J.A. Johnstone, Burning up the Beampipe, MI-0035 (1990).

[7]Y.S. Touloukian, E.H. Buyco, Specific Heat - Metallic Elements and Alloys, Thermophysical Properties of Matter, vol.4, pgs: 1-5 (Al) 16-20 (Be), 181-183 (Re), 258-262 (Ti), 699-701 (AISI 304), 702-704(AISI 305), 708-710(AISI 316), 711-713(AISI 347). (1970).

[8]Y.S. Touloukian, R.W. Powell, C.Y. Ho, P.G. Klemens, Thermal Conductivity - Metallic Elements and Alloys, Thermophysical Properties of Matter, vol.1, pgs: 9 (Al), 24 (Be), 291 (Re), 414 (Ti), & 1212-1215 (Stainless Steels). (1970).

[9]Y.S. Touloukian, D.P. DeWitt, Thermal Radiative Properties - Metallic Elements and Alloys, Thermophysical Properties of Matter, vol.7, pgs: 8-11 (Al), 71-73 (Be), 562-564 (Re), 726-728 (Ti), & 1218-1230 (polished Stainless Steels). (1970).