ELECTRICAL MODELING OF MAIN INJECTOR SEXTUPOLE MAGNETS

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1. Introduction

The electrical models for the main injector sextupole magnet is obtained based on three terminal device impedance matrix measurement. The measurement data are analyzed and curve fitted into their equivalent circuits by using circuit simulation program Spice.

2. Electrical Measurement

The sextupole magnet is a three-terminal device. Fig 1 depicts the sextupole magnet three-terminal representation. Terminal 1 and 2 are the coil bus terminals and terminal 3 is the magnet case ground. The electrical characteristics of the magnet at non saturation can be described by its admittance matrix. The equations for this three-terminal device network can be written as

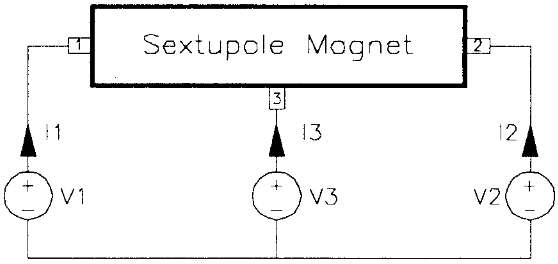


Fig 1 Sextupole Three-Terminal Representation

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad (1)$$

The 3×3 matrix on the right hand side of Eqn. (1) is called shorted circuit admittance matrix of the considered three-terminal sextupole magnet. The elements in the shorted circuit admittance matrix are frequency dependent variables. The way to measure this 3×3 matrix elements is depicts in figure 2. The excitation source used here is a high power frequency generator (Elgar Model 500) that can output current up to 5 amps. the output voltage can be adjusted from 0 to 150 Vrms, and the frequency can be varied from 10 Hz to 10KHz. A Tektronix current probe, which has the bandwidth of DC to 50 MHz, was used for current measurement. Both voltage and current as well as phase shifted between

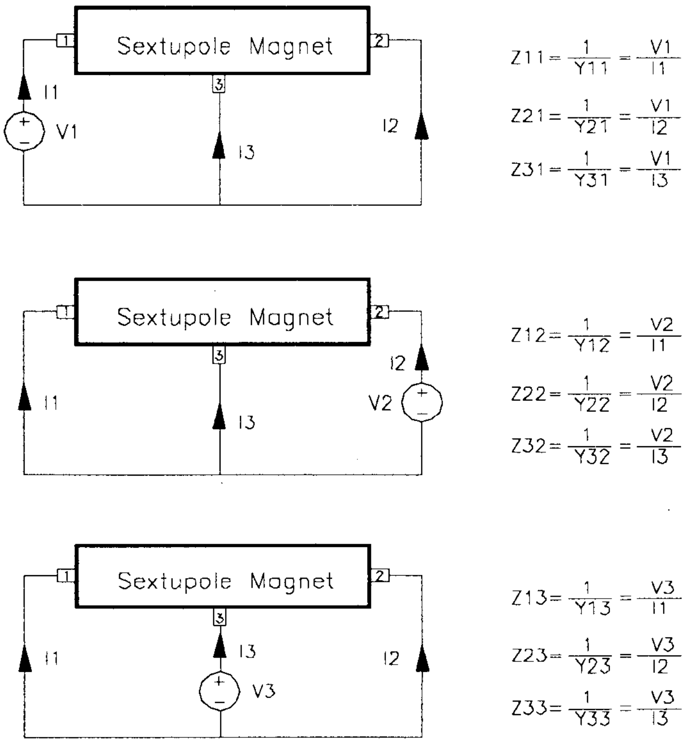


Fig 2 Admittance Matrix Measurement

voltage and current were measured by Tektronix scope. Ratio of peak to peak value of current to voltage was obtained for the shorted circuit admittance at the frequency of interest. Scope measurement for voltage and current made sure the signals being measured were not distorted.

3. Measurement Data Fitting

DC Resistance of the coil bus is measured at 18 °C. The resistance then is scaled to the resistance at 40 °C by using the formula:

$$R(40^o C) = R(18^o C)\cdot[1+0.004/^o\ C\cdot(40^o C-18^o C)] \qquad (2)$$

Coil Impedance measurement is performed by Z_{11}, Z_{22}, Z_{12}, Z_{21}. Z_{11} and Z_{22} are equal both in magnitude and phase for the frequency up to 10 KHz. Z_{12} and Z_{21} are also the same both in magnitude and phase. Z_{11} and Z_{12} are the same in magnitude but 180° out of phase because the reverse direction of the current. This implies the Sextupole is symmetrical since impedance looking into both coil bus terminals is equal.

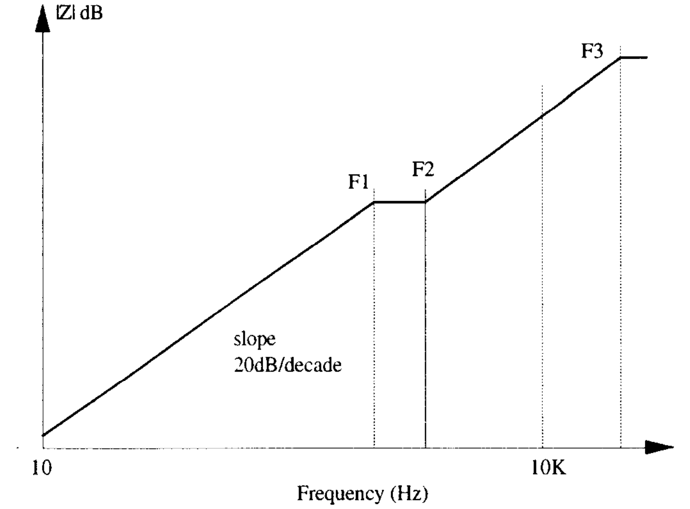
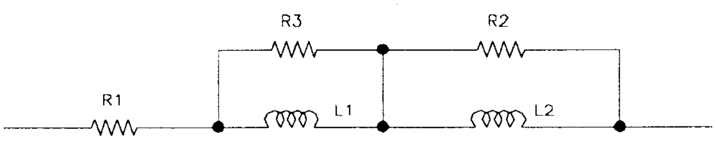


Fig 3 Coil Impedance Data Asymptote Representation

By analyzing the coil impedance measurement data Z_{11} and Z_{22}. The data can be represented by straight-line asymptotes as shown in Fig 3. The bus DC resistance has effect on coil impedance at very low frequency(<< 10 Hz), therefore it is not shown in the impedance asymptote representation here. The coil bus impedance is inductive at the frequency between 10 Hz and F1, F2 and F3 because the slope of the magnitude asymptote line is 20 dB/decade and the phase is 90°. The coil bus becomes small resistive at the frequency between *F1* and *F2*, and above *F3*. In the other word, the inductance of the coil decreases as the frequency increases.

An electrical circuit shown in Fig 4 can be used to represent the straight-line asymptote characteristics in Fig 3. The corner frequency break points are approximately given by the formula in figure 4.



$$F1=\frac{R2}{2*\pi*L2} \qquad F2=\frac{Ra}{2*\pi*La} \qquad F3=\frac{R3}{2*\pi*L1}$$

where $Lc=\frac{L1*L2}{L1+L2}$ $\qquad Ra=\frac{R2*R3}{R2+R3}$

Fig 4 Circuit For Fig 3 Straight-Line Asymptote

Bus Capacitance Measurement is obtained by Z_{13}, Z_{31}, Z_{23}, Z_{32}, and Z_{33}. Z_{33} measures the total bus to ground capacitance. Z_{13} and Z_{31} measures the capacitance between terminal 1 and ground while terminal 2 is shorted to ground. Similarly, Z_{23} and Z_{32} measures the capacitance between terminal 2 and ground while terminal 1 is grounded. The Z_{13}, Z_{31}, Z_{23}, Z_{32}, and Z_{33} are capacitance measurement because the slope of the measurement data is -20dB/decade in the magnitude plot. The capacitance is determined by the following formula

$$C=\frac{1}{2\cdot\pi\cdot f\cdot|Z|} \qquad (3)$$

where f is the excitation frequency in Hz and Z is the impedance in ohms.

4. Sextupole Electrical Model

The impedance matrix measurement data for the sextupole are given in Appendix A. The DC resistance is measured at 18 °C and the value is 15.62 mΩ. The resistance at 40 °C is calculated to be 17 mΩ by using Eqn (2). R_1 in Fig 4 is the DC resistance.

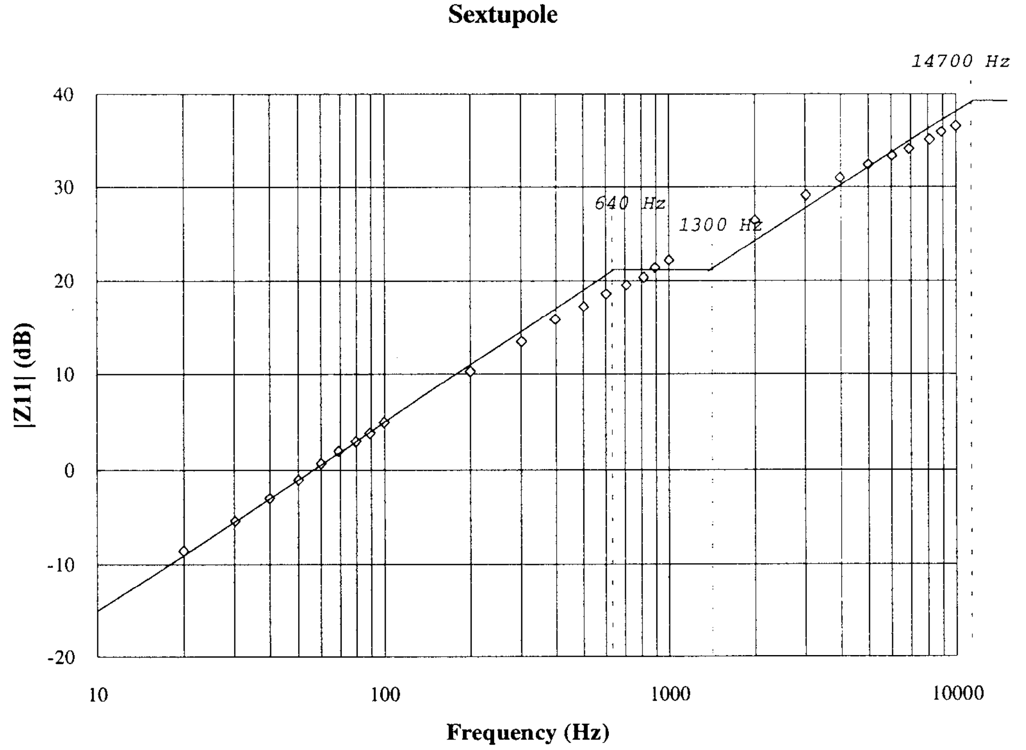


Fig 5 Straight Line Asymptote Representation of Sextupole Coil Impedance

Coil Impedance measurement is obtained by Z_{11}, Z_{22}, Z_{12}, Z_{21}. They all have the same magnitude as function of frequency. Figure 5 shows Z_{11} measurement data where the data can be represented by straight line asymptotes. The parameters in the circuit (Fig 4) can be determined according to the data. Referring to Fig 5, the corner frequency break points, the total coil inductance at low frequency (10 Hz), the inductance at high frequency (10KHz) are obtained as:

$F1$=640 Hz $F2$=1300 Hz $F3$=14700 Hz

Total Inductance @ 10 Hz: $L_T=\frac{1}{2\pi f}\log^{-1}[\frac{|Z_{11}|}{20}]$ = 2.8 mH where $|Z_{11}|$ = -15.1 dB

Inductance @ 5 KHz: $L_1 = \frac{1}{2\pi f}\log^{-1}[\frac{|Z_{11}|}{20}] = 1.3\ \text{mH}$ where $|Z_{11}| = 32.2$ dB

The value of the circuit elements can be calculated since the total coil inductance, inductance at the frequency of $F2 < f < F3$, and the corner frequency break points are known.

L2 Inductance: $L_2 = L_T - L_1 = 1.5\ \text{mH}$

R2: $R2 = 2 \cdot \pi \cdot F1 \cdot L2 = 6\ \Omega$

R3: $R4 = 2 \cdot \pi \cdot F3 \cdot L1 = 120\ \Omega$

Bus Capacitance is measured by Z_{15}, Z_{51}, Z_{25}, Z_{52}, and Z_{55}. The total bus to magnet case ground capacitance (C_T) is measured by Z_{55}. Z_{15} and Z_{51} measures the capacitance (C_1) between terminal 1 and ground while terminal 2 is shorted to ground. Similarly, Z_{25} and Z_{52} measures the capacitance (C_2) between terminal 2 and ground while terminal 1 is grounded.

Total Bus to Ground Capacitance: $C_T = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{55}|} = 3.8\ \text{nF}$

Terminal 1 to Ground Capacitance: $C_1 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{15}|} = 1.9\ \text{nF}$ or

$C_1 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{51}|} = 1.9\ \text{nF}$

Terminal 2 to Ground Capacitance: $C_2 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{25}|} = 1.9\ \text{nF}$ or

$C_2 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{52}|} = 1.9\ \text{nF}$

The sextupole magnet electrical model is shown in Fig 6. The simulation results are given in Appendix A. The model matches the measurement data up to 10 KHz.

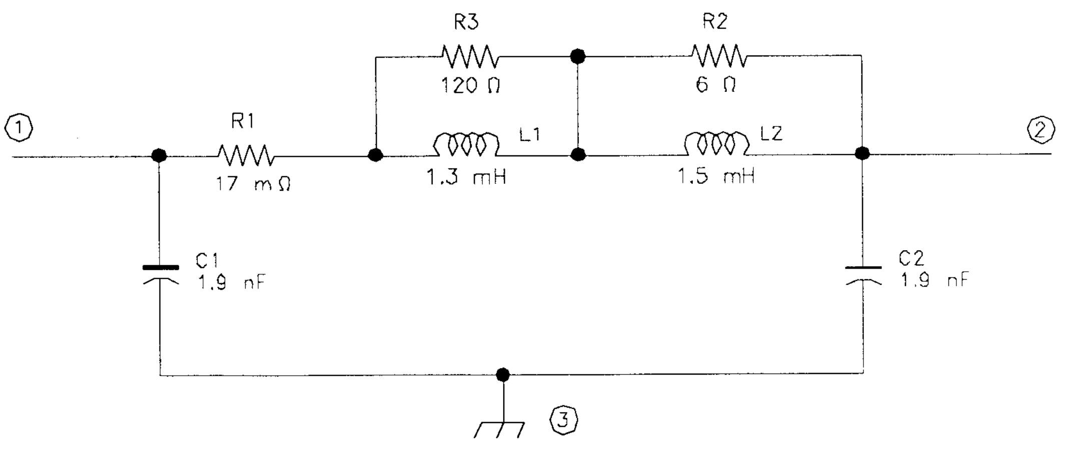


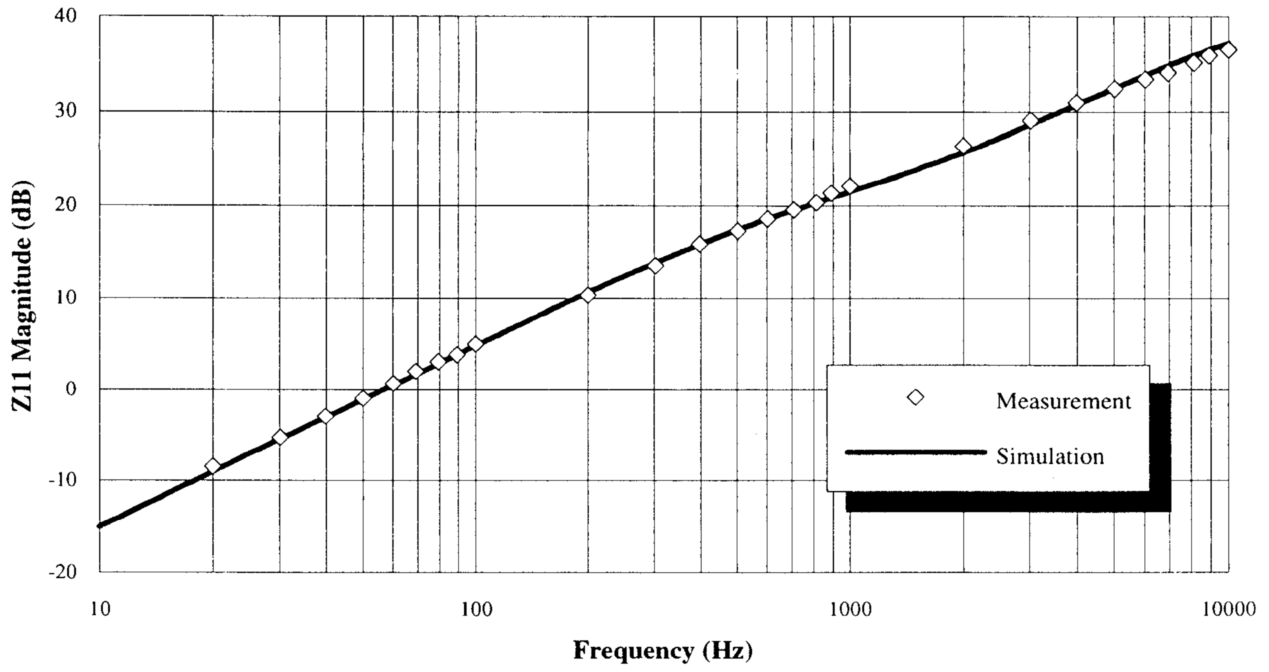
Fig 6 Main Injector Sextupole Electrical Model

5. Conclusion

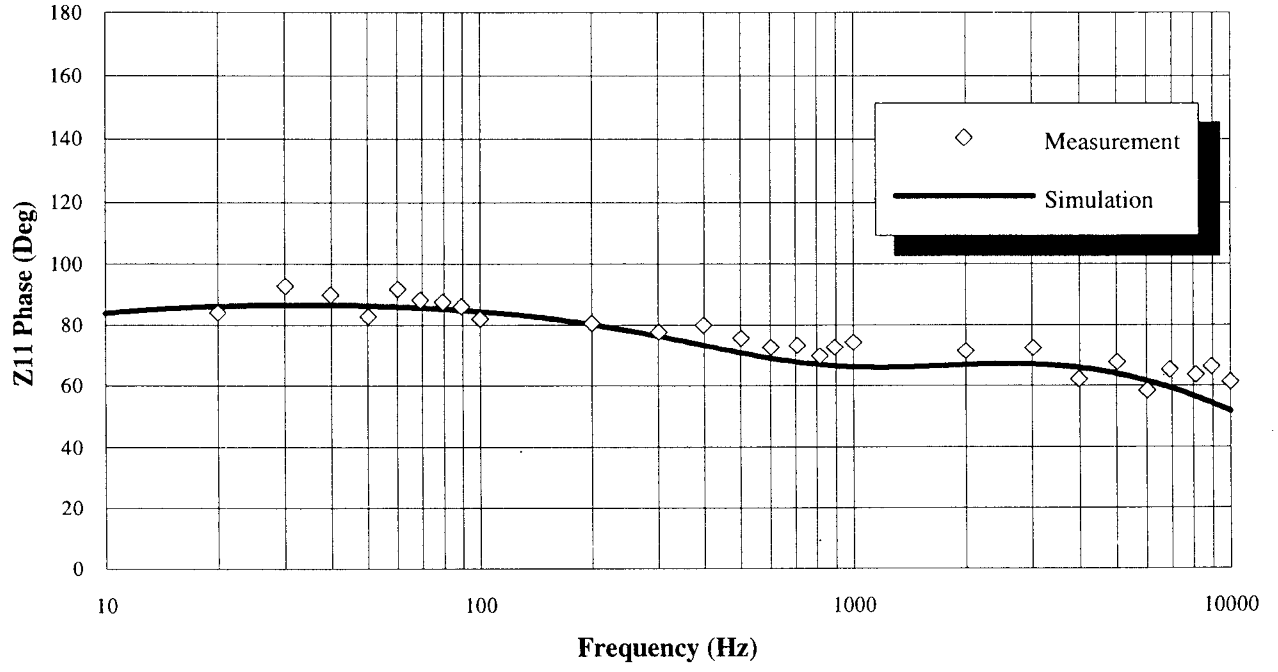
The sextupole electrical model is obtained based on the impedance matrix measurement. Spice simulation result shows the accuracy of the models. The electrical model can be used as a sub circuit to build a sextupole ring model to study the transient and frequency response of the system.

Main Injector Sextupole Magnet

Z11 MAGNITUDE PLOT

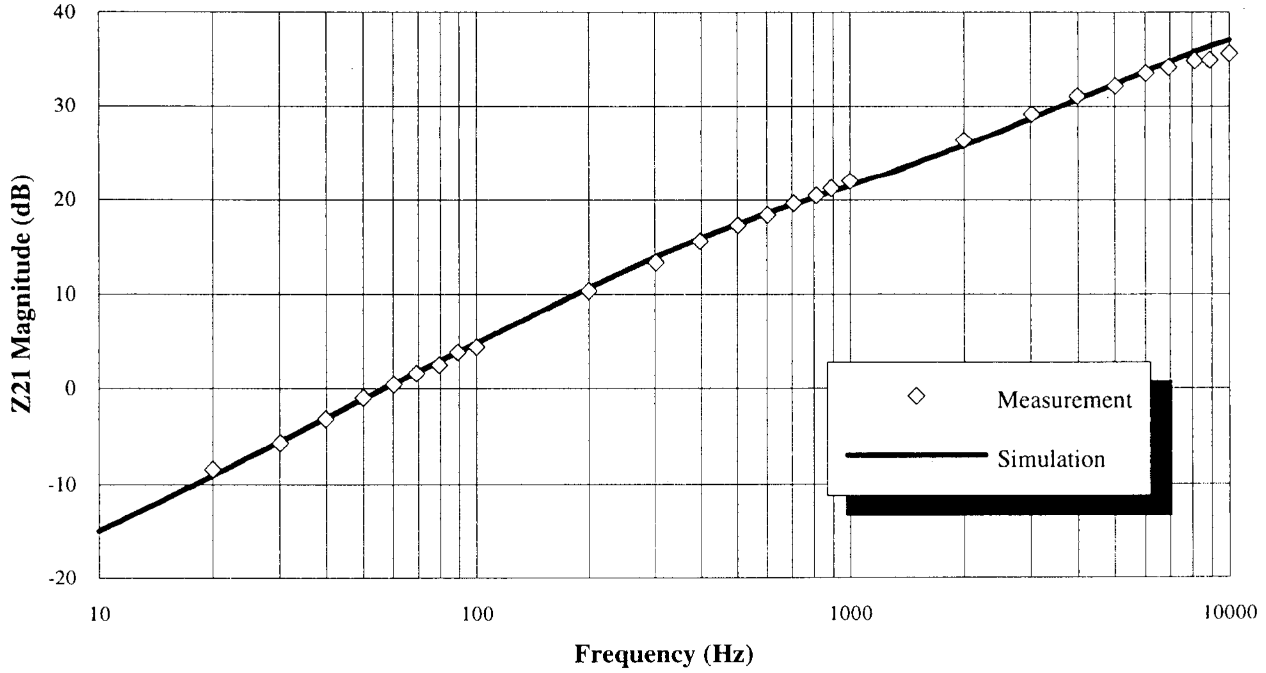


Z11 PHASE PLOT

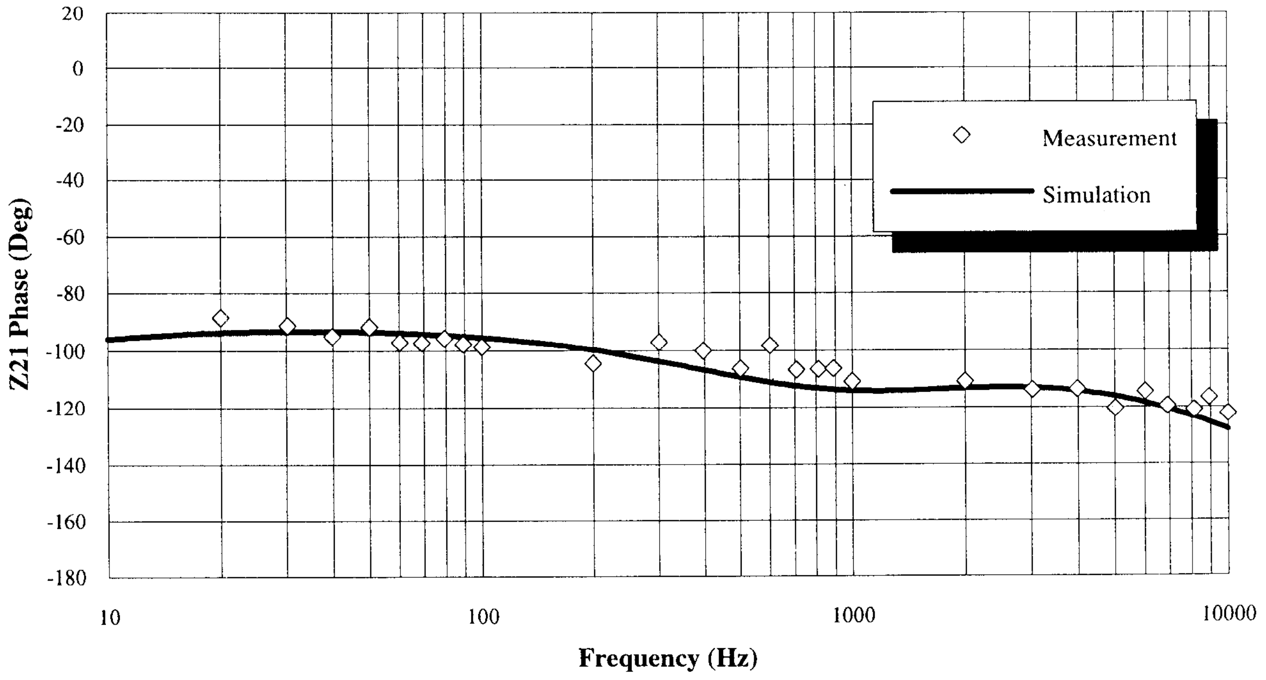


Main Injector Sextupole Magnet

Z21 MAGNITUDE PLOT



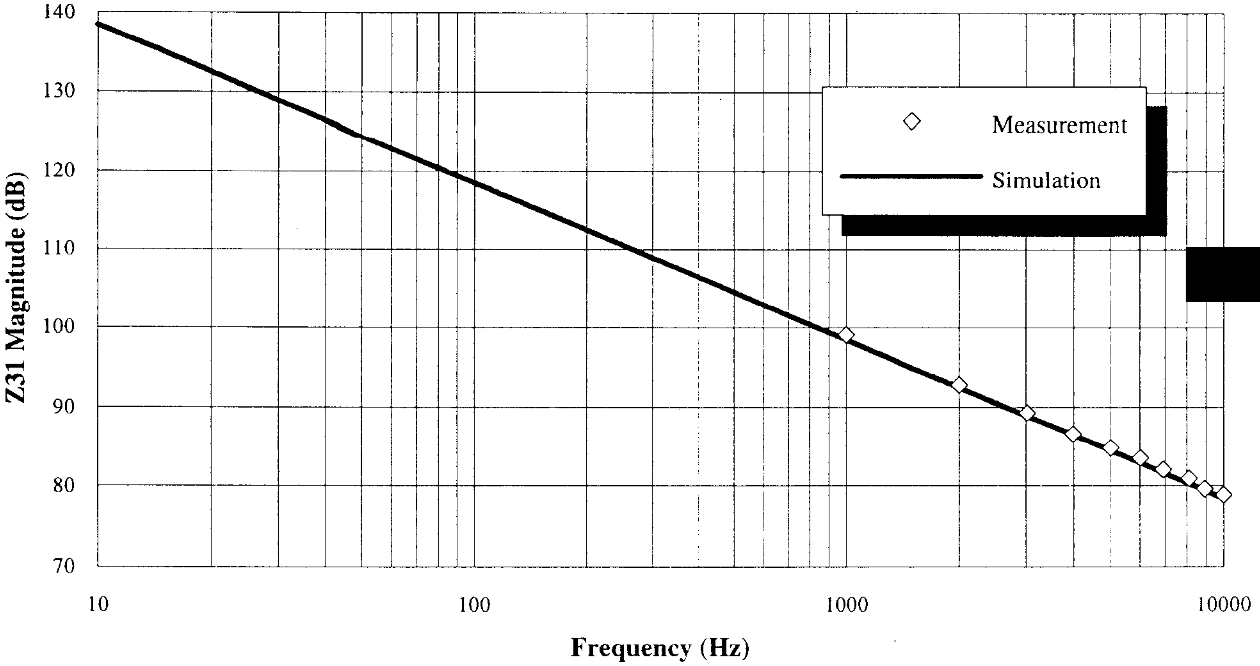
Z21 PHASE PLOT



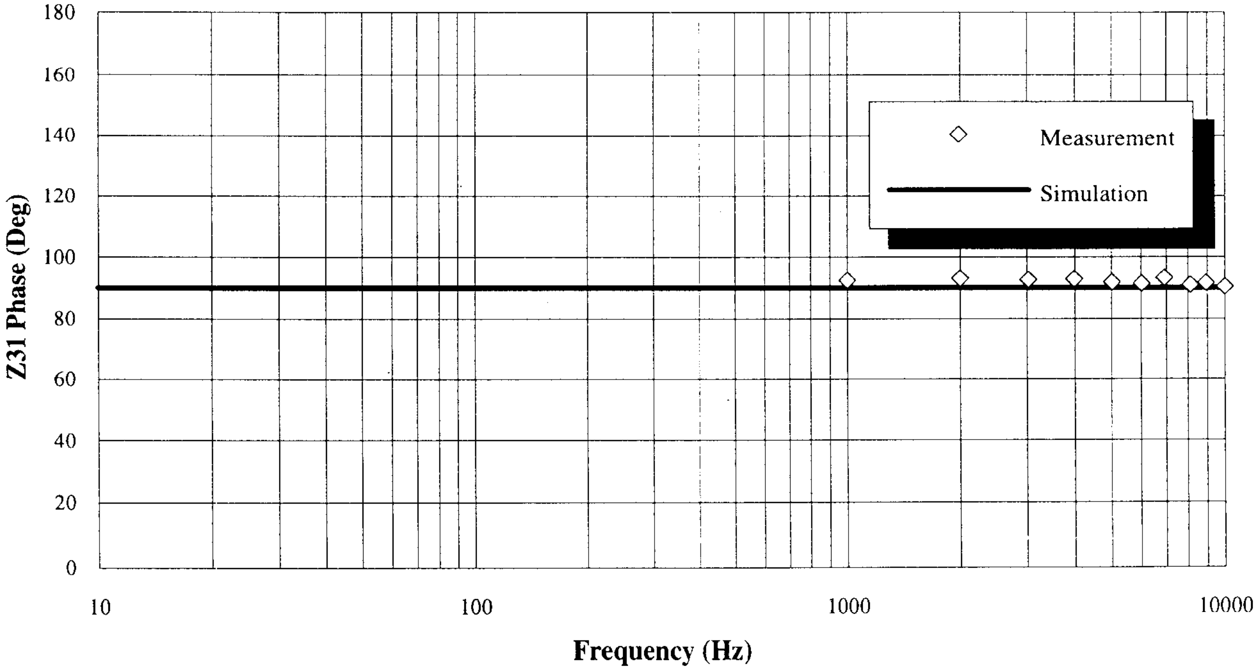
Appendix A-3

Main Injector Sextupole Magnet

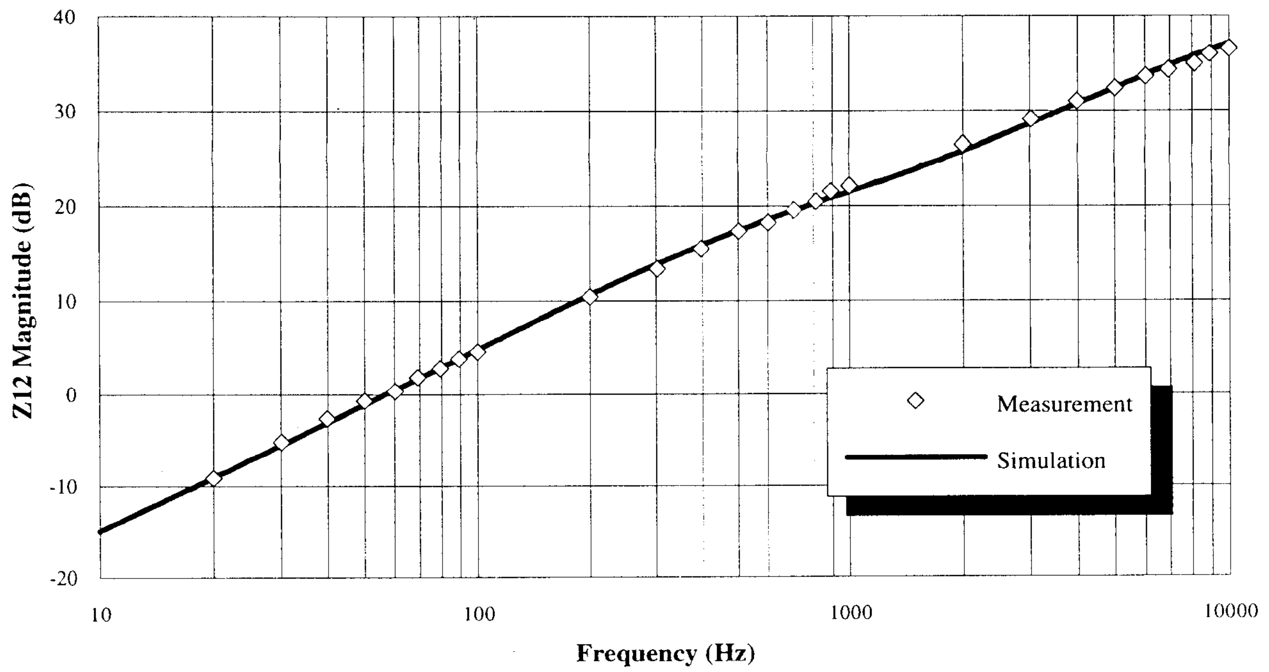
Z31 MAGNITUDE PLOT



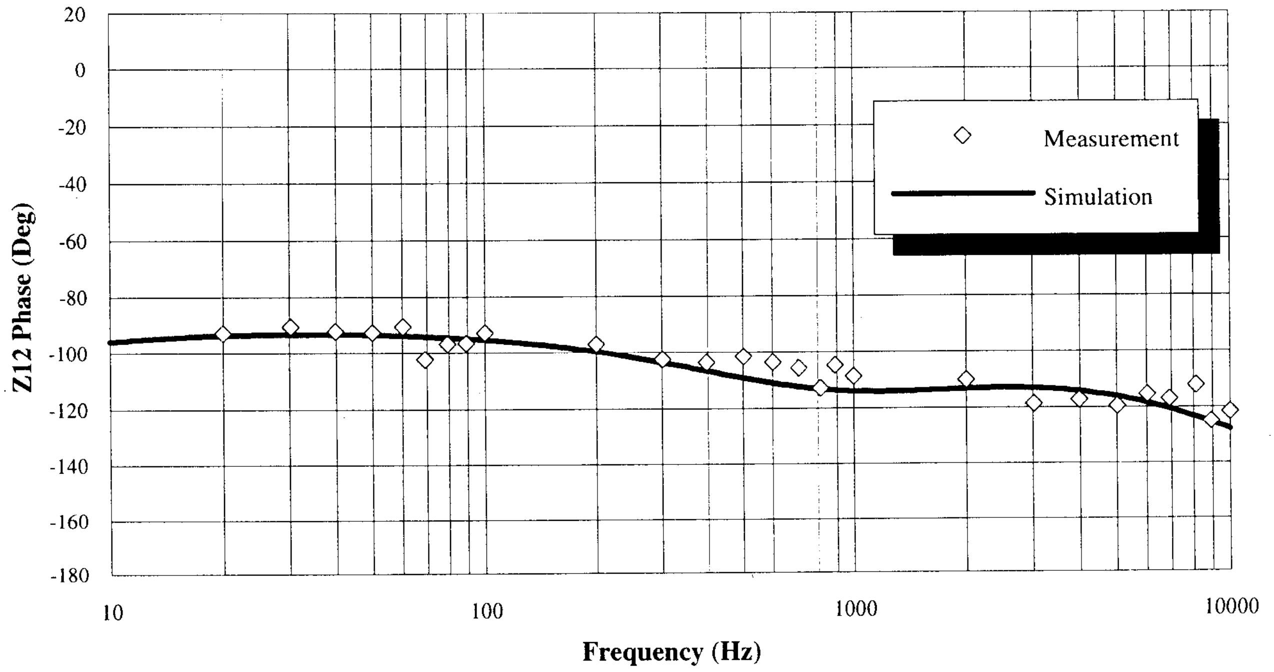
Z31 PHASE PLOT



Z12 MAGNITUDE PLOT



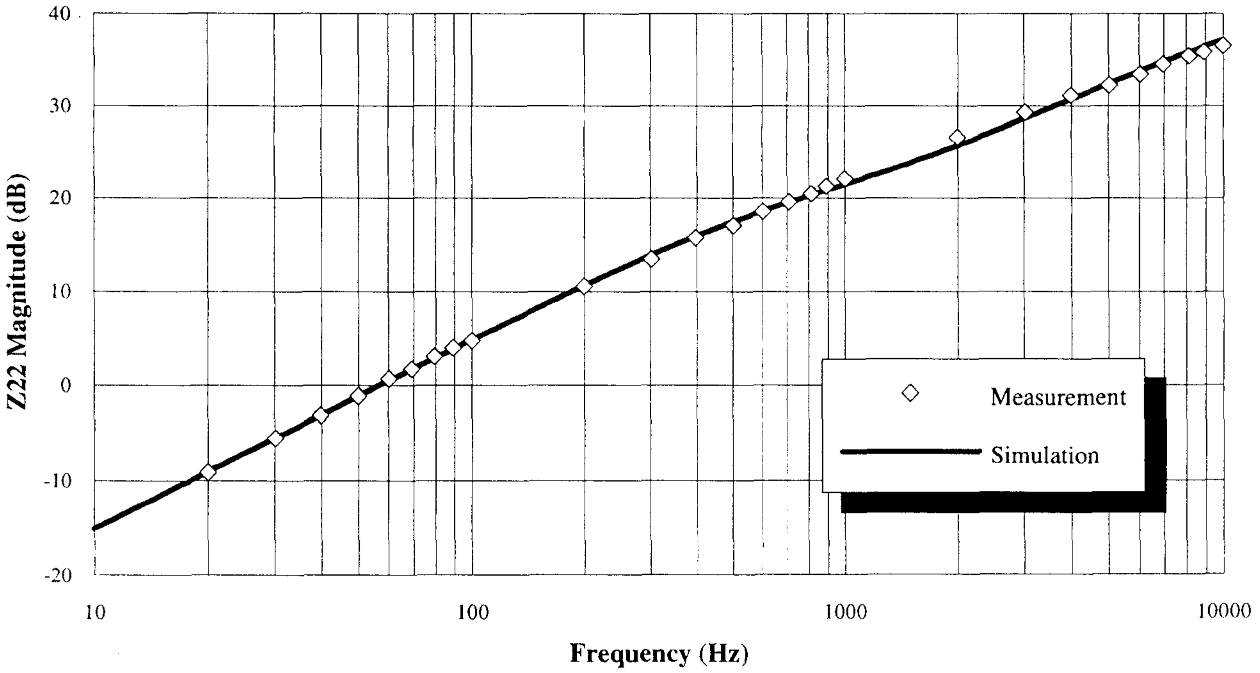
Z12 PHASE PLOT



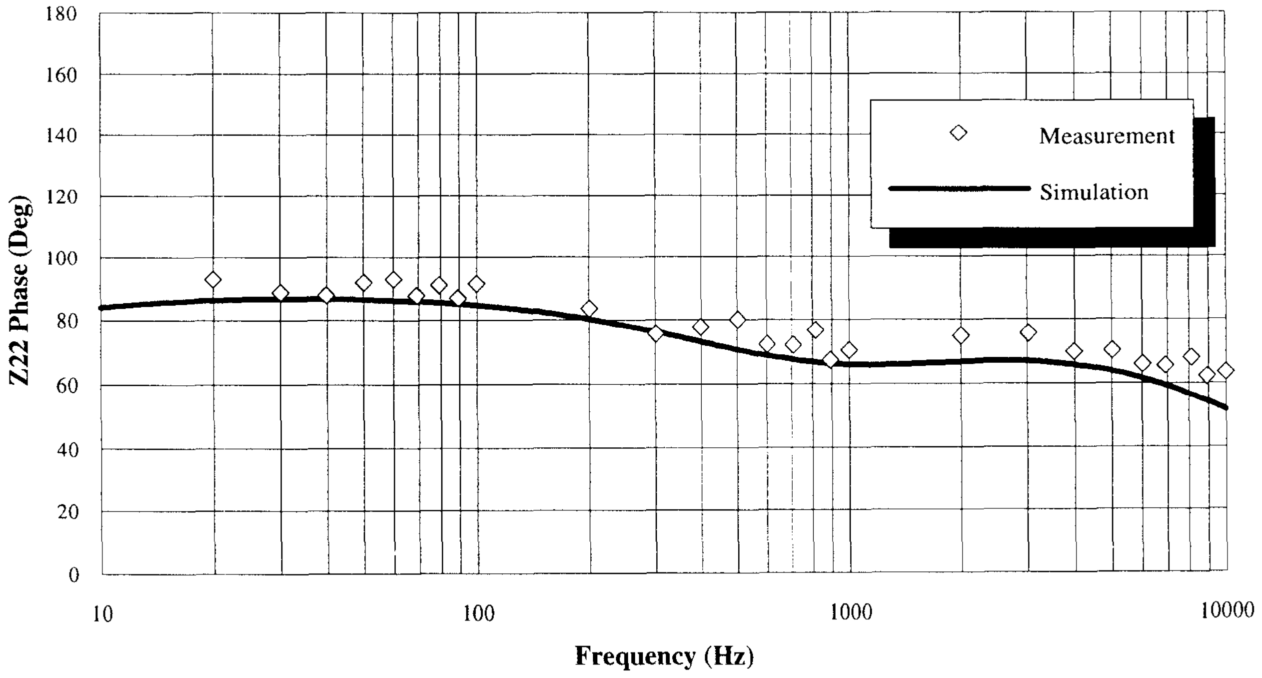
Appendix A-5

Main Injector Sextupole Magnet

Z22 MAGNITUDE PLOT

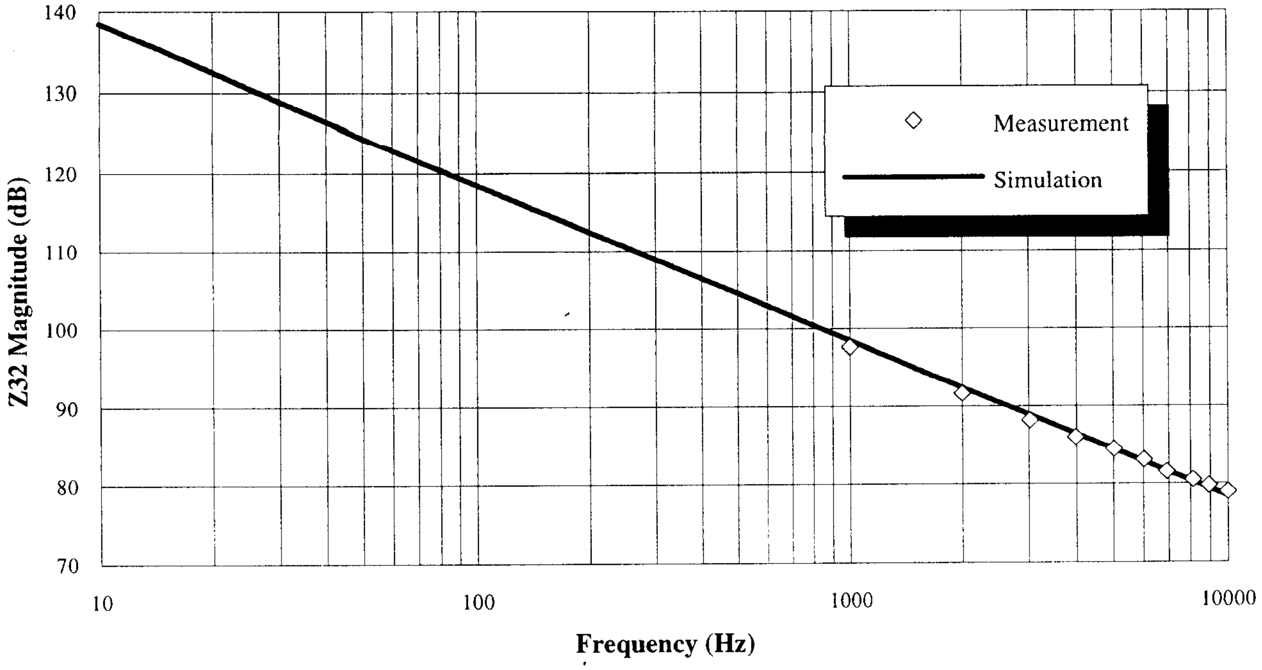


Z22 PHASE PLOT

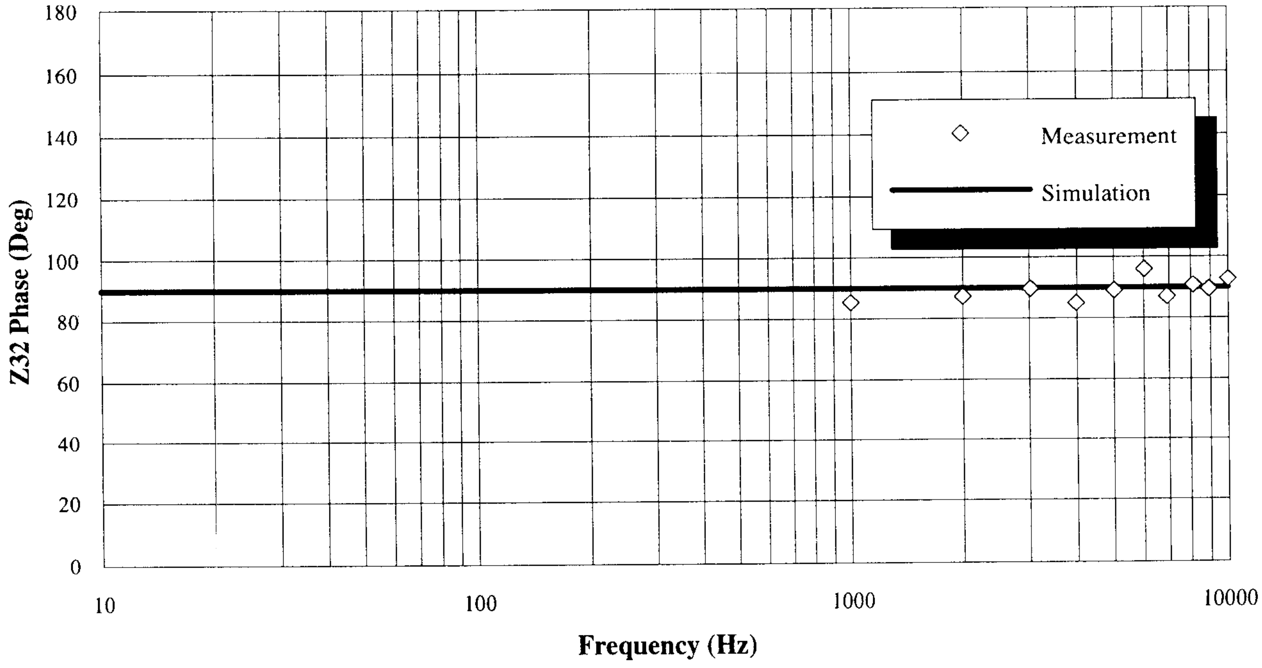


Main Injector Sextupole Magnet

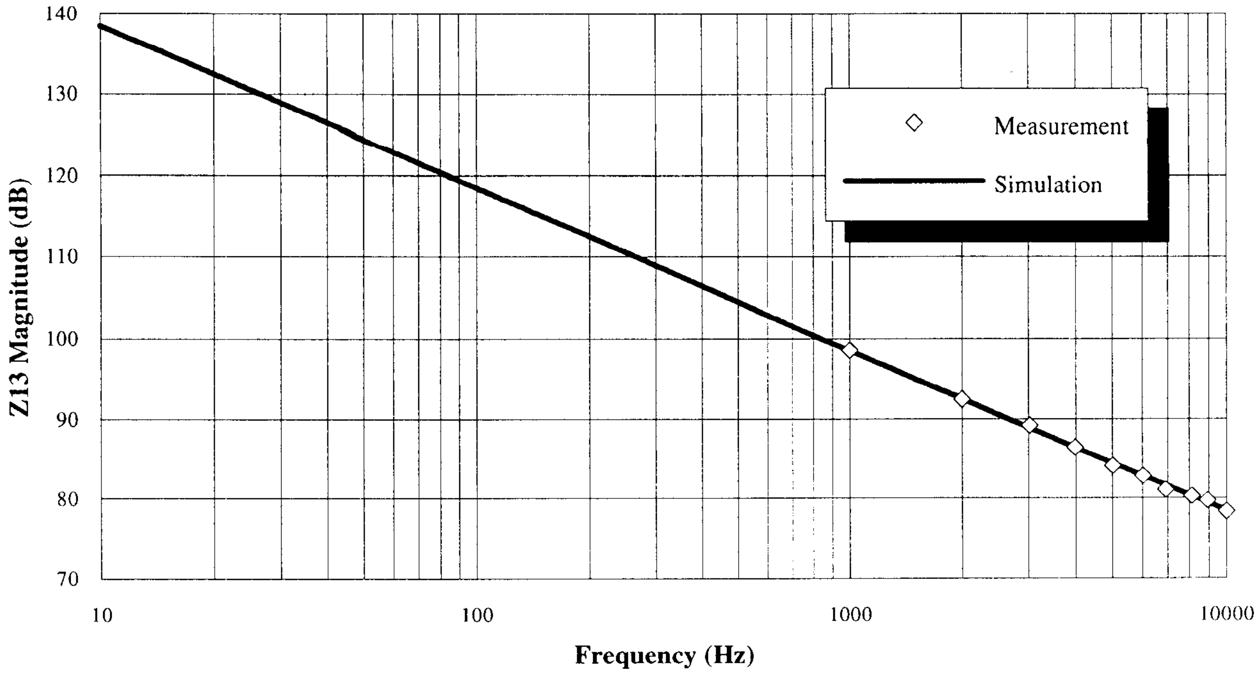
Z32 MAGNITUDE PLOT



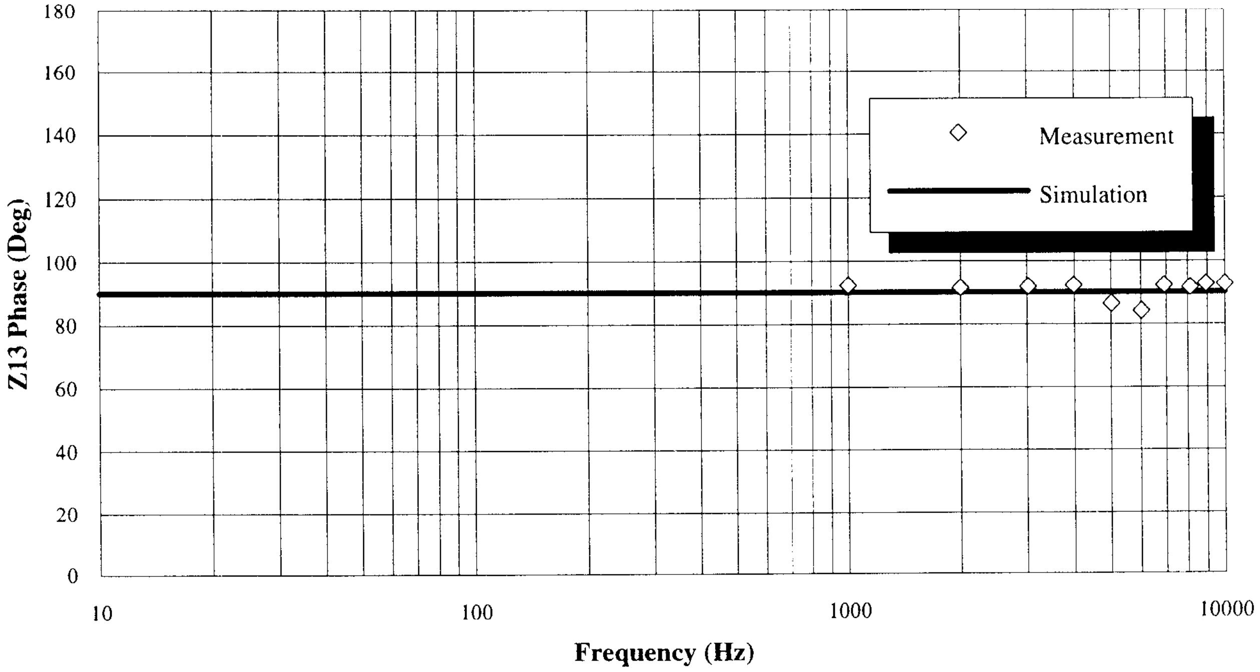
Z32 PHASE PLOT



Z13 MAGNITUDE PLOT

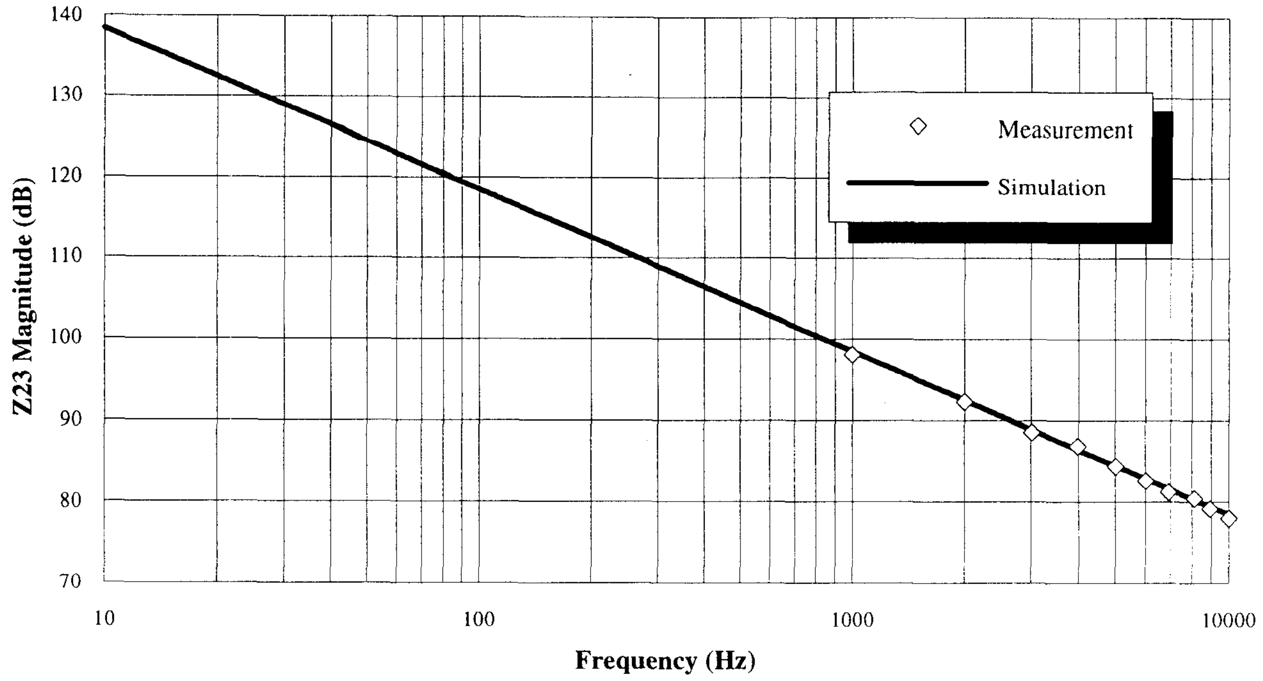


Z13 PHASE PLOT

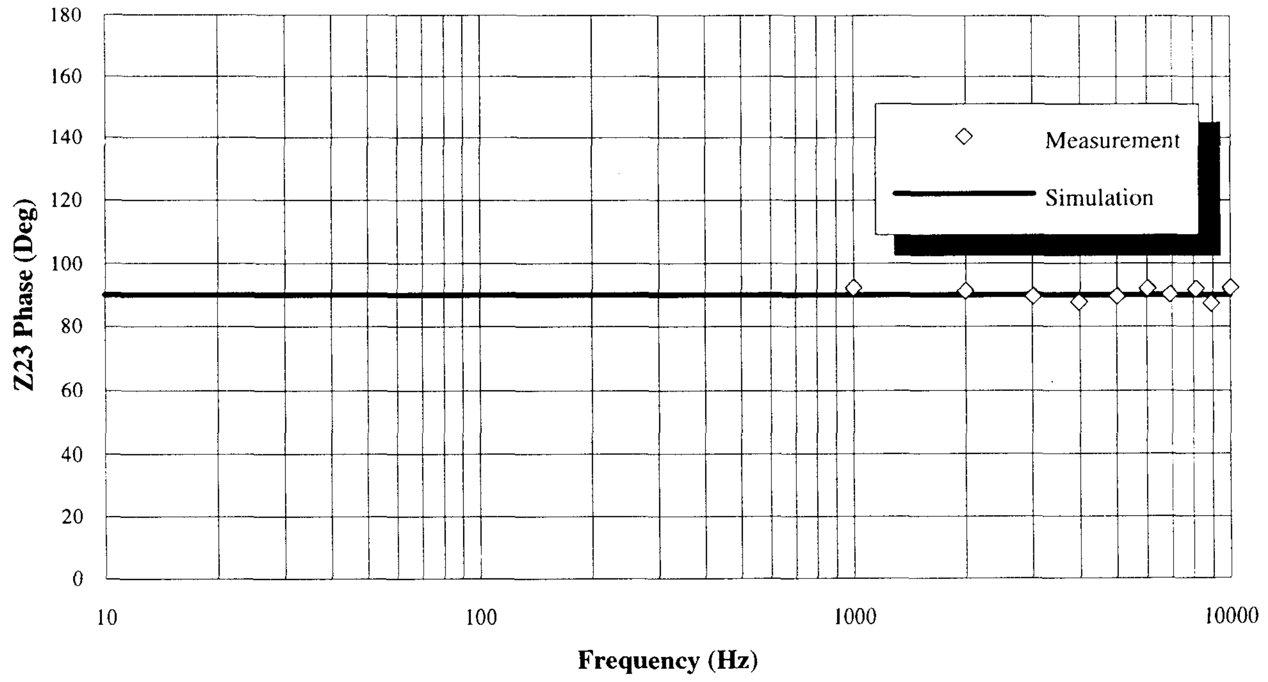


Appendix A-8

Main Injector Sextupole Magnet



Z23 MAGNITUDE PLOT



Z23 PHASE PLOT