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IMPEDANCES AND BEAM STABILITY ISSUES OF THE FERMILAB RECYCLER RING

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Abstract

The longitudinal and transverse coupling impedances of the Fermilab Recycler Ring are estimated and found to be dominated by the space-charge contributions. The beam that contains 2.53×10^{12} anti-protons will be safe from longitudinal microwave instability. A small tune spread, such as $\Delta\nu_\beta = 0.01$ generated by an octupole, will be required to damp the transverse microwave instability. Resistive-wall instability will not occur at the operation chromaticity of $\xi = -3$, even when the coasting beam is squeezed to bunches of full length 1 μs each.

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I. INTRODUCTION

The Fermilab Recycler Ring [1] will be built on top of the Fermilab Main Injector sharing the same tunnel. It main function is to recycle the anti-protons after a store in the Tevatron and to provide a storage for the anti-protons after they are accumulated and cooled in the Accumulator. The Recycle will be made from permanent magnets so that its storage of anti-protons will not be affected by a sporadic discontinuity of the electric power, for example during a thunder storm. The Recycler is therefore operated at only a single energy, which has been chosen to be the injection energy $E = 8.93827$ GeV of the Main Injector. The lattice closely follows that of the Main Injector, having a circumference of 3319.414 m or a mean radius of $R = 528.301$ m. The revolution frequency is therefore $f_0 = \omega_0/2\pi = 89.8159$ kHz. Except for injection stacking and cooling, the beam, which contains $N = 2.53 \times 10^{12}$ anti-protons, will not be bunched.

In this paper, we try to estimate the coupling impedances of the Recycler vacuum chamber, both longitudinal and transversely. The results will be used to investigate the various instabilities of the anti-proton beam.

II. COUPLING IMPEDANCES

The lattice is the Recycler is almost identical to that of the Main Injector. Also the beam pipe, which is elliptic $1.75'' \times 3.75''$ (4.445 cm $\times$ 9.525 cm), does not differ very much from that of the Main Injector. As a result, many of the impedance estimates for the Main Injector [2] can be directly translated to the Recycler.

A. Space Charge

The largest longitudinal space charge impedance per revolution harmonic is seen by particles in the center of the beam and is given by [3]

$$\frac{Z_\parallel}{n} = i\frac{Z_0}{2\beta\gamma^2}g_\parallel \ , \tag{2.1}$$

where $Z_0 \approx 120\pi$ Ω is the free-space impedance, β and γ are the relativistic factors of the beam. For a beam of uniform density and radius a inside a circular beam pipe of radius b, the geometric factor g can be expressed as

$$g_\parallel = 1 + 2\ln\frac{b}{a} \ , \tag{2.2}$$

whereas for a rectangular beam pipe of full height h and full width w, [4]

$$g_{\|} = 1 + 2\ln\left[\frac{2h}{\pi a}\tanh\left(\frac{\pi w}{2h}\right)\right] . \tag{2.3}$$

The 95% normalized emittance is $10\pi \times 10^{-6}$ m and the minimum beta function is 10 m. This lead to a minimum rms beam radius of $\sigma = 1.33$ mm in both horizontal and vertical directions. Since the beam is more or less bi-Gaussian distributed, we take $a = 1.735\sigma = 2.31$ mm, so that the result would be equivalent to Eq. (2.2) for the uniformly distributed situation, giving $g_{\|} = 5.53$. For the rectangular-pipe approximation, we take $h = 2b = 4.445$ cm, and $w = 9.525$ cm to obtain $g_{\|} = 6.01$. So it is fair to conclude that $(Z_{\|}/n)_{\rm sp\ ch} \approx i12$ Ohms, which remains constant drops off only when frequency reaches $f \ll \gamma c/2\pi b \approx 20$ GHz, with c being the velocity of light. In fact, $(Z_{\|}/n)_{\rm sp\ ch}$ is not very sensitive to the transverse beam size.

The horizontal and vertical space charge impedances are

$$Z_{\rm H,V} = i\frac{RZ_0}{\beta^2\gamma^2}g_{\rm H,V} , \tag{2.4}$$

where the geometric factors are

$$g_{\rm H,V} = \frac{1}{a^2} - \frac{1}{b^2} , \tag{2.5}$$

if the beam pipe is circular, or

$$g_{\rm H,V} = \frac{1}{a^2} - \frac{8}{h^2}(\xi_1^{\rm H,V} - \epsilon_1^{\rm H,V}) , \tag{2.6}$$

if the beam pipe is rectangular. In the above, $R = 528.30$ is the mean radius of the Recycler, the superscripts or subscripts H and V refer to the horizontal and vertical direction, and ϵ_1 and ξ_1, called the incoherent non-penetrating electric image coefficients, are found from the conformal mapping technique and depend only on the ratio w/h. For the ratio $w/h = 2.14$, we find $\xi_1^{\rm H} - \epsilon_1^{\rm H} = 0.2115$ and $\xi_1^{\rm V} - \epsilon_1^{\rm V} = 0.4113$. Since the beam size is very much less than the size of the beam pipe, only the a^{-2} term dominates; giving $g_{\rm H,V} = 1.85 \times 10^5$ for a circular pipe, and 1.87×10^5 and 1.86×10^5, respectively, for the horizontal and vertical direction of a rectangular pipe. Thus, we can conclude $(Z_{\rm H,V})_{\rm sp\ ch} \approx i410$ MΩ/m, which again only rolls off after ~ 20 GHz. Note that this estimate is very sensitive to the transverse beam size.

B. Space Charge Tune Shifts

We are also interested in the coherent and incoherent space-charge tune shifts, which, on the one hand, may provide Landau damping to beam instabilities, and, on the other hand, may limit the operation of the ring in the tune plane. The tune shifts result from direct space charge forces, electric image charges, and magnetic image currents inside the vacuum chamber walls and magnet laminations. They create electromagnetic fields which alter the transverse focusing force on the beam particles thus changing their tunes. The tune shifts are usually characterized [5, 6] by the electric coherent and incoherent image coefficients $\xi_1^{\rm H,V}$ and $\epsilon_1^{\rm H,V}$ and by the magnetic coherent and incoherent image coefficients $\xi_2^{\rm H,V}$ and $\epsilon_2^{\rm H,V}$, which depend on the geometry of the vacuum chamber and magnet laminations.

Summing up all of the above contributions to the tune shift [7], we arrive at the following expression for the total coherent tune shifts

$$\Delta\nu_{\rm coh}^{\rm H,V} = -\frac{4Nr_0R}{\pi\nu_{\rm H,V}\gamma}\left(\frac{\xi_1^{\rm H,V}}{\beta^2h^2} + \frac{\epsilon_2^{\rm H,V}}{g^2}\mathcal{F} - \frac{\xi_1^{\rm H,V} - \epsilon_1^{\rm H,V}}{h^2}\right) . \tag{2.7}$$

The first term is due to the electric image force from the wall, the second is due to the DC magnetic image force from the dipole pole faces, the third is due to the AC magnetic image forces from the wall as a result of the transverse betatron oscillations of the beam. Since we are dealing only with coasting beam only in the Recycler, the effect due to longitudinal bunching does not show up. In Eq. (2.7), $N = 2.53 \times 10^{12}$ is the total number of anti-protons in the beam, $r_0 = 1.535 \times 10^{-18}$ m is the classical proton radius, $g = 2''$ is the full magnet pole gap distance, $\mathcal{F} \approx 0.5$ is fraction of the ring circumference covered by dipole magnets, and $\nu_{\rm H} = 26.225$ and $\nu_{\rm V} = 26.215$ are the horizontal and vertical tunes.

Similarly the incoherent tune shifts are given by the expression [7]

$$\Delta\nu_{\rm inc}^{\rm H,V} = -\frac{4Nr_0R}{\pi\nu_{\rm H,V}\gamma}\left(\frac{\epsilon_1^{\rm H,V}}{\beta^2h^2} + \frac{\epsilon_2^{\rm H,V}}{g^2}\mathcal{F}\right) - \frac{Nr_0}{4\gamma^2\beta}\frac{6}{\epsilon_{N,95\%}} . \tag{2.8}$$

The incoherent tune shifts do not have a contribution from the AC magnetic field created by the transverse motion of the beam similar to the third term in Eq. 2.7. However, there is an extra term which represents the contribution from the direct space charge forces and is dependent on $\epsilon_{N,95\%}$, the 95% normalized transverse emittance. [8]. Here, a Gaussian distribution in both the longitudinal and vertical directions has been assumed.

With $\epsilon_2^H = -\epsilon_2^V = -\pi^2/24$ for infinite parallel plates, $\xi_1^H = 0.0118$, $\xi_1^V = 0.6110$, $\epsilon_1^H = -\epsilon_1^V = -0.1997$ for rectangular approximation, the computed tune shifts are:

	Horizontal	Vertical
Coherent	$+0.00189$	-0.00193
Incoherent	-0.00015	-0.00396

These tune shifts are relatively small and would not affect the operation of the Recycler in the tune diagram.

C. Resistive Wall

The longitudinal and transverse impedances of the elliptical beam pipe are: [4]

$$Z_{\|} = (1-i)\frac{2\rho R}{h\delta}F_{\rm L}\ , \tag{2.9}$$

$$Z_{\rm H,V} = (1-i)\frac{16\rho Rc}{\omega h^3\delta}F_{\rm H,V}\ , \tag{2.10}$$

where $\rho = 74\ \mu\Omega$-cm is the resistivity of the stainless steel that composes the beam pipe, and δ is the skin depth at frequency $\omega/2\pi$. The form factors are, respectively, $F_{\|,{\rm H,V}} = 0.9829$, 0.4018, and 0.8233. Therefore we get

$$\begin{aligned} Z_{\|}/n &= (1-i)12.0\times n^{-1/2}\ \Omega\ , \\ Z_{\rm H} &= (1-i)10.5\times(n+\nu_{\rm H})^{-1/2}\ {\rm M}\Omega/{\rm m}\ , \\ Z_{\rm V} &= (1-i)21.6\times(n+\nu_{\rm V})^{-1/2}\ {\rm M}\Omega/{\rm m}\ , \end{aligned} \tag{2.11}$$

where $\omega/\omega_0 = n$, $n+\nu_{\rm H}$, or $n+\nu_{\rm V}$ is the harmonic of the frequency under consideration.

D. BPM's

Not much information has been given for the BPM's. Here, we assume that they are similar to those in the Main Injector. For the 208 BPM's of the Main Injector, we get [2]

$$\begin{aligned} Z_\parallel/n &= -i0.095\ \Omega\ , \\ Z_{\rm H} &= -i2.66\ {\rm k}\Omega/{\rm m}\ , \\ Z_{\rm V} &= -i5.15\ {\rm k}\Omega/{\rm m}\ , \end{aligned} \tag{2.12}$$

when the frequency $f < c/(4\pi\ell) = 60$ MHz and $\ell = 40$ cm is the length of each stripline. These impedances are much lower than other components in the ring. This is mainly due to the relatively narrow width (1 cm) of the stripline pickups. Thus the BPM's are not expected to be a problem.

E. Bellows

There are many bellows in the Recycler. For the former *unshielded* design of the bellows in the Main Injector, one obtains using TBCI, [9] which assumes a circular beam pipe, a total longitudinal impedance of [2] $Z_\parallel/n = 0.466\ \Omega$ at the first peak near 4 GHz and $Z_\parallel/n \approx 0.3\ \Omega$ at the second. The transverse impedance appears as a broad resonance of $Z_\perp = 0.77$ MΩ/m centered around 8 GHz. Since the bellows in the Recycler are all shielded, we expect the impedance would be much lower than these values.

F. RF System

The Recycler RF system, which is required to generate longitudinal phase-space gymnastics, is composed of four 50-Ω gaps driven by wide-band solid-state amplifiers. The bandwidth of the net system is from 10 kHz to 100 MHz. Therefore, a particle is seeing 200 Ω of resistance. However, this impedance is purely resistive, and there is a feedback loop that monitors the beam current so that the right amount of energy will be delivered back to the beam to compensate the loss. As a result, the net impedance seen by the particle is essentially zero. The gain of the feedback rolls off at ~ 20 MHz. After that the beam still sees the 200 Ω resistance. Therefore the highest impedance per harmonic occurs at ~ 20 MHz. Therefore we expect at frequency f (in MHz)

$$\frac{Z_\parallel}{n} \approx 0.9 \times \frac{20}{f}\ \Omega \qquad 20\ {\rm MHz} < f < 100\ {\rm MHz}\ . \tag{2.13}$$

G. Vacuum Valves

The code MAFIA [10] had been employed [2] to study the impedances of some earlier designs of the vacuum valves of the Main Injector. The results are mainly resonances. For example, a design without pill-box cavity, gives a resonance at 1.3 GHz with $Z_{\|} = 52$ kΩ and $Q = 1600$ and another one at 2.9 GHz with $Z_{\|} = 13.4$ kΩ and $Q = 2700$. Since these resonances are rather narrow, we expect the contribution of the 30 vacuum valves not to add up, but rather produce broad bands of $Z_{\|}/n \approx 3.6\ \Omega$ near 1.5 GHz and 0.42 Ω near 2.5 GHz, which can be appreciable. There are also transverse resonances: one in the horizontal direction at 3.0 GHz with $Z_{\rm H} = 42$ kΩ/m and $Q = 2500$, and one in the vertical direction at 3.1 GHz with $Z_{\rm V} = 32$ kΩ/m and $Q = 2500$. We have also assumed the contributions of the 30 vacuum valves do not add up.

There are also small imaginary parts of the impedances, which arrive from the direct addition of the contributions of the 30 valves. If the parallel-circuit resonance model is assumed, we obtained $\mathcal{I}m\, Z_{\|}/n \approx 0.007\ \Omega$, $\mathcal{I}m\, Z_{\rm H} \approx 5$ kΩ/m, and $\mathcal{I}m\, Z_{\rm V} \approx 0.4$ kΩ/m at frequencies below the resonances.

H. Kickers

There will be only two kickers, each of length 1 m, in the Recycler. The rise times and fall times are 1 to 2 μs, which are much longer than the 395 ns rise time of the kickers in the Main Injector. For this reason, a thicker coating can be applied onto the ceramic beam pipe to carry the image current. At this moment, the thickness coating is set at one skin depth of stainless steel at 10 MHz. Therefore, we do not expect the kickers to generate any impedance larger than that of the resistive walls of the beam pipe.

I. Lambertsons

The injection and extraction rates of the Recycler are so slow that laminations in the Lambertsons will not be necessary. Instead block steel will be used. This implies the elimination of the high resistivity due to the exposed laminations, and the impedance is now the same as that of the resistive walls of the beam pipe.

J. Summary

We find that the both the longitudinal and transverse impedances of the Recycler are dominated by space charge which is capacitive. Although there are inductive contributions from BPM's, wall resistivity, vacuum valves, etc, they are insufficient to counteract the space-charge effect. If one wishes to quote broad-band impedances, they will be the space-charge impedances:

$$\begin{aligned} Z_{\parallel}/n &= i12\ \Omega\ , \\ Z_{\rm H,V} &= i410\ \mathrm{M}\Omega/\mathrm{m}\ , \end{aligned} \tag{2.14}$$

which stay flat and start to roll off only after ~ 20 GHz. It is worth pointing out that the estimation of $Z_{\rm H,V}$ may have been crude, because it depends on the beam size like a^{-2}.

III. LONGITUDINAL INSTABILITIES

We would like to see whether the intense coasting anti-proton beam at $E =$ 8.9 GeV will be stable inside the Recycler Vacuum chamber. One fast instability is the microwave instability. Because the beam is unbunched, this instability can be driven at any frequencies, for example, way below cutoff. The Keil-Schnell criterion [11] for this not to happen is

$$\frac{|Z_{\parallel}|}{n} \lesssim \frac{|\eta|(E/e)}{I}\left(\frac{\Delta E}{E}\right)^2_{\rm FWHM}\ , \tag{3.1}$$

where $\eta = \gamma_t^{-2} - \gamma^{-2}$ is the slippage factor and I the beam current. With the transition gamma $\gamma_t = 20.71$, one gets $\eta = -0.008688$. With $N = 2.53 \times 10^{12}$ anti-protons, the current is $I = 0.0364$ Amp. When the beam is cooled, the maximum energy spread is $\Delta E_{\rm max} = 0.5$ MeV. If we assume a parabolic distribution of energy, $(\Delta E/E)_{\rm FWHM} = \sqrt{2}(\Delta E/E)_{\rm max} = 0.7911 \times 10^{-4}$. The limit of stability is therefore

$$\frac{|Z_{\parallel}|}{n} \lesssim 13.3\ \Omega\ . \tag{3.2}$$

Our broad-band impedance is mainly due to space charge. Theoretically, a pure capacitive impedance cannot drive microwave instability below transition. However, for a parabolic energy distribution, the stability region in the traditional U-V plot is closed. Thus a large enough capacitive impedance plus a small resistive impedance can make the beam unstable. In Sec. II.A the longitudinal impedance has been very well estimated to be $Z_{\|}/n = i12\ \Omega$, because it is not sensitive to the beam size. Therefore, we may conclude that the beam will be stable against longitudinal microwave growth. Even if our estimate is not accurate enough, the growth rate should be very slow.

Sometimes the anti-protons are bunched by the barrier voltages of the RF system. Since the bunch area remains constant, the larger momentum spread will help the individual bunch be more stable. At some moment, the Recycler may contain a number of stacked bunches. However, the barrier voltages make each bunch look almost squared in the longitudinal phase space with very small gaps between bunches. We can therefore approximate these bunches as a coasting beam. This is the situation before cooling takes place and the momentum spread will be very much larger than 0.5 MeV. As a result, these bunches will definitely be Landau damped.

Even if we view these bunches as bunches instead of a coasting beam, there will not be any possibility of having longitudinal coupled-bunch instabilities. The cavities of Recycler RF system do not look like the usual cavities with sharp parasitic resonances. Instead, they resemble more closely kickers. This eliminates the possibility of driving couple-bunch modes. Even if there are other sharp resonances from other components, the growth rates will be reduced to a large extent by the form factor. This is because of the long bunch length, so that the phase of the resonance changes tremendously when the bunch passes the component from the head to the tail. This phase change dilutes the coherent effect of the coupling.

IV. TRANSVERSE INSTABILITIES

Transverse microwave instability can also occurs. The criterion for transverse microwave stability is [12]

$$|Z_\perp| \lesssim \frac{4\nu_\beta(E/e)}{IR}\left(\frac{\Delta E}{E}\right)_{\rm FWHM}|(n+\nu_\beta)\eta-\xi|\ , \tag{4.1}$$

where $\nu_\beta \approx 26.215$ is the betatron tune, ξ is the chromaticity, and $n+\nu_\beta$ is the instability driving harmonic under consideration. The Landau damping embedded in Eq. (4.1) comes from the momentum spread of the beam. Equation (4.1) gives the limit

$$|Z_\perp| \lesssim 3.86\,|(n+\nu_\beta)\eta-\xi|\ \ {\rm M}\Omega/{\rm m}\ . \tag{4.2}$$

Instability is possible only for the slow wave. In this notation, this happens when $n+\nu_\beta<0$. Since $\eta<0$ and the chromaticity will be operated at $\xi=-3$, the zero of $|(n+\nu_\beta)\eta-\xi|$ occurs only in the fast-wave region which will not lead to instability.

In Sec. II.C, we obtain the vertical wall resistive impedance $|Z_{\rm V}|=21.6\sqrt{2}(n+\nu_\beta)^{-1/2}$ MΩ/m. Substitution into Eq. (4.2) shows that there is an instability when $n+\nu_\beta\lesssim 8$, or at a frequency $f\lesssim 720$ kHz. Here, the chromaticity $\xi=-3$ is the dominant contribution of Eq. (4.2). Because the betatron tune is $\nu_\beta=26.215$, which is above an integer, this growth actually will not occur and the beam is stable.

In the situation that the tune is below an integer, the criterion of stability in Eq. (4.2) applies. The growth rate without taking into account of Landau damping can be readily computed, [13]

$$\tau^{-1}=-\frac{c^2}{2\nu_\beta\omega_0(E/e)}\frac{I\,\mathcal{R}e\,Z_\perp}{2\pi R}\ . \tag{4.3}$$

The highest growth occurs when $n+\nu_\beta$ which is negative and closest to zero. As an illustration, let us take this residual tune as -0.3. Assuming that the wall of the beam pipe is still larger than the skin depth at this frequency, $(\mathcal{R}e\,Z_{\rm V})_{\rm wall}\approx-39.4$ MΩ/m. Therefore, $\tau^{-1}\approx 147$ s^{-1}, or a growth time of 6.81 ms. This growth is very slow and can easily been damped by having a small tune spread not resulting from the momentum spread of the beam. Such a tune spread can be supplied by an octupole and only an amount of $\Delta\omega/\omega_\beta\gtrsim 1.0\times10^{-5}$ or $\Delta\nu_\beta\gtrsim 0.00027$ will be required. Even the incoherent space-charge tune spread computed in Sec. II.B will be sufficient. Therefore, a feedback damper is not necessary. Note that this is different from the situation of transverse coupled-bunch instability driven by the resistive wall. There, an octupole-providing tune spread is usually insufficient.

The transverse space charge impedance is $Z_\perp=i410$ MΩ/m. According to Eq. (4.2), it will drive an instability at $\xi=-3$ when $-(n+\nu_\beta)\lesssim 1.2\times10^4$ or $|f|\lesssim 1.1$ GHz.

Theoretically, pure reactive impedance will not lead to any growth of the transverse microwave instability. But for a parabolic energy distribution that gives a closed stability region in the U-V plot, the reactive impedance can shift the coherent frequency of the microwave growth outside the tune distribution of the beam so that all Landau damping will be lost. Then a small resistive component of the impedance will lead to a coherent growth. To see whether the space-charge impedance is really harmful, we compute the shift in frequency,

$$\delta\omega = \frac{c^2}{2\nu_\beta \omega_0 (E/e)} \frac{I\,\mathcal{I}m\,Z_\perp}{2\pi R} \approx 1540\ \mathrm{s}^{-1}\ . \tag{4.4}$$

We find a tune spread of $\Delta\omega/\omega_\beta \gtrsim 1.2 \times 10^{-4}$ or $\Delta\nu_\beta \gtrsim 0.0028$ will be sufficient to enclose the shift. Again, such a tune spread can be provided sufficiently by the vertical incoherent tune spread due to space charge. The horizontal incoherent tune spread due to space charge will not be large enough. On the safe side, we suggest the utilization of an octupole that can produce a tune spread of ~ 0.01.

Let us examine the situation when all the anti-protons are squeezed into a square-like bunch of total length $\tau_L = 1\ \mu$s. Since the phase-space area is conserved, the maximum energy spread becomes 5.57 MeV. Such a bunch has a bunch spectrum extended to roughly ± 1 MHz or $\pm 11 f_0$, which will excite the azimuthal $m = 0$ mode longitudinally along the bunch. For the $m = 1,\ 2,\ 3,\ \cdots$ modes, the excitation spectra peak at $\pm(m+1)/\tau_L$ or $\pm 11(m+1) f_0$. The Recycler is designed to run at a chromaticity of $\xi = -3$, implying that the spectra of all modes will be shifted to the right (or positive-frequency or fast-wave side) by

$$f_\xi = \frac{\xi}{\eta} f_0 = 345 f_0\ . \tag{4.5}$$

This means that the spectra of modes up to $m = 29$ will shifted to the fast-wave side and all these modes will be stable. The main reason for this to happen is the long length of the bunch, so that the spectrum for each mode does not spread out very much.

V. CONCLUSION

We have made estimation of the coupling impedances of the Recycler ring and found that they are dominated by space charge. We have examined the longitudinal instabilities and found that microwave instability will not occur if there are only $N = 2.53 \times 10^{12}$ anti-protons in the beam. Longitudinal coupling-bunch instability during injection stacking does not appear to be possible, because of the long bunch lengths (or short bunch gaps) and the lack of sharp resonances in the RF system. Transverse instability, on the other hand, cannot be Landau damped by the momentum spread in the beam. However, it can be cured by a small spread in the betatron tunes either coming from space charge or supplied by an octupole.

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