Octupoles for differential chromaticity settings in the Tevatron

Tanaji Sen
Fermilab, MS 220
Batavia, IL 60510

April 23, 2002

1 Introduction

The proton lifetime at injection energy depends sensitively on the chromaticities. Too low a chromaticity setting can make the beam unstable due to the weak head-tail instability. The anti-proton apparently does not suffer from this instability - this may be attributed to the lower intensity. If the anti-proton chromaticity is too high however, the lifetime suffers. A means to create a chromaticity split between the beams may therefore be useful. The existing octupole circuits, which are not presently used, may be used for this purpose. In this note we will consider the different families of octupoles which may be used.

There are 67 octupoles on 4 different circuits. Table 1 lists the name and number of each type of octupole.

Octupole Name	Number
TOZD	24
TOZF	12
TOF39S	7
TOD39S	8
TOF39C	8
TOD39C	8

Table 1: Octupoles in the Tevatron. (TOF39S, TOD39S) are on the same bus but the TOD39S are rotated by 45°, thus reversing its polarity. Similarly (TOF39C, TOD39C).

Figure 1 shows the layout of the 4 families of octupoles in the Tevatron. A more detailed sketch showing the spacing between the octupoles is in Figure 6 in Appendix A.

While using the octupoles for chromaticity adjustment, we need to ensure that they do not introduce undesirable effects. The side effects of octupoles include: (i) second order chromaticity, (ii) additional coupling and (iii) nonlinear effects which can be characterized by additional amplitude dependent tune shifts, resonances and perhaps also interference with existing nonlinearities. We will choose the octupole families to minimize these effects.

2 Octupoles for Chromaticity Correction

Sextupoles are used to correct the chromaticity created by the quadrupoles but the chromaticity is the same for both protons and anti-protons. Sextupoles are used however to create a differential tune split between the beams via feed-down of the different orbits. Similarly octupoles can be used to create chromaticity differences between the two beams via feed-down of the orbit.

The field due to a octupole in terms of the multipole components (b_3, a_3) is

$$B_y + iB_x = \frac{B_0(b_3 + ia_3)}{R_{ref}^3}[x^3 - 3xy^2 + i(3x^2y - y^3)] \tag{2.1}$$

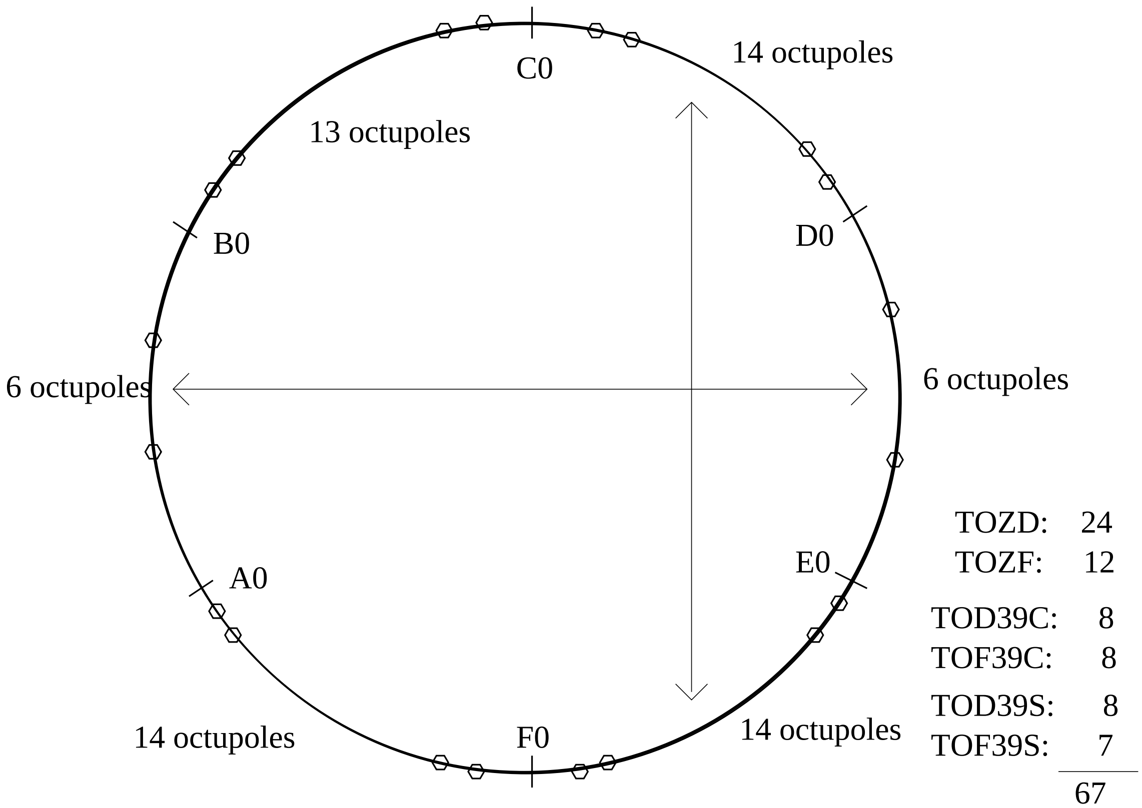

Figure 1: Layout of the octupoles in the Tevatron. The distribution of octupoles between A0 and B0 is identical to that between D0 and E0 and similarly the distribution between C0 to D0 and between E0 to F0 are identical. A more detailed layout can be seen in Appendix A.

The multipole components b_n are related to the integrated strengths $K_n L$ used in MAD as

$$k_n \equiv K_n L = \frac{n!}{R_{ref}^n \rho} b_n L = \frac{1}{(B\rho)} \frac{\partial^n B_y}{\partial x^n} L \tag{2.2}$$

Closed orbit offsets $\Delta x, \Delta y$ result in feed-down normal and skew sextupoles

$$b_2^{eff} = \frac{3}{R_{ref}}[b_3 \Delta x - a_3 \Delta y] \ , \qquad a_2^{eff} = \frac{3}{R_{ref}}[b_3 \Delta y + a_3 \Delta x] \tag{2.3}$$

Sextupoles at locations with non-zero dispersion (D_x, D_y) act as quadrupoles for particles with non-zero relative momentum deviation δp.

$$b_1^{eff} = \frac{2}{R}(b_2 D_x - a_2 D_y)\delta_p \ , \qquad a_1^{eff} = \frac{2}{R}(b_2 D_y + a_2 D_x)\delta_p \tag{2.4}$$

Hence the effective quadrupolar (normal and skew) strengths of normal octupoles are

$$b_1^{eff} = \frac{6}{R^3} b_3 [D_x \Delta x - D_y \Delta y]\delta_p \ , \qquad a_1^{eff} = \frac{6}{R^3} b_3 [D_y \Delta x + D_x \Delta y]\delta_p \tag{2.5}$$

From the expression for the tune change due to a gradient error, it follows that the change in chromaticities due to octupoles at locations with dispersion is

$$\begin{aligned} \nu_x^{(1)} &= \frac{1}{4\pi} \sum_j (\beta_x k_3 [D_x \Delta x - D_y \Delta y])_j \\ \nu_y^{(1)} &= -\frac{1}{4\pi} \sum_j (\beta_y k_3 [D_x \Delta x - D_y \Delta y])_j \end{aligned} \tag{2.6}$$

Octupole Family	$\langle x_{co}\rangle, \langle y_{co}\rangle$ [mm]	$\langle \beta_x\rangle, \langle \beta_y\rangle$ [m]	$\langle D_x\rangle, \langle D_y\rangle$ [m]
ZF	-0.819, 0.148	93.6, 30.2	3.67, -0.02
ZD	-0.271, 0.136	30.5, 92.5	2.07, -0.01
F39S	0.237, -1.023	101.1, 31.0	4.71, 0.017
D39S	2.945, 0.248	85.2, 29.4	4.76, 0.054
F39C	-0.175, 0.438	88.1, 30.9	3.12, 0.011
D39C	-1.645, -1.248	98.1, 29.6	3.11,-8.7$\times 10^{-5}$

Table 2: Average Twiss functions in the octupoles.

where

With all 4 octupole families energized, change in proton chromaticities are

$$\begin{aligned}\nu_x^{(1)}(p) &= k_{3,ZF}S_x(ZF,p) + k_{3,ZD}S_x(ZD,p) + k_{3,39S}[S_x(F39S,p) - S_x(D39S,p)] \\ &\quad +k_{3,39C}[S_x(F39C,p) - S_x(D39C,p)] \qquad (2.7)\\ \nu_y^{(1)}(p) &= k_{3,ZF}S_y(ZF,p) + k_{3,ZD}S_y(ZD,p) + k_{3,39S}[S_y(F39S,p) - S_y(D39S,p)] \\ &\quad +k_{3,39C}[S_y(F39C,p) - S_y(D39C,p)] \qquad (2.8)\end{aligned}$$

where e.g

$$S_x(ZF,p) = \frac{1}{4\pi}(\sum_j \beta_{x,j}[D_x\Delta x(p) - D_y\Delta y(p)]_j)_{ZF} \qquad (2.9)$$

$$S_y(ZF,p) = -\frac{1}{4\pi}(\sum_j \beta_{y,j}[D_x\Delta x(p) - D_y\Delta y(p)]_j)_{ZF} \qquad (2.10)$$

Here we have assumed that the D39S and D39C are rotated by 45° and have strengths opposite to F39S and F39C respectively. The change in anti-proton chromaticities are given by similar expressions with the orbit offsets $(\Delta x(p), \Delta y(p))$ replaced by the offsets of the anti-protons $(\Delta x(\bar{p}), \Delta y(\bar{p}))$ in the octupoles. The changes in chromaticities will be different in the two species because of their different orbits.

Figure 2 shows the closed orbits of the protons in the octupoles while Figure 3 shows the sum of the proton and anti-proton orbits in each plane at all the quadrupoles. We note that due to the helical orbits, the protons and anti-protons are roughly on opposite sides of the central axis of the octupoles.

In principle 4 families of octupoles required to correct all 4 quantities $\nu_x^{(1)}(p), \nu_y^{(1)}(p), \nu_x^{(1)}(\bar{p}), \nu_y^{(1)}(\bar{p})$. In practice, 2 families of octupoles suffice because of the anti-symmetry of the orbits

$$\Delta x(\bar{p}) \approx -\Delta x(p), \qquad \Delta y(\bar{p}) \approx -\Delta y(p)$$

which imply that if 2 families are used to change the proton chromaticities, then

$$\nu_x^{(1)}(\bar{p}) \approx -\nu_x^{(1)}(p), \qquad \nu_y^{(1)}(\bar{p}) \approx -\nu_y^{(1)}(p) \qquad (2.11)$$

2.1 Effectiveness of Octupoles

We have a choice of octupole families to use. The minimum strengths are required when the families with the largest value of the functions S_x, S_y, defined above, are used. Table 2 shows the average Twiss functions in each family. We note that only the ZD octupole family is near the D quadrupoles where the vertical beta functions are larger than the horizontal beta functions. All others are next to F quadrupoles with larger horizontal beta functions.

The orbits change sign in the octupoles, as Figure 2 shows. Thus octupoles in the same family may not be in phase with the helical orbit which may lead to opposing contributions from members in the family. This can be corrected by choosing the polarity of each octupole such that all members of a family contribute with the same

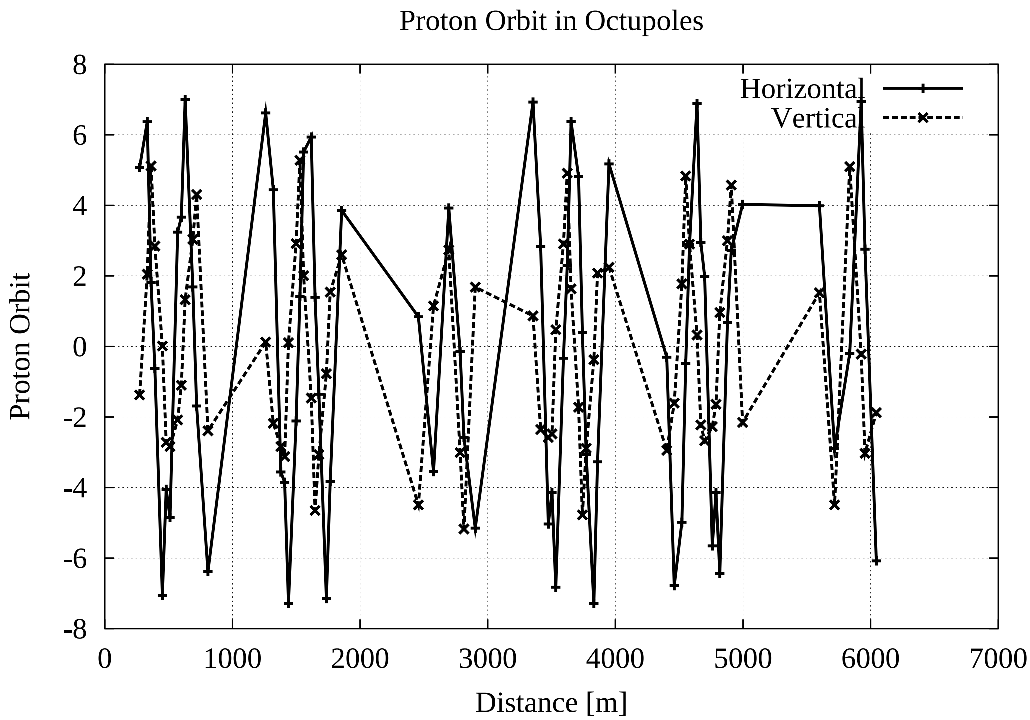

Figure 2: Horizontal and vertical closed orbit of protons at the locations of the 67 octupoles in the Tevatron.

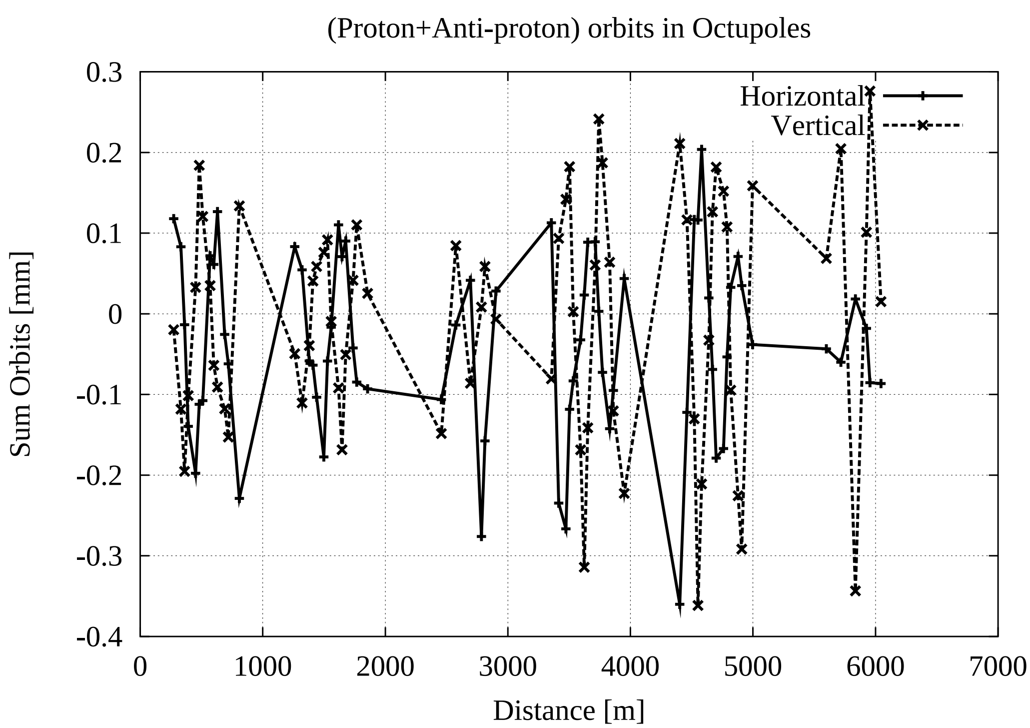

Figure 3: Sum of the proton and anti-proton closed orbits at the octupoles.

	No Change $(\langle S_x\rangle, \langle S_y\rangle) \times 10^{-2}$	Polarity change (if required) $(\langle S_x\rangle, \langle S_y\rangle) \times 10^{-2}$
ZF	(-1.70, 0.65)	(12.6, -4.13)
ZD	(-0.10, 0.35)	(-1.29, 4.0)
F39S	(0.82, -0.27)	(8.9, -2.8)
D39S (oppos.)	(10.7, -3.72)	(-20.6, 7.1)
F39C	(3.03, -0.87)	(13.3, -4.7)
D39C (oppos.)	(-4.11, 1.25)	(-8.2, 2.5)

Table 3: Effective parameters $\langle S_x\rangle$, $\langle S_y\rangle$ (defined in Equation (2.12)) of each family for two cases.

sign. Thus we define effective parameters for each family as, for example for ZF,

$$\begin{aligned}\langle S_x(ZF,p)\rangle &= \frac{1}{4\pi N_{ZF}}\sum_j (w_j\beta_x[D_x\Delta x(p) - D_y\Delta y(p)])_j \\ \langle S_y(ZF,p)\rangle &= -\frac{1}{4\pi N_{ZF}}\sum_j (w_j\beta_y[D_x\Delta x(p) - D_y\Delta y(p)])_j \end{aligned} \tag{2.12}$$

The weighting function $w_j = 1$ if there is no change in polarity, otherwise $w_j = \mathrm{sign}(D_x\Delta x(p) - D_y\Delta y(p))$ when all octupoles in the family contribute in phase. N_{ZF} is the number of ZF octupoles and the sum extends over all members of this family. Table 3 shows the effective parameters of each family for these two cases. It is evident that choosing the polarity of the octupole according to the sign of $D_x\Delta x(p) - D_y\Delta y(p)$ increases the effectiveness of that family by an order of magnitude in most cases.

3 Second Order Chromaticity

Here we consider the second order chromaticity generated when we use octupoles to create a split in linear chromaticity between protons and anti-protons. The tune shifts of off-momentum particles with orbit offsets $(\Delta x, \Delta y)$ in the octupoles are

$$\begin{aligned}\nu_x &= \frac{1}{4\pi}\sum_j (\beta_x k_3[D_x\Delta x - D_y\Delta y])_j\delta_p \\ \nu_y &= -\frac{1}{4\pi}\sum_j (\beta_y k_3[D_x\Delta x - D_y\Delta y])_j\delta_p \end{aligned} \tag{3.1}$$

The orbit offsets are composed of two parts: one due to the closed orbit determined by the separator settings and the other due to dispersion

$$\Delta x = \Delta x_{sep} + D_x\delta_p, \qquad \Delta y = \Delta y_{sep} + D_y\delta_p \tag{3.2}$$

With the contribution from the dispersions alone written separately, the tune shifts to second order in δ_p are,

$$\begin{aligned}\nu_x &= \frac{1}{4\pi}\sum_j (\beta_x k_3)_j\left\{[D_x\Delta x_{sep} - D_y\Delta y_{sep}]\delta_p + [D_x^2 - D_y^2]\delta_p^2\right\}_j \equiv \nu_x^{(0)} + \nu_x^{(1)}\delta_p + \frac{1}{2}\nu_x^{(2)}\delta_p^2 \\ \nu_y &= -\frac{1}{4\pi}\sum_j (\beta_y k_3)_j\left\{[D_x\Delta x - D_y\Delta y])_j\delta_p + [D_x^2 - D_y^2]\delta_p^2\right\}_j \equiv \nu_y^{(0)} + \nu_y^{(1)}\delta_p + \frac{1}{2}\nu_y^{(2)}\delta_p^2 \end{aligned} \tag{3.3}$$

This identifies the second chromaticity contribution from the octupoles

$$\begin{aligned}\nu_x^{(2)} &= \frac{1}{2\pi}\sum_j (\beta_x k_3[D_x^2 - D_y^2])_j \\ \nu_y^{(2)} &= -\frac{1}{2\pi}\sum_j (\beta_y k_3[D_x^2 - D_y^2])_j \end{aligned} \tag{3.4}$$

This expression does not depend on the orbit offsets, hence the second order chromaticity due to the octupoles will be the same for both protons and anti-protons.

4 Detuning and resonance driving terms

The field due to an octupole is given by Equation (2.1). The change in the Hamiltonian due to the octupole field is

$$\Delta H = \frac{R}{4!} K_3(\theta)[x^4 - 6x^2y^2 + y^4] \tag{4.1}$$

Here the independent variable is θ, R is the machine radius and the octupole strength parameter K_3 is defined as in Equation (2.2).

Introducing the action angle coordinates

$$x = \sqrt{2\beta_x J_x}\cos\phi_x\ , \qquad y = \sqrt{2\beta_y J_y}\cos\phi_y \tag{4.2}$$

The angles ϕ_x, ϕ_y can be split into two parts, a periodic part α_x, α_y and a part ψ_x, ψ_y which advances linearly with the azimuth θ,

$$\begin{aligned} \phi_x &= \alpha_x(\theta) + \psi_x(\theta) \\ \alpha_x &= \int_0^s \frac{ds'}{\beta_x(s')} - \nu_x\theta + \phi_{x,0} \\ \psi_x &= \nu_x\theta \end{aligned} \tag{4.3}$$

The distance $s = R\theta$. Here we have included the initial phase $\phi_{x,0}$ in the definition of the periodic part of the phase α_x. Similarly α_y and ψ_y are defined.

Expanding the coordinates in terms of the actions and angles, the perturbation can be written as

$$\begin{aligned} \Delta H &= \sum_{m_x=0,\pm2,\pm4}\ \sum_{m_y=0,\pm2,\pm4} U_{m_x,m_y} e^{i[m_x\alpha_x + m_y\alpha_y]} e^{i[m_x\psi_x + m_y\psi_y]} \\ &\equiv \sum_{m_x=0,\pm2,\pm4}\ \sum_{m_y=0,\pm2,\pm4} V_{m_x,m_y} e^{i[m_x\psi_x + m_y\psi_y]} \end{aligned} \tag{4.4}$$

where

$$U_{m_x,m_y} = \frac{R}{4!}K_3(\theta)\sum_{l=0}^{2} J_x^l J_y^{2-l} g_{m_x,m_y,l}(\beta_x,\beta_y) \tag{4.5}$$

Since $V_{m_x,m_y} = U_{m_x,m_y}e^{i[m_x\alpha_x + m_y\alpha_y]}$ is periodic in θ, it can be expanded in a Fourier series,

$$V_{m_x,m_y} = \sum_{p=-\infty}^{\infty} W_{m_x,m_y;p}e^{-ip\theta} \equiv \sum_{l=0}^{2} J_x^l J_y^{2-l} \sum_{p=-\infty}^{\infty} C_{m_x,m_y,l;p}e^{-ip\theta} \tag{4.6}$$

and the perturbation is

$$\Delta H = \sum_{m_x}\sum_{m_y}\sum_{p} W_{m_x,m_y;p}e^{i[(m_x\nu_x + m_y\nu_y - p)\theta]} \tag{4.7}$$

Resonances are associated with the slowly varying terms in the Hamiltonian, i.e when the following conditions are satisfied

$$\Delta_p \equiv m_x\nu_x + m_y\nu_y - p \ll 1 \tag{4.8}$$

Octupoles can drive the following six families of resonances,

$$2\nu_x = p, \qquad 2\nu_y = p \tag{4.9}$$

$$2\nu_x + 2\nu_y = p, \qquad 2\nu_x - 2\nu_y = p \tag{4.10}$$

$$4\nu_x = p \qquad 4\nu_y = p \tag{4.11}$$

From Equation (4.6) it follows that the coefficients $W_{m_x,m_y;p}$ are obtained by taking the inverse Fourier transform of V_{m_x,m_y}. Hence it follows that the resonance driving terms are

$$C_{m_x,m_y,l;p} = \frac{R}{2\pi 4!}\int_0^{2\pi} d\theta K_3(\theta) g_{m_x,m_y,l}(\beta_x,\beta_y)\exp[i\{m_x\phi_x + m_y\phi_y - \Delta_p\theta\}] \tag{4.12}$$

We will assume that we are close to a particular resonance specified by (m_x, m_y, p) so that $\Delta_p \ll 1$. We will assume that the integral over the distribution of octupoles can be replaced by a discrete sum with the assumption that the phases and beta functions are nearly constant over the length of a single octupole. The resonance driving terms in a form suitable for numerical evaluation are

$$C_{m_x,m_y,l;p} = \frac{1}{2\pi 4!}\sum_n k_3(n) g_{m_x,m_y,l}(\beta_{x,n},\beta_{y,n})\exp[i\{m_x\phi_{x,n} + m_y\phi_{y,n} - \Delta_p\theta_n\}] \tag{4.13}$$

where $k_3(n) = K_3 L_n$ is the integrated strength of the nth octupole.

The non-zero driving terms associated with the octupoles are

$$\begin{aligned}
C_{\pm 202;p} &= \frac{1}{2\pi 4!}\sum_n k_3(n)\beta_x^2(n)\exp[i(\pm 2\phi_{x,n} - \Delta_p\theta_n)] \\
C_{\pm 201;p} &= -\frac{3}{2\pi 4!}\sum_n k_3(n)\beta_x(n)\beta_y(n)\exp[i(\pm 2\phi_{x,n} - \Delta_p\theta_n)] \\
C_{0\pm 21;p} &= -\frac{3}{2\pi 4!}\sum_n k_3(n)\beta_x(n)\beta_y(n)\exp[i(\pm 2\phi_{y,n} - \Delta_p\theta_n)] \\
C_{0\pm 20;p} &= \frac{1}{2\pi 4!}\sum_n k_3(n)\beta_y^2(n)\exp[i(\pm 2\phi_{y,n} - \Delta_p\theta_n)] \\
C_{\pm 2\pm 21;p} &= -\frac{3}{2(2\pi)4!}\sum_n k_3(n)\beta_x(n)\beta_y(n)\exp[i(\pm 2\phi_{x,n} \pm 2\phi_{y,n} - \Delta_p\theta_n)] \\
C_{\pm 402;p} &= \frac{1}{4(2\pi)4!}\sum_n k_3(n)\beta_x^2(n)\exp[i(\pm 4\phi_{x,n} - \Delta_p\theta_n)] \\
C_{0\pm 40;p} &= \frac{1}{4(2\pi)4!}\sum_n k_3(n)\beta_y^2(n)\exp[i(\pm 4\phi_{y,n} - \Delta_p\theta_n)]
\end{aligned} \tag{4.14}$$

The detuning is determined by the zeroth harmonic part of the perturbed Hamiltonian

$$\Delta H(m_x = 0, m_y = 0) = \sum_l J_x^l J_y^{2-l}\sum_{p=-\infty}^{\infty} C_{m_x,m_y,l;;p}\exp(-ip\theta) \tag{4.15}$$

The change in tunes are found from:

$$\begin{aligned}
\Delta\nu_x &= \frac{1}{2\pi}\int d\theta\frac{\partial}{\partial J_x}\Delta H(m_x = 0, m_y = 0) = \sum_l lC_{00l0}J_x^{l-1}J_y^{2-l} \\
\Delta\nu_y &= \frac{1}{2\pi}\int d\theta\frac{\partial}{\partial J_y}\Delta H(m_x = 0, m_y = 0) = \sum_l (2-l)C_{00l0}J_x^{l}J_y^{1-l}
\end{aligned} \tag{4.16}$$

The coefficients are

$$\begin{aligned}
C_{0000} &= \frac{1}{k2\pi 4!}\frac{3}{2}\sum_n k_3(n)\beta_{y,n}^2 \\
C_{0010} &= -6\frac{1}{2\pi 4!}\sum_n k_3(n)\beta_{x,n}\beta_{y,n} \\
C_{0020} &= \frac{1}{2\pi 4!}\frac{3}{2}\sum_n k_3(n)\beta_{x,n}^2
\end{aligned} \tag{4.17}$$

Hence the action dependent tune shifts are

$$
\begin{aligned}
\Delta\nu_x &= \frac{1}{16\pi} J_x \sum_n k_3(n)\beta_{x,n}^2 - \frac{1}{8\pi} J_y \sum_n k_3(n)\beta_{x,n}\beta_{y,n} \\
\Delta\nu_y &= -\frac{1}{8\pi} J_x \sum_n k_3(n)\beta_{x,n}\beta_{y,n} + \frac{1}{16\pi} J_y \sum_n k_3(n)\beta_{y,n}^2
\end{aligned} \tag{4.18}
$$

The resonant condition is satisfied when

$$
m_x(\nu_{x,0} + \Delta\nu_x) + m_y(\nu_{y,0} + \Delta\nu_y) = p
$$

where $(\nu_{x,0}, \nu_{y,0})$ are the bare tunes. This defines a family of resonant actions $(J_{x,res}, J_{y,res})$ which lie on the lines defined by

$$
[\sum_n k_3(n)\beta_{x,n}(m_x\beta_{x,n} - 2m_y\beta_{y,n})]J_x + [\sum_n k_3(n)\beta_{y,n}(m_y\beta_{y,n} - 2m_x\beta_{x,n})]J_y = 16\pi(p - m_x\nu_{x,0} - m_y\nu_{y,0}) \tag{4.19}
$$

5 Select Results on Octupole Families

All possible combinations of two family solutions (six in all), three family solutions (four in all) and the single four family solution have been examined and with all possible choices of polarities (no change or a change according to the sign of $D_x\Delta x(p) - D_y\Delta y(p)$ in a octupole). The results are tabulated in Appendix B. Here in this section we choose to show only those solutions which require low octupole strengths and may therefore be practical. We will also assume (perhaps somewhat confusingly) that the D39S and D39C octupoles are not rotated by 45° so that the decision on whether to change polarity or not according to the sign of $D_x\Delta x(p) - D_y\Delta y(p)$ is based with respect to the un-rotated configuration.

Table 2 in Section 2 shows that only the ZD family of octupoles is at locations where the vertical beta functions are large. This is also the family with the largest number of octupoles. Hence this family must be essential in any scheme where two or more families are used to create the chromaticity split.

In addition to the strengths required to create the desired chromaticity split, we also look at the second order chromaticities, detuning with amplitude and resonance driving terms. While the second order chromaticity and resonance driving terms are always required to be small, the tune spread may be required to be small or large depending on the purpose. If the concern is that the additional tune spread created will cause the beam distribution to span some resonances, then clearly a small value is better. If however the intent is to stabilize the beam against collective effects due to impedances, then a larger tune spread is required. For this reason we have included possible choices with 3 families of octupoles, since these lead to somewhat different tune spreads than the 2 family solutions.

We define the detuning coefficients as

$$
C_{xx} \equiv \frac{\partial\Delta\nu_x}{\partial J_x} = \frac{1}{16\pi}\left[k_{3,ZF}(\sum w_n\beta_{x,n}^2)_{ZF} + k_{3,39S}(\sum w_n\beta_{x,n}^2)_{39S} + \ldots\right] \tag{5.1}
$$

$$
C_{xy} \equiv \frac{\partial\Delta\nu_x}{\partial J_y} = \frac{\partial\Delta\nu_y}{\partial J_x} = -\frac{1}{8\pi}\left[k_{3,ZF}(\sum w_n\beta_{x,n}\beta_{y,n})_{ZF} + k_{3,39S}(\sum w_n\beta_{x,n}\beta_{y,n})_{39S} + \ldots\right] \tag{5.2}
$$

$$
C_{yy} \equiv \frac{\partial\Delta\nu_y}{\partial J_y} = \frac{1}{16\pi}\left[k_{3,ZF}(\sum w_n\beta_{y,n}^2)_{ZF} + k_{3,39S}(\sum w_n\beta_{y,n}^2)_{39S} + \ldots\right] \tag{5.3}
$$

These coefficients will be shown as well the tune shifts at three representative amplitudes.

Figures 4 and 5 show the tune footprints of protons and anti-protons with and without octupoles. The octupole strengths were chosen to generate a tune spread and did not involve any changes in polarity. Without octupoles the horizontal tune spread for anti-protons (about 7E-3) is larger than for protons. This larger spread should help to stabilize the anti-protons against instabilities at injection. With the octupole strengths chosen, we observe that the vertical tune spread of both species increases while the horizontal tune spread shrinks.

In the following we calculate the octupole strengths required to create the desired chromaticity splits. The relevant parameters for the calculations are shown in Table 4. Tables 5, 6 and 7 show the octupole strengths, detuning and tune shifts and the resonance driving terms respectively for those select cases where the octupole strengths are small.

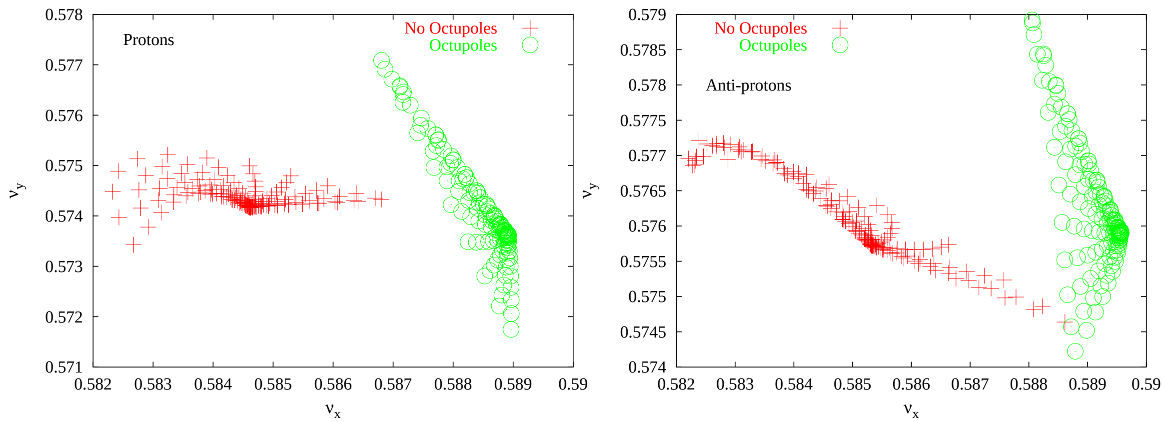

Figure 4: Left: Footprints of the proton beam with and without octupoles. Octupole settings were $k_3(ZF) = 5$, $k_3(ZD) = 3$. Right: Footprints of anti-protons with the same octupole strengths.

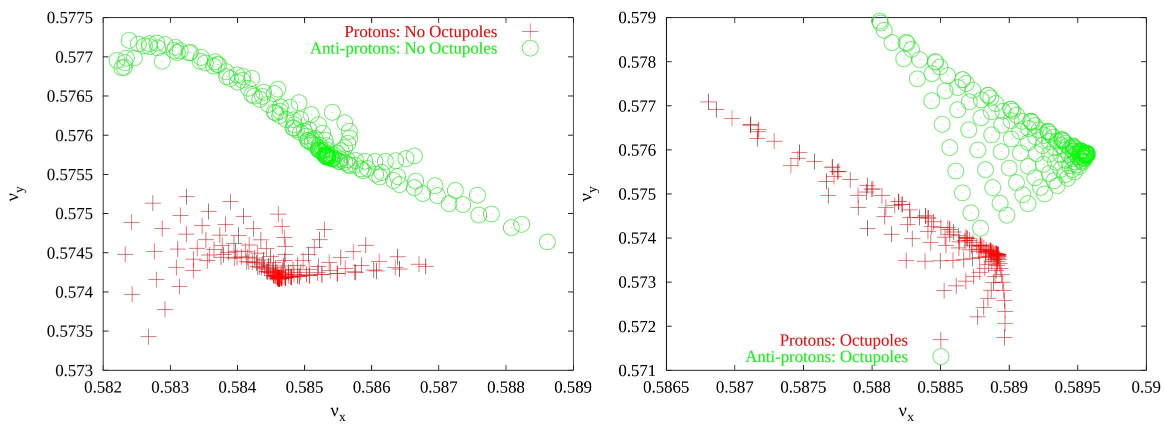

Figure 5: Left: Footprints of the proton and anti-proton beams without octupoles. Right: Footprints with octupoles. Octupole strengths were the same as above.

Energy [GeV]	150
Linear tune	20.585, 20.576
Linear lattice chromaticity w/out octupoles	15.04, 14.96
Desired change in proton chromaticities $\nu_x^{(1)}, \nu_y^{(1)}$	+5, +5

Table 4: Relevant parameters used in calculating octupole strengths. The lattice parameters were extracted from a MAD lattice file for the Tevatron (ca. August 2001).

Case	Families	$k_3(ZF)$	$k_3(ZD)$	$k_3(39S)$	$k_3(39C)$	$(\nu_x^{(2)}, \nu_y^{(2)}) \times 10^4$
			2 Families			
2.1	ZF(pc) & ZD(pc)	4.89	7.75	0	0	(1.79, -1.70)
2.2	ZD(pc) & 39S	0	7.94	-9.35	0	(0.39, -1.37)
2.3	ZD(pc) & 39S (pc)	0	7.84	3.27	0	(0.44, -1.24)
2.4	ZD(pc) & 39C	0	7.49	0	12.82	(0.31, -1.25)
2.5	ZD(pc) & 39C (pc)	0	7.79	0	4.30	(0.39, -1.27)
			3 Families			
3.1	ZF(pc), ZD(pc) & 39S	5.64	7.72	1.43	0	(2.01, -1.75)
3.2	ZF(pc), ZD(pc) & 39S(pc)	11.29	7.64	-4.27	0	(3.56, -2.29)
3.3	ZF(pc), ZD(pc)& 39C	1.76	7.58	0	8.21	(0.84, -1.41)
3.4	ZD(pc), 39S(pc) & 39C	0	7.65	1.49	7.00	(0.37, -1.25)

Table 5: Integrated octupole strengths required to create the chromaticity split in different cases. The abbreviation pc denotes a polarity change according to $D_x \Delta x(p) - D_y \Delta y(p)$. The proton chromaticities were corrected to +5 units, the resulting anti-proton chromaticities were found to be close to -5 units in each case. The last column shows the 2$^{\mathrm{nd}}$ order chromaticities.

Case	Families	$(C_{xx}, C_{xy}, C_{yy}) \times 10^4$	$(\Delta\nu_x, \Delta\nu_y)\times 10^{-3}$		
			(6,0) σ	(0, 6) σ	(6, 6) σ
		2 Families			
2.1	ZF(pc) & ZD(pc)	(1.38, -2.75, 3.27)	3.88, -7.75	-7.75, 9.22	-3.87, 1.47
2.2	ZD(pc) & 39S	(0.11, -2.07, 3.25)	0.30, -5.84	-5.84, 9.15	-5.54, 3.31
2.3	ZD(pc) & 39S (pc)	(0.44, -2.14, 3.20)	1.23, -6.02	-6.02, 9.01	-4.79, 2.99
2.4	ZD(pc) & 39C	(-0.047, -1.95, 3.08)	-0.13, -5.48	-5.48, 8.66	-5.61, 3.18
2.5	ZD(pc) & 39C (pc)	(0.22, -2.08, 3.19)	0.62, -5.85	-5.85, 8.98	-5.23, 3.13
		3 Families			
3.1	ZF(pc), ZD(pc) & 39S	(1.57, -2.86, 3.28)	4.43, -8.04	-8.04, 9.23	-3.62, 1.19
3.2	ZF(pc), ZD(pc) & 39S(pc)	(2.61, -3.56, 3.37)	7.33, -10.00	-10.00, 9.48	-2.68, -0.52
3.3	ZF(pc), ZD(pc)& 39C	(0.47, -2.24, 3.15)	1.31, -6.30	-6.30, 8.86	-4.99, 2.56
3.4	ZD(pc), 39S(pc) & 39C	(0.17, -2.03, 3.13)	2.28, -6.40	-6.40, 9.26	-4.21, 2.86

Table 6: Detuning with amplitude and tune shifts at three amplitudes due to octupole strengths shown in Table 5.

Case	Families	2nd order resonance driving terms ($\times 10^2$) $C_{202;41}$ (cosine, sine)	$C_{201;41}$ (cosine, sine)	$C_{021;41}$ (cosine, sine)	$C_{020;41}$ (cosine, sine)
		2 Families			
2.1	ZF(pc) & ZD(pc)	(2.60, -1.44)	(5.32, 0.45)	(1.39, 7.52)	(-2.01,-1.04)
2.2	ZD(pc) & 39S	(5.83, -74.2)	(-0.012, 71.3)	(-33.5, 67.8)	(2.00, -7.50)
2.3	ZD(pc) & 39S (pc)	(-3.74, 25.6)	(9.83, -25.6)	(16.2, -16.0)	(-3.46, 1.54)
2.4	ZD(pc) & 39C	(-111., -3.83)	(113, 3.09)	(98.8, 52.2)	(-12.3, -5.88)
2.5	ZD(pc) & 39C (pc)	(-38.1, -1.49)	(43.0, 0.73)	(35.4, 21.4)	(-5.51, -2.51)
		3 Families			
3.1	ZF(pc), ZD(pc) & 39S	(2.11, 9.71)	(6.13, -10.4)	(6.74, -1.70)	(-2.62, -0.052)
3.2	ZF(pc), ZD(pc) & 39S(pc)	(10.9, -36.7)	(-0.58, 34.4)	(-18.0, 38.2)	(-0.109, -4.42)
3.3	ZF(pc), ZD(pc)& 39C	(36.9, -0.71)	(-27.4, -0.35)	(-28.0, -6.0)	(1.11, 0.42)
3.4	ZD(pc), 39S(pc) & 39C	(70.7, 46.0)	(-62.1, -45.5)	(-41.1, -63.3)	(2.70, 6.70)
Case	Families	4th order resonance driving terms ($\times 10^2$) $C_{221;82}$ (cosine, sine)	$C_{2-21;0}$ (cosine, sine)	$C_{402;82}$ (cosine, sine)	$C_{042;82}$ (cosine, sine)
		2 Families			
2.1	ZF(pc) & ZD(pc)	(-0.96, 3.74)	(-57.1, 35.9)	(-0.31, -0.43)	(1.56, -1.00)
2.2	ZD(pc) & 39S	(-0.34, 0.65)	(-40.5, 31.9)	(0.89, 1.99)	(1.52, -1.02)
2.3	ZD(pc)& 39S (pc)	(-1.61, 3.43)	(-43.0, 30.4)	(-0.34, -0.88)	(1.59, -0.92)
2.4	ZD(pc) & 39C	(1.42, 0.62)	(-37.4, 31.1)	(-2.29, 0.79)	(1.38, -0.71)
2.5	ZD(pc)& 39C (pc)	(-0.38, 2.03)	(-41.0, 31.1)	(-0.78, 0.17)	(1.52, -0.87)
		3 Families			
3.1	ZF(pc), ZD(pc)& 39S	(-1.06, 4.22)	(-59.6, 36.5)	(-0.49, -0.80)	(1.57, -1.00)
3.2	ZF(pc), ZD(pc)& 39S(pc)	(-0.12, 4.15)	(-75.5, 43.1)	(-0.27, 0.17)	(1.52, -1.10)
3.3	ZF(pc), ZD(pc)& 39C	(-1.68, 4.69)	(-63.1, 37.3)	(0.29, -0.80)	(1.62, -1.09)
3.4	ZD(pc), 39S(pc) & 39C	(-3.71, 5.39)	(-46.9, 29.9)	(1.01, -2.05)	(1.74, -1.07)

Table 7: 2nd and 4th order resonance driving terms due to octupoles with strengths shown in Table 5.

	Ranking by case
Strengths	
2 family solutions	(2.1, 2.3, 2.5) | (2.2, 2.4)
3 family solutions	3.1 | (3.3, 3.4) | 3.2
2^{nd} order chromaticities	
$\nu_x^{(2)}$	(2.4, 3.4, 2.2, 2.5, 2.3, 3.3) | (2.1, 3.1, 3.2)
$\nu_x^{(2)}$	(2.3, 2.4, 3.4, 2.5, 2.2, 3.3) | (2.1, 3.1, 3.2)
Detuning with amplitude	
C_{xx}	(2.4, 2.2, 3.4, 2.5, 2.3, 3.3) | (2.1, 3.1, 3.2)
C_{xy}, C_{yy}	All are comparable
2^{nd} order resonances	
$2\nu_x$	2.1 | 3.1 | (2.3, 3.3, 3.2, 2.5) | (2.2, 2.4, 3.4)
$2\nu_y$	(3.1, 2.1) | (2.3, 3.3) | (2.5, 3.2) | (3.4, 2.2) | 2.4
4^{th} order resonances	
$2\nu_x + 2\nu_y$	2.2 | (2.4, 2.5) | (2.1, 2.3) | (3.2, 3.1) | 3.3 | 3.4
$2\nu_x - 2\nu_y$	2.4 | (2.5, 2.2, 2.3) | 3.4 | (2.1, 3.1) | 3.3 | 3.2
$4\nu_x$	(3.2, 2.1) | (2.5, 3.3, 3.1, 2.3) | (2.2, 3.4, 2.4)
$4\nu_y$	All are comparable

Table 8: Comparison of 2 and 3 family solutions by case. The corresponding families for the different cases are shown in Table 5. The rankings are shown in increasing order of the parameter, e.g. Case 2.1 requires the lowest strengths and 2.4 the largest strengths amongst the 2 family solutions. Families that are comparable are included within brackets while those solutions that differ significantly are separated by vertical bars.

We observe that the 3 family solutions require somewhat larger strengths than the 2 family solutions. In general the 3 family solutions create a larger horizontal detuning (C_{xx}) while the cross-detuning C_{xy} and the vertical detuning C_{yy} are comparable. The driving terms of the one dimensional resonances $2\nu_x$, $2\nu_y$, $4\nu_x$, and $4\nu_y$ do not help to distinguish between the 2 and 3 family solutions. The $2\nu_x \pm 2\nu_y$ resonance driving terms are mostly smaller with the 2 family solutions. These observations can be quantified for each figure of merit. Table 8 shows the ranking of the different solutions.

6 Summary

We found solutions using 2 and 3 families of octupoles to create chromaticity splits of 10 units in each plane between the protons and anti-protons. All solutions with moderate octupole strengths require that the polarity of each octupole be chosen according to the sign of $D_x\Delta x(p) - D_y\Delta y(p)$ in that octupole. Solutions without any change in polarity require very large octupole strengths to create the required chromaticity splits.

Along with the chromaticity split, we also examined other figures of merit including the second order chromaticity generated, the detuning with amplitude and the strengths of 2nd and 4th order resonance driving terms.

We find that the ZD family of octupoles is needed in any solution. Possible 2 family combinations are (ZD, ZF), (ZD, 39S), (ZD, 39C) with the first combination most likely to be the best. If a large horizontal detuning is required, then the 3 family solution (ZD, ZF and 39S) would be most effective.

A Appendix: Layout of all octupoles

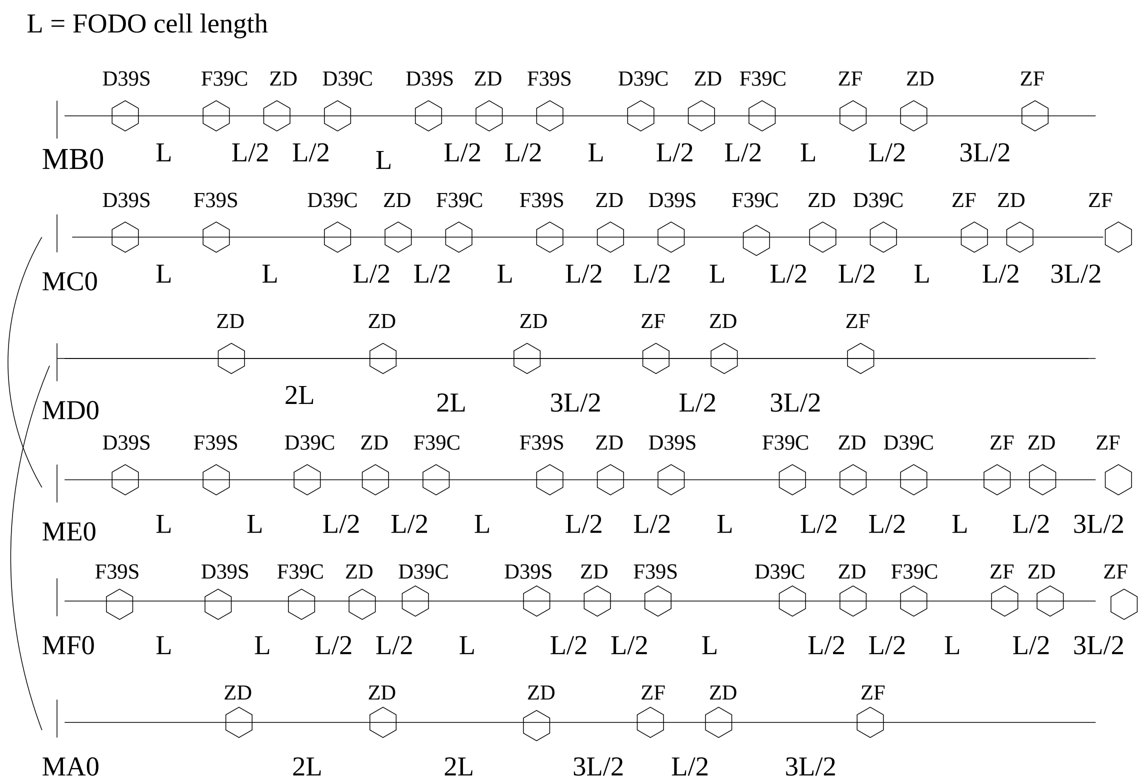

Figure 6: Sketch of all the 67 octupoles in the Tevatron with distances between them. Note that only the ZD octupoles are near D quadrupoles, all others are next to F quadrupoles.

B Appendix: Results with all families

The proton chromaticities are corrected to +5 units in each case. In the tables below, N/A indicates that the octupole strengths k_3 exceed 100 units.

B.1 2 Families

1. ZF and ZD

Polarity Change	$k_3(ZF)$	$k_3(ZD)$	$\nu'_x(\bar{p}), \nu'_y(\bar{p})$
None	-35.54	92.33	-4.56, -4.21
ZF	4.71	86.94	-5.23, -4.05
ZD	-37.05	8.25	-4.22, -5.19
ZF & ZD	4.89	7.75	-4.94, -4.96

2. ZD and 39S

Polarity Change	$k_3(ZD)$	$k_3(39S)$	$\nu'_x(\bar{p}), \nu'_y(\bar{p})$
None	89.00	-8.98	-4.89, -4.13
ZD	7.94	-9.35	-4.58, -5.07
39S	87.89	3.14	-5.20, -4.05
ZD & 39S	7.84	3.27	-4.90, -4.97
ZD & 39S (D39S oppos.)	8.07	-7.35	-4.96, -4.96

"D39S oppos." implies that the polarity of D39S is chosen opposite to the sign of $D_x \Delta x(p) - D_y \Delta y(p)$ in that family, assuming that the D39S octupoles are not rotated by 45°. If they are rotated by this angle, then the sign convention would reverse meaning, i.e. "D39S oppos." would imply that the polarity of a D39S octupole has the same sign as $D_x \Delta x(p) - D_y \Delta y(p)$ in that octupole.

3. ZD and 39C

Polarity Change	$k_3(ZD)$	$k_3(39C)$	$\nu'_x(\bar{p}), \nu'_y(\bar{p})$
None	84.10	12.34	-5.05, -4.14
ZD	7.49	12.82	-4.76, -5.02
39C	87.35	4.14	-1.26, -5.48
ZD & 39C	7.79	4.30	-0.81, -6.45
ZD & 39C (D39C oppos.)	8.56	18.46	16.35, -12.19

4. 39S and 39C

Polarity Change	$k_3(39S)$	$k_3(39C)$	$\nu'_x(\bar{p}), \nu'_y(\bar{p})$
None	N/A		
39S	N/A		
39C	N/A		
39S (D39S oppos.)	N/A		
39C (D39C oppos.)	N/A		
39S & 39C	N/A		
39S & 39C (D39C oppos.)	N/A		

5. ZF and 39C

Polarity Change	$k_3(ZF)$	$k_3(39C)$	$\nu'_x(\bar{p}), \nu'_y(\bar{p})$
None	N/A		
ZF	N/A		
39C	N/A		
ZF & 39C	N/A		
39C (D39C oppos.)	N/A		

6. ZF and 39S

Polarity Change	$k_3(ZF)$	$k_3(39C)$	$\nu'_x(\bar{p}), \nu'_y(\bar{p})$
None	N/A		
ZF	N/A		
39S	N/A		
ZF & 39S	N/A		
ZF & 39S(D39S oppos.)	N/A		

B.2 3 Families

1. ZF, ZD and 39S

Polarity Change	$k_3(ZF)$	$k_3(ZD)$	$k_3(39S)$	$\nu'_x(\bar{p}), \nu'_y(\bar{p})$
None	11.91	87.89	-11.99	-5.0, -4.10
ZF	1.51	88.34	-6.10	-5.0, -4.10
(a) ZD	43.93	7.58	-20.44	-5.0, -4.92
(b) ZD	21.08	7.77	-14.67	-4.78, -5.0
39S	-10.95	89.26	2.18	-5.0, -4.10
(a) ZF & ZD	5.64	7.72	1.43	-5.0, -4.94
(b) ZF & ZD	3.11	7.82	-3.42	-4.81, -5.0
(a) ZF, ZD & 39S	11.29	7.64	-4.27	-5.0, -4.94
(b) ZF, ZD & 39S	-10.15	8.02	10.06	-4.81, -5.0
ZF, ZD & 39S (D39S oppos.)	-14.22	9.00	-28.71	-5.0, -4.97

2. ZF, ZD and 39C

Polarity Change	$k_3(ZF)$	$k_3(ZD)$	$k_3(39C)$	$\nu'_x(\bar{p}), \nu'_y(\bar{p})$
None	-3.37	84.88	11.17	-5.0, -4.14
(a) ZF	-1.15	83.40	15.37	-5.0, -4.16
ZD	4.93	7.39	14.52	-4.83, -5.0
39C	-40.28	93.0	-0.55	-5.0, -4.04
(a) ZF & ZD	6.37	7.83	-3.87	-5.0, -4.94
(b) ZF & ZD	1.76	7.58	8.21	-4.83, -5.0
ZF, ZD & 39C	4.96	7.75	-0.058	-5.0, -4.94
ZF, ZD & 39C (D39C oppos.)	4.91	7.75	-0.049	-5.0, -4.94

3. ZD, 39S and 39C

Polarity Change	$k_3(ZD)$	$k_3(39S)$	$k_3(39C)$	$\nu'_x(\bar{p}), \nu'_y(\bar{p})$
None	85.54	-2.64	8.71	-5.0, -4.13
ZD	7.27	4.47	18.95	-4.84, -5.0
39S	82.93	-0.97	16.14	-5.0, -4.16
39C	89.06	-9.26	-0.13	-5.0, -4.09
(a) ZD, 39S	8.08	5.54	-8.90	-5.0, -4.94
(b) ZD, 39S	7.65	1.49	7.00	-4.82, -5.0
(a) ZD & 39S (D39S oppos.)	8.19	-8.89	-2.68	-5.0, -4.95
(b) ZD & 39S (D39S oppos.)	7.71	-2.74	8.04	-4.83, 5.0
(a) ZD & 39C	7.96	-10.40	-0.48	-5.0, -4.91
(b) ZD & 39C	7.95	-9.82	-0.22	-4.77, -5.0
(a)ZD & 39C (D39C oppos.)	7.93	-9.54	-0.37	-5.0, -4.93
(b) ZD & 39C (D39C oppos.)	7.94	-9.44	-0.18	-4.78, -5.0
ZD, 39S, 39C	7.84	3.35	-0.010	-5.0, -4.94
ZD, 39S(D39S oppos.) & 39C (D39C oppos.)	8.07	-7.37	-0.036	-5.0, -4.95

4. ZF, 39S and 39C

Polarity Change	$k_3(ZF)$	$k_3(39S)$	$k_3(39C)$	$\nu_x'(\bar{p}), \nu_y'(\bar{p})$
None	N/A			
ZF	N/A			
39S	N/A			
39C	N/A			
ZF, 39S	N/A			
ZF, 39S (D39S oppos.)	N/A			
ZF, 39C	N/A			
ZF, 39C(D39C oppos.)	N/A			
ZF, 39S & 39C	N/A			
ZF, 39S(D39S oppos.), 39C(D39C oppos.)	N/A			

B.3 4 Families

Polarity Change	$k_3(ZF)$	$k_3(ZD)$	$k_3(39S)$	$k_3(39C)$
None	N/A			
ZF	N/A			
ZD	-50.72	6.07	50.53	64.55
39S	N/A			
39C	N/A			
ZF & ZD	N/A			
ZF, ZD & 39S	N/A			
ZF, ZD & 39S (D39S oppos.)	-37.26	10.32	-60.84	4.33
ZF, ZD & 39C	16.98	7.24	25.19	0.97
ZF, ZD & 39C (D39C oppos.)	24.35	7.03	37.79	1.23
ZF, ZD, 39S, 39C	N/A			
ZF, ZD, 39S(D39S oppos.), 39C (D39C oppos.)	-30.54	10.40	-60.71	0.054
ZF, ZD(ZD oppos.), 39S(D39S oppos.), 39C(D39C oppos.)	N/A			