MI0198

Characteristics of the RRv9 Lattice As Determined from Tracking and Calculations of the Effects of the High Order Multiple Moments Listed in MI170

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Introduction

A recent version of the lattice for the proposed Recycler Ring has been designated v9.[1] This note is an attempt to describe some of the characteristics of the lattice as observed in tracking studies performed using TEVLAT [2]

TEVLAT does not allow for finite length sextupoles and so the MAD version has to be modified for use in TEVLAT. This is done by splitting each of the magnets with the built in sextupole components and putting an equivalent strength zero length sextupole at the ends of the divided magnet. The tunes are not changed by this modification, there is a small change in the chromaticities, when compared to MAD, of less than 1 unit.

There is one other small change made. MAD treats the magnets as *rbends*. I have replaced them with *sbends* and added small zero length quadrupoles at the ends of the magnets to account for the different edge focusing.

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[1] I wish to thank Dave Johnson for making the details of this lattice available to me.

[2] A. Russell, private communication

Figure 1 shows the lattice functions as computed in MAD, and as computed in TEVLAT. The agreement is excellent.

The Bare Lattice

Tunes

The nominal zero amplitude fractional tune for the lattice RRv9 is $\nu_x = 0.425$ and $\nu_y = 0.415$. The tunes however, due to the strong sextupole fields used to correct the chromaticity are amplitude dependent. Figure 2 show the variation in tune ν_x as a function of the horizontal amplitude. [3] A fit to these results gives

$$\delta\nu_x = (-1.95 \pm 0.03) \cdot 10^{-5} \epsilon_{xn}/\pi$$

where ϵ_{xn} is the normalized emittance in $mmmr$. As the amplitude increases the tune ν_x approaches the fifth integer resonance at $\nu_x = 0.4$. This observed dependence would imply a dynamic aperture $\epsilon_{xn} =\approx 1250\pi mmmr$. Short term tracking ($\approx$ 1000turns) gives an upper limit to the dynamic aperture of $\approx 665\pi mmmr$. The corresponding tune for this amplitude is ≈ 0.412 suggesting that the resonance width is ≈ 0.012. The fifth order resonance is clearly seen in the plot of the trajectory, in phase space, for particles, whose initial amplitude is near the boundary of the dynamic aperture (figure 3).

The situation when the initial amplitude is in the y direction is more complicated. The sextupole field couples the initial vertical amplitude into the horizontal plane. There will then be a change in both the horizontal and vertical tunes. Figure 4 gives the variation of the tunes ν_x and ν_y as a function of the initial vertical amplitude. The change in the horizontal tune is fit by the expression

$$\delta\nu_x = (-5.2 \pm 0.3) \cdot 10^{-5} \epsilon_{yn}/\pi$$

The change in the vertical tune $\delta\nu_y$ is also fit with only a linear dependence on the amplitude. The fit yields

$$\delta\nu_y = (-4.6 \pm 0.3) \cdot 10^{-5} \epsilon_{yn}/\pi$$

[3] This tracking is done with the intial value of x'=0 mr.

As the vertical emittance increases both tunes decrease and approach the fifth. Unlike the case where we had only a horizontal emittance, the phase space plot in both the vertical and horizontal planes shows a slight indication of the pattern characteristic of a fifth order resonance. The patterns suggest a *chaotic* boundry and there is, of course, a large amount of coupling.

Dynamic Aperture

For the purposes of this note we will consider the dynamic aperture the maximum amplitude particle which survives tracking for N turns. Survival will mean that the particle, during the tracking, never goes outside a circle of radius $75mm$.[4]

The number of turns in the tracking, N, should be as large as possible. In the Recycler Ring 10^5 turns corresponds to about 1 second for the beam. In the absence of the non-linear fields, which are due to the sextupole component of the gradient magnets, particles with a normalized emittance, ϵ_n of $\approx 900\pi mmmr$ would remain within the $75mm$ aperture used as the beam limits in the tracking. The non-linear sextupole field used to correct the chromaticity of the beam reduces the dynamic aperture.

The sextupole fields induces an amplitude dependent tune shift and when the tune passes through a resonance the particles are lost. As shown above the tune shifts appear to be linear in the amplitude of the particles and depend on both ϵ_{xn} and ϵ_{yn}. Figure 5 is a plot of the distribution of emittances for which a particle survives 1024 turns. The dependence on ϵ_{xn} and ϵ_{yn} is complicated, the configuration with $\epsilon_{xn} = \epsilon_{yn}$ producing a small dynamic aperture.

Though dynamic apertures are generally stated in terms of the linear emittances ϵ_{xn} and ϵ_{yn} specifying the emittance is not sufficient to determine if a particle will be stable.

The tracking done above began with the particle at each value of the emittance having the initial value of x'=y'=0. Clearly this is not the only possible choice. To study the effect of different starting points on the stability of the particles with the same linear emittance but with 10 different values of the initial phase in phase space were tracked for 1024 turns. Only particles

[4]The physical aperture in the Recycler will be $\pm 47.625mm$ horizontally, and $\pm 21.225mm$ vertically. These physical apertures correspond to normalized emittances of $\epsilon_{xn} \approx 365\pi mmmr$ and $\epsilon_{yn} \approx 75\pi mmmr$

with $\epsilon_{xn} = \epsilon_{yn}$ were studied. A particle with an initial emittance $\epsilon_{xn} = \epsilon_{yn} = 220\pi mmmr$ **and** $x' = y' = 0$ is stable for 10^4 turns. Increasing the number of turns to 10^5 did not result in any change in the dynamic aperture, the particle stable for 10^4 turns is stable for 10^5 turns.

The tracking was then repeated for 10 different values on each (the (x, x') and the (y, y')) phase ellipse for each value of the emittance. A particle is regarded as stable for given values of ϵ_{xn} and ϵ_{yn} only if it is stays within the aperture for all the different starting points on the phase ellipse. With this criteria, and tracking with $\epsilon_{xn} = \epsilon_{yn}$ for ≈ 1000 turns we find a dynamic aperture of $\epsilon_{xn} = \epsilon_{yn} = 120\pi mmmr$. It should be noted that this value is smaller than the value found by tracking for 10^4 turns with $x' = y' = 0$. Increasing the number of turns to 10^4 gives a dynamic aperture of $\epsilon_{xn} = \epsilon_{yn} = 110\pi mmmr$.

The dynamic aperture can only be calculated for a small fraction of the possible configurations in phase space for the particles in the beam. In addition the modeling only incorporates a small fraction of the forces that the particles encounter. For these reasons I personally consider any calculation of the dynamic aperture optimistic vis a vis the true dynamic aperture in a machine. I would use the most pessimistic configuration tried as an upper limit of the true dynamic aperture. I would therefore conclude that for the bare lattice, the lattice with only sextupoles to correct for the chromaticity, the dynamic aperture does not exceed 110 $\pi mmmr$.

The Lattice With High Order Multipoles.

Introduction

MI note MI0170 (Draft #3)[5] provides values for the maximum allowed values for the systematic and random components of the normal and skew multipoles of the Recycler Ring combined function magnets. The values are shown in table I. I recognize that this is a draft MI and the values are going to change, nonetheless the multipole moments described here provide a reference set of numbers. This short note attempts to discuss the implications of these values on the performance of the lattice RRv9.

[5] A Proposed Magnetic Field Quality Specification for Recycler Ring Combined Function Magnets. S.D. Holmes and C.S. Mishra, Nov. 5, 1996

Systematic Quadrupole Moment

Any systematic quadrupole moment will change the tune. If b_{1s} is the systematic quadrupole moment [6] the tune shifts will be:

$$\delta\nu_x = +0.044 * b_{1s}$$
$$\delta\nu_x = -0.045 * b_{1s}.$$

These are clearly quite large for the value of $b_{1s} = 1.0$ in MI170. In addition to the tune shifts the systematic quadrupole will produce a beta wave (figure 6). The phase trombone will need to not only correct the tune but also match the modified lattice functions in the arcs.

If the lattice functions are not corrected the systematic quadrupole b_{1s} will also produce a change in the chromaticity of ≈ -0.5 units.

Systematic Skew Quadrupole Moment

The systematic skew quadrupole moment a_1 splits the horizontal and vertical tunes. The value given for $a_1 = 1.0$ in MI170 will increase the value of ν_x by $+0.003$ and correspondingly decrease the value of ν_y by -0.003. The total tune split $\nu_x - \nu_y = +0.006$. As usual skew quadrupole correctors will be required to bring the tunes together.

Systematic Sextupole Moment

The systematic sextupole moment will change the chromaticity of the lattice. For the value of b_{2s} of $+0.5$ units I calculated a change in ζ_x of $\approx +3$ units and a change in ζ_y of ≈ -2 units. These are quite large compared with the nominal values of -2 units.

Systematic Octupole Moment

The octupole moments in the magnets will produce an amplitude dependent tune shift. A simple calculation of the effect gives for the lattice:

$$\delta\nu_x = +112.3 \cdot 10^{-6} \cdot (\epsilon_{xn}/\pi)mmmr - 110.9 \cdot 10^{-6} \cdot (\epsilon_{yn}/\pi)mmmr)$$

[6]The moments are given in Fermilab units where the given value is 10^4 times the relative contribution (relative to the bend field) to the field at a radius of 1"

$$\delta\nu_y = -110.9 \cdot 10^{-6} \cdot (\epsilon_{xn}/\pi) mmmr + 120.0 \cdot 10^{-6} \cdot (\epsilon_{yn}/\pi) mmmr$$

The actual tune shifts observed will also include the contribution from the effect of the sextupoles in second order.

Tracking studies, including the effect of the sextupole component of the magnets, but not the contribution from systematic b_2 described in MI170, give for the amplitude dependence of the tune:

$$\delta\nu_x = +91.7 \cdot 10^{-6} \cdot (\epsilon_{xn}/\pi) mmmr \qquad (\epsilon_{yn} = 0.0)$$

$$\delta\nu_y = +84.5 \cdot 10^{-6} \cdot (\epsilon_{yn}/\pi) mmmr \qquad (\epsilon_{xn} = 0.0)$$

A betatron oscillation in the vertical plane will couple, because of the sextupole component of the magnetic field, into the horizontal plane and therefore we also see a shift in the horizontal tune:

$$\delta\nu_x = -175.2 \cdot 10^{-6} \cdot (\epsilon_{yn}/\pi) mmmr \qquad (\epsilon_x = 0.0)$$

Dynamic Aperture

The inclusion of the high order multipoles shown in table I, except for the systematic b_1, into the tracking calculations results in a reduced dynamic aperture. For $\epsilon_{xn} = \epsilon_{yn}$ the bare lattice yielded a dynamic aperture of 220 $\pi mmmr$ when for tracking 10,000 turns with the initial x'=y'=0 and 110 $\pi mmmr$ when we start at 10 points on each phase ellipse. With the high order multipoles included the corresponding values for the dynamic aperture are $\approx 100\pi mmmr$ when x'=y'=0 and 85 $\pi mmmr$ when 10 starting points are chosen on each phase ellipse.

Conclusion

Tracking studies suggest using the multipoles listed in MI170 draft #3, that the dynamic aperture, on momentum, before considering the effects of misalignments resulting in closed orbit distortions and for a particular seed for assigning the high order multipoles, will be adequate for Recycler operations. Care must be taken however when determining the dynamic aperture that the initial conditions for a given value of the emittance be varied in order to find the true region of stability.

Table I

Allowed Systematic and Random Multipole Components in the Combined Function Magnets

Multipole Component	Normal (Systematic)	Normal (Random)	Skew (Systematic)	Skew (Random)
Quadrupole	1.0	1.0	1.0	0.1
Sextupole	0.5	1.0	-	0.5
Octupole	0.5	0.5	-	0.5
10-pole	0.2	0.5	-	0.5
12-pole	0.1	0.5	-	0.5
14-pole	0.1	0.5	-	0.5
16-pole	0.1	0.5	-	0.5
18-pole	0.1	0.5	-	0.5