Fermilab

Doc Number: MI-0223
Version: 1.1

# Ferrite and Compensator Symmetry Requirements for Gradient Magnets


Bruce C. Brown
Beams Division, Main Injector Department
*Fermi National Accelerator Laboratory* *
*P.O. Box 500*
*Batavia, Illinois 60510*


12/19/97

## Contents



*Operated by the Universities Research Association under contract with the U. S. Department of Energy

**Abstract**

Formulas are derived which relate strength and asymmetry between magnet top and bottom poles of ferrite and compensator to the strength and temperature compensation of the magnetic field and the skew quadrupole moment and its temperature dependence in dipole or gradient dipole magnets. Applying these formulas will allow one to judge to what extent the symmetry must be maintained separately for ferrite and compensator and the interaction between compensation and asymmetry. We find for Recycler Ring materials, if $\alpha \approx 1/40$ is the ratio of the skew quad to the top-bottom asymmetry in magnetic potential, to keep $|a_1| < 1$ unit at operating temperature, we need the asymmetry of the ferrite, $\delta_F < 36$ units. Since the compensator contributes much less field change, $\delta_C < 394$ units is sufficient. This symmetry will result in $da_1/dT$ less than 0.02 units$^o$C which is sufficient for Recycler Ring requirements.

# 1 Introduction

For Recycler Gradient magnets, one must achieve balance between the top and bottom pole potential in order to satisfy the required limits on skew quadrupole moments[1]. It has been found that the smaller pieces of NiFe compensator provide more convenient quantity for final tuning of top-bottom symmetry than the available larger unit of ferrite. But the correct mix of ferrite and compensator is required on each pole in order to achieve balance at all temperatures. Formulas which relate the strength and skew harmonics and their temperature dependence to the symmetry of the pole excitation are derived below and some cases of interest are examined.

# 2 Basic Formulas

The correct formulation of the asymmetry problems for magnets such as the Recycler Gradient magnets is slightly more complicated than the formulas which will be considered here. We will ignore some of the skew moment effect on pole potential and consider only the simple formulas derived previously[2] for the symmetric case. We write

$$B_1 = \frac{1}{gP}[B_{rF}(A_{BF} + A_{TF}) - B_{rC}(A_{BC} + A_{TC})] \tag{1}$$

where $B_1$ is the dipole field, $g$ is the gap height, $P$ is the permeance of the magnet assembly, $B_{rF}(B_{rC})$ is the remanence of the ferrite (compensator) and $A_{BF}, A_{TF}, A_{BC}, A_{TC}$ are the areas of the bottom and top ferrite and compensator. The temperature variation is found by taking the temperature derivative

$$gp\frac{dB_1}{dT} = [\frac{dB_{rF}}{dT}(A_{BF} + A_{TF}) - \frac{dB_{rC}}{dT}(A_{BC} + A_{TC})]. \tag{2}$$

For convenience, we usually examine the normalized temperature dependence

$$\frac{1}{B_1}\frac{dB_1}{dT} = \frac{[\frac{dB_{rF}}{dT}(A_{BF} + A_{TF}) - \frac{dB_{rC}}{dT}(A_{BC} + A_{TC})]}{[B_{rF}(A_{BF} + A_{TF}) - B_{rC}(A_{BC} + A_{TC})]}. \tag{3}$$

The related formula for finding the normalized skew quadrupole ($a_1$) is

$$a_1 = \alpha\frac{[B_{rF}(A_{BF} - A_{TF}) - B_{rC}(A_{BC} - A_{TC})]}{[B_{rF}(A_{BF} + A_{TF}) - B_{rC}(A_{BC} + A_{TC})]} \tag{4}$$

with a temperature derivative of

$$\begin{aligned}\frac{da_1}{dT} &= \alpha\frac{[\frac{dB_{rF}}{dT}(A_{BF}-A_{TF})-\frac{dB_{rC}}{dT}(A_{BC}-A_{TC})]}{[B_{rF}(A_{BF}+A_{TF})-B_{rC}(A_{BC}+A_{TC})]} \\ &-\alpha\frac{[B_{rF}(A_{BF}-A_{TF})-B_{rC}(A_{BC}-A_{TC})]}{[B_{rF}(A_{BF}+A_{TF})-B_{rC}(A_{BC}+A_{TC})]}\frac{[\frac{dB_{rF}}{dT}(A_{BF}+A_{TF})-\frac{dB_{rC}}{dT}(A_{BC}+A_{TC})]}{[B_{rF}(A_{BF}+A_{TF})-B_{rC}(A_{BC}+A_{TC})]}\end{aligned} \tag{5}$$

where $\alpha$ is a geometric property with a value of about $1/40$.

## 3 Normalized Formulas

In order to gain perspective on the practical application of these formulas, let us define some coefficients which relate the ferrite and compensator quantities. To keep formulas simple, we will write them as if the ferrite and compensator properties were constant with only the areas varying. Since the variations are small, this will provide suitable guidelines, despite the approximations required. Let

$$C_s \equiv \frac{B_{rC}}{B_{rF}}, \quad C_T \equiv \frac{dB_{rC}/dT}{dB_{rF}/dT}, \tag{6}$$

$$A_F \equiv A_{BF} + A_{TF}, \quad C_A \equiv \frac{A_{BC} + A_{TC}}{A_{BF} + A_{TF}}, \tag{7}$$

$$\delta_F \equiv \frac{A_{BF} - A_{TF}}{A_{BF} + A_{TF}}, \quad \delta_C \equiv \frac{A_{BC} - A_{TC}}{A_{BC} + A_{TC}}. \tag{8}$$

This allows us to write our formulas as

$$B_1 = \frac{1}{gP} B_{rF} A_F [1 - C_s C_A] \tag{9}$$

$$\frac{1}{B_1}\frac{dB_1}{dT} = \frac{1}{B_{rF}}\frac{dB_{rF}}{dT}\frac{1 - C_T C_A}{1 - C_s C_A} \tag{10}$$

$$a_1 = \alpha \frac{\delta_F - C_s C_A \delta_C}{1 - C_s C_A} \tag{11}$$

$$\frac{da_1}{dT} = \alpha \frac{1}{B_{rF}}\frac{dB_{rF}}{dT}\left[\frac{\delta_F - C_T C_A \delta_C}{1 - C_s C_A} - \frac{\delta_F - C_s C_A \delta_C}{1 - C_s C_A}\frac{1 - C_T C_A}{1 - C_s C_A}\right] \tag{12}$$

$$\frac{da_1}{dT} = \alpha \frac{1}{B_{rF}}\frac{dB_{rF}}{dT}\frac{1}{1 - C_s C_A}[(1 - \frac{1 - C_T C_A}{1 - C_s C_A})\delta_F - (C_T C_A - C_s C_A \frac{1 - C_T C_A}{1 - C_s C_A})\delta_C \tag{13}$$

$$\frac{da_1}{dT} = \alpha \frac{1}{B_{rF}}\frac{dB_{rF}}{dT}\frac{C_T C_A - C_s C_A}{(1 - C_s C_A)^2}[\delta_F - \delta_C] \tag{14}$$

# 4 Evaluations of Physical Quantities

There is some variability in materials properties which requires different quantities of ferrite and compensator to achieve the same design properties during the series production of hybrid permanent magnets. For the Recycler materials, we have approximate values of the relevant quantities which will permit us to evaluate the variability of the compensation of field and skew quad by the formulas above.

The following approximate values will suffice for this discussion. We know the area ratio of ferrite to compensator for RGF magnets circa November 1997. A well compensated RGF uses 11.5 compensator strips ($6'' \times 2'' \times 0.05''$) to compensate a pair (2 high) of $1'' \times 4'' \times 6''$ ferrite bricks thus we calculate $C_A = .14375$. For compensation we know $C_T C_A = 1$ so $C_T = 6.96$. We have measured the normalized temperature dependence of ferrite and find it to be quite constant (over different batches) at

$$\frac{1}{B_{rF}}\frac{dB_{rF}}{dT} = -.0018/^o\,C \tag{15}$$

The value $B_{rF} = 3900$ gauss has been reported by the vendor[3]. From this we calculate $dB_{rF}/dT = -7.02$ gauss/$^o$C. $dB_{rC}/dT = C_T dB_{rF}/dT = -48.84$ gauss/$^o$C. The value of $B_{rC}$ is less well known but is about 2500 gauss. With these values we calculate $C_s = .64$. Combining these we have $C_s/C_T = .0921$.

The normalized temperature dependence of the magnet strength, $\frac{1}{B_1}\frac{dB_1}{dT}$, is the quantity used to evaluation measurements of compensation. When multiplied by $10^4$ it is reported as units per degree Centigrade. Define the compensation ratio as the normalized temperature dependence of the magnet divided by the normalized temperature dependence of an uncompensated magnet.

$$R_c = \frac{\frac{1}{B_1}\frac{dB_1}{dT}}{\frac{1}{B_{rF}}\frac{dB_{rF}}{dT}} = \frac{1 - C_T C_A}{1 - C_s C_A} \tag{16}$$

This quantity normalizes the compensation to the ferrite property. Solving for $C_A$ we find

$$C_A = \frac{1 - R_c}{C_T - C_s R_c}. \tag{17}$$

If we demand the the temperature compensation of the strength (Eq. 10) be good to $1 \times 10^{-4}/^o$C we can calculate the following related quantities:

$$-0.0001 < \frac{1}{B_1}\frac{dB_1}{dT} < 0.0001 \, (\frac{1}{B_1}\frac{dB_1}{dT} = 0 \pm 0.0001) \tag{18}$$

$$0.056 > R_c > -0.056 \, (R_c = 0 \mp 0.056) \tag{19}$$

$$0.1365 < C_A < 0.151 \, (C_A = 0.14375 \pm 0.00721) \tag{20}$$

$$0.087 < C_s C_A < 0.097 \, (C_s C_A = .09215 \pm 0.00462) \tag{21}$$

$$0.949 < C_T C_A < 1.050 \, (C_T C_A = 1 \pm 0.05018) \tag{22}$$

We graphically understand the relations between the limits for the compensation and symmetry by revisiting Equations 11 and 14. Consider plotting values of $\delta_F$ on the $x$ (horizontal) axis with $\delta_C$ on the $y$ (vertical) axis. If we express the linear equation as

$$J = Kx + Ly \tag{23}$$

we will find an intercept on the $x$ axis at $J/K$ with the intercept on the y axis at $J/L$. The limits for $a_1$ or $da_1/dT$ provide the additional information required to limit the asymmetries $\delta_F$ and $\delta_C$. From Equation 11 we identify

$$J = \frac{a_1(1 - C_s C_A)}{\alpha}, \; K = 1, \; L = -C_s C_A \tag{24}$$

for intercepts at

$$\frac{a_1(1-C_sC_A)}{\alpha}, \frac{-a_1(1-C_sC_A)}{\alpha C_sC_A} \tag{25}$$

$$\pm\, 0.00363 \pm 0.000018, \pm 0.0394 \pm 0.00207 \tag{26}$$

on the $\delta_F$ and $\delta_C$ axes, respectively, for $|a_1| < 0.0001$. From Equation 14 we identify

$$J = \frac{da_1/dT}{1/B_1\, dB_1/dT} \frac{(1-C_sC_A)^2}{\alpha(C_TC_A - C_sC_A)},\ K = 1,\ L = -1 \tag{27}$$

for intercepts at

$$\frac{da_1/dT}{1/B_1\, dB_1/dT} \frac{(1-C_sC_A)^2}{\alpha(C_TC_A - C_sC_A)}, - \frac{da_1/dT}{1/B_1\, dB_1/dT} \frac{((1-C_sC_A)^2)}{\alpha(C_TC_A - C_sC_A)} \tag{28}$$

$$\pm\, 0.0403 \pm 0.00232, \mp 0.0403 \pm 0.00232 \tag{29}$$

for $|da_1/dT| < 2 \times 10^{-6}$.

We display these in graphic form in Figures 1 and 2. In Figure 1, asymmetries which are permitted while satisfying requirements for $a_1$ fall inside the lines shown. The ferrite or the compensator are permitted the same contribution to the flux asymmetry so their asymmetry limits are in inverse proportion to their flux contribution. Figure 2 shows the lines which bound the asymmetry permitted by the temperature dependence of $a_1$. The temperature dependence limit which is used is set by the maximum asymmetry permitted by the $a_1$ requirement and is quite adequate for Recycler needs.

# 5 Conclusions

## 5.1 Warning - Imprecise Formulation

This calculation has been performed without accounting for the change in magnetic potential in asymmetric cases by flux coupling between the poles. This is unlikely to change these results, but will make this a poor model for some other calculations. A more careful formulation would separately consider three equipotential surfaces: bottom pole, top pole, and return flux shell and the fluxes which couple them. It is like a problem of capacitors which couple three nodes.

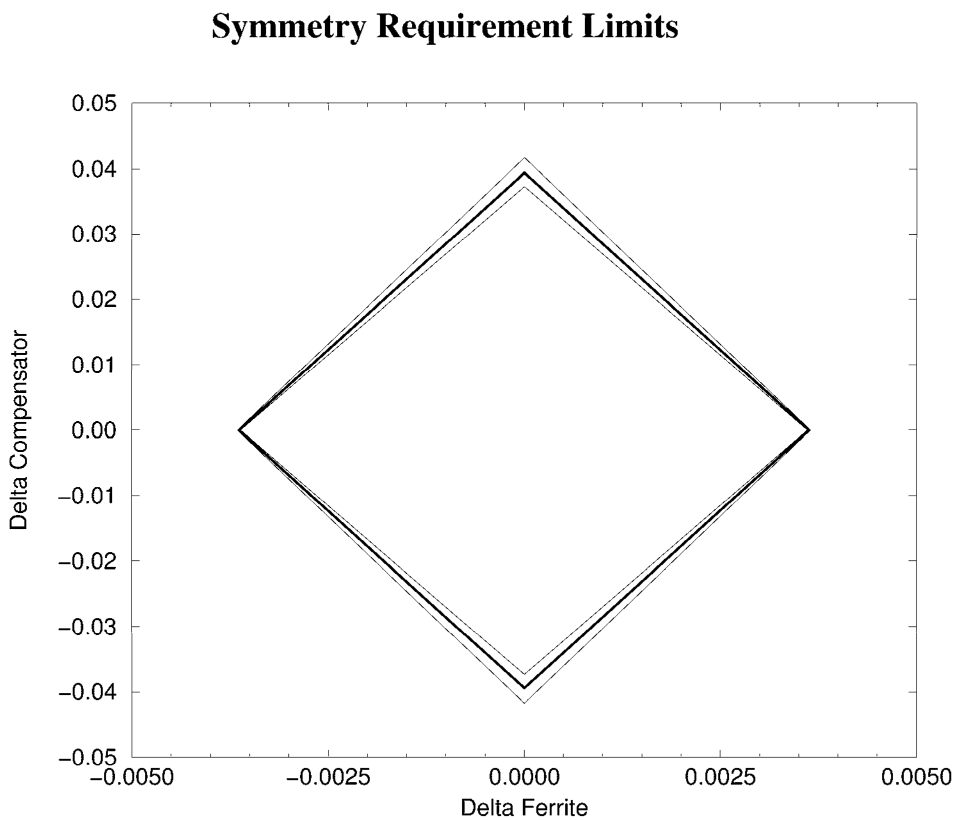


Figure 1: Symmetry requirements imposed by limits of $|a_1| < 0.0001$. Heavy line (center) corresponds with case of precise temperature compensation of strength with light lines for compensation at allowed extremes. Delta ($\delta_F, \delta_C$ is defined as the quantity bottom minus top divided by the sum. Note the $\times 10$ scale ratio for the two axes.

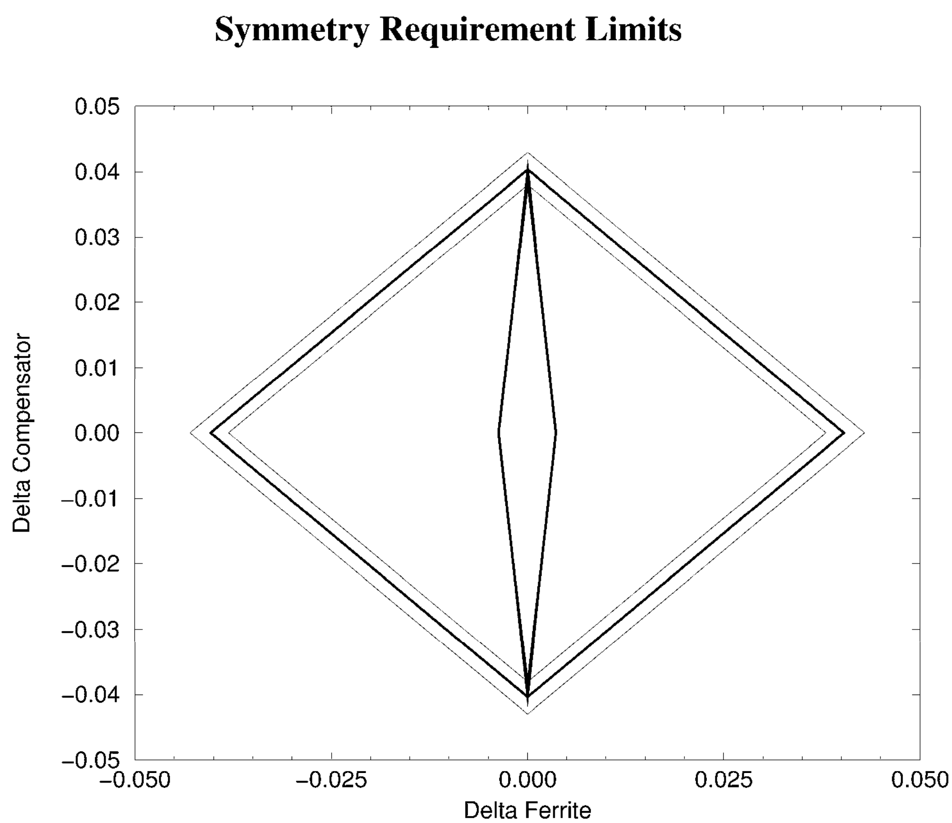


Figure 2: Symmetry requirements imposed by limits of $|da_1/dT| < 2 \times 10^{-6}$ are shown as rectangular figure. Inner figure shown $a_1$ limits again. Heavy line (center) corresponds with case of precise temperature compensation of strength with light lines for compensation at allowed extremes. Delta $(\delta_F, \delta_C$ is defined as the quantity bottom minus top divided by the sum.

### 5.2 Summary of Results

We have examined some magnet properties which results from using ferrite and compensator of known properties to construct a dipole or gradient magnet. Four variables describe the material placement: the areas of the top pole and bottom pole ferrite and compensator $(A_{BF}, A_{TF}, A_{BC}, A_{TC})$. These four variables allow us to adjust four quantities. These are $B_1$, the field strength, and $a_1$, the skew quadrupole (controlled by the top-bottom magnetic potential asymmetry) and the temperature dependence of these quantities. We find it convenient to describe the four variables by suitable sums and asymmetries as the total ferrite, $A_F$, the ratio of compensator to ferrite, $C_A$, and the top-bottom asymmetry of the ferrite ($\delta_F$) and compensator ($\delta_C$).

Using the formulas derived to relate these quantities, we can determine the requirements for material selection. The overall strength and temperature compensation is set independent of the asymmetries. The fractional asymmetries in ferrite and compensator placement which are permitted can be determined based on the allowed $a_1$ and $da_1/dT$. Lines in Figures 1 and 2 show the maximum allowed asymmetries. Since the overall potential must be symmetric to about 0.4%, and since the ferrite mostly sets the strength, it is not surprising that the ferrite asymmetry must be held to <0.4%. The asymmetry in the compensator permitted to maintain the limit on $a_1$ is larger by the ratio of the contribution of strength from ferrite to compensator: $C_s C_A$. If we take this limit on allowed asymmetry of the compensator, it implies that $|da_1/dT| < 2 \times 10^{-6}/^o\mathrm{C}$. Recycler performance requirements would be less restrictive. When approximately compensated, each pole will see temperature effects nearly equal from ferrite and compensator, so it should be no surprise that the symmetry requirement from $da_1/dT$ are symmetric between compensator and ferrite. This is much less restrictive than the requirement based on $a_1$ itself. These limits vary only slightly when compensation is not perfect ($R_c \neq 0$). Thus we find that a magnet which is sufficiently symmetric at the center of its operating range will be adequately symmetric for any useful temperature.

## References

[1] S.D. Holmes, N. Gelfand, D. E. Johnson, and C. S. Mishra. Magnetic Field Quality Specifications for Recycler Ring Combined Function and Quadrupole Magnets. Main Injector Note MI-0170, Fermilab, July 1997.

[2] Bruce C. Brown. Design Formulas for the Strength, Compensation and Trimming of Hybrid Permanent Magnets. FERMILAB-Conf 96/273, Fermilab, October 1996.

[3] William B. Fowler, Bruce Brown, and James Volk. Experience With the Procurement of Ferrite and Temperature Compensator for Permanent Magnets for Accelerators. In *Proceedings of the 1997 Particle Accelerator Conference (to be published)*, 1997. Also available as FERMILAB-Conf-97/248.

# A Alternative Expressions

We can derive results explicitly in terms of observable compensation, $R_c$, rather than the compensator to ferrite ratio, $C_A$. We define the compensation ratio as the normalized temperature dependence of the magnet divided by the normalized temperature dependence of an uncompensated magnet.

$$R_c = \frac{\frac{1}{B_1}\frac{dB_1}{dT}}{\frac{1}{B_{rF}}\frac{dB_{rF}}{dT}} = \frac{1 - C_T C_A}{1 - C_s C_A} \tag{30}$$

This is the quantity is determined when magnet compensation is measured. Since we want the magnet to have, *e.g.*, $< 1$ unit/$^o$C temperature variation and ferrite has about 18 units/$^o$C, We will want $R_c < 1/18$. From the measured compensation, we can now calculate the ferrite to compensator area ratio as

$$C_A = \frac{1 - R_c}{C_T - C_s R_c}. \tag{31}$$

and

$$C_s C_A = \frac{\frac{C_s}{C_T}(1 - R_c)}{1 - \frac{C_s}{C_T} R_c}. \tag{32}$$

$$C_T C_A = \frac{1 - R_c}{1 - \frac{C_s}{C_T} R_c}. \tag{33}$$

We apply these result to the skew quadrupole formulas.

$$a_1 = \alpha \frac{\delta_F - \delta_C \frac{\frac{C_s}{C_T}(1-R_c)}{1-\frac{C_s}{C_T}R_c}}{1 - \frac{\frac{C_s}{C_T}(1-R_c)}{1-\frac{C_s}{C_T}R_c}} \tag{34}$$

For the temperature dependence we similarly find

$$\frac{da_1}{dT} = \alpha \frac{1}{B_{rF}} \frac{dB_{rF}}{dT} [ \frac{\delta_F - \frac{1-R_c}{1-\frac{C_s}{C_T}R_c}\delta_C}{1 - \frac{\frac{C_s}{C_T}(1-R_c)}{1-\frac{C_s}{C_T}R_c}} - \frac{\delta_F - \frac{\frac{C_s}{C_T}(1-R_c)}{1-\frac{C_s}{C_T}R_c}\delta_C}{1 - \frac{\frac{C_s}{C_T}(1-R_c)}{1-\frac{C_s}{C_T}R_c}} \frac{1 - \frac{1-R_c}{1-\frac{C_s}{C_T}R_c}}{1 - \frac{\frac{C_s}{C_T}(1-R_c)}{1-\frac{C_s}{C_T}R_c}} ] \quad (35)$$