Dispersion-Wave Generation of a High $\eta/\sqrt{\beta}$ Insert in the Recycler

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Implementation of Palmer cooling of pbars in the Recycler requires constructing a lattice insert wherein beam-size is dominated by the momentum spread. In the standard Recycler arcs $\eta/\sqrt{\beta} \approx 0.275\ m^{1/2}$ [$\beta_{(max)} = 51.84m$ & $\eta_{(max)} = 1.99m$], which is inadequate for cooling purposes. The creation of large dispersion via very strong bending magnets (à la Accumulator) has been studied by Norm Gelfand[1]. While he showed it was possible to achieve $\eta/\sqrt{\beta} \approx 3\ m^{1/2}$ with this technique, the resulting insert would not fit in the tunnel.

An alternative approach, presented here, builds a dispersion wave by altering the cell phase advance over a sizable arc segment plus deliberately mismatching the dispersion at the entrance to these special cells. The permanent, gradient magnet structure of the Recycler, however, imposes significant limitations on the enhancements attainable by this approach:

(1) With the Recycler lattice defined by gradient magnets, cell bend & focusing centers are fixed relative to one another. This leaves only the adjustment of phase advance per cell as an option for dispersion manipulation. In a separated function lattice, alterations in cell lengths & regrouping of bends could also be explored;

(2) The permanent magnet nature of the machine essentially determines that the tunes are unalterable. The Recycler phase trombone is capable of tune adjustments at the level of only a few tenths of a unit[2]. Consequently, any ring modifications to create a high $\eta/\sqrt{\beta}$ insert must not simultaneously alter the machine tunes to any appreciable extent. This is not a difficulty of course in an electromagnet ring, where the main quad circuits can always be used to re-establish the optimum tunes.

To a lesser extent the physical aperture is also a nuisance. The Recycler beampipe dimensions of $2^5/_8$" (H) x $1^7/_8$" (V) (compared to 5" x 2" in the MI) are sufficiently restrictive that large variations of the beam size from nominal can not be accommodated.

THE $\eta/\sqrt{\beta}$ INSERT:

The overall layout of the insert is illustrated in Fig. 1. There are 3 sections. The majority of the insert is constructed from Recycler arc cells modified to decrease the phase advance per cell to 50°

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[1] Norman M. Gelfand, "A Low-β, High Dispersion Insert for the Recycler Ring", MI-0175, 1996.
[2] Dave Johnson, private communication.

in the horizontal plane and thereby increase the (matched) dispersion to 4.5 m from the 2.0 m of the regular arcs cells, while leaving β_{max} essentially unchanged at 53m. A $\lambda/4$ transformer matches $\beta_{x,y}$ and $\alpha_{x,y}$ between the regular 85° cells & the modified 50° arc cells, while (nearly) optimizing the mis-match between the dispersions. With $\eta \approx 0$ & $\eta' \approx 0$ at the entrance to the special cells, dispersion grows to ≈ 9m (twice the 'matched' value) 180° downstream. Five special quadrupoles contained within a single cell length comprise the final section of the low β- high η insert. The mirror image of these sections matches the optics back into the standard Recycler arc.

Compared to standard cells spanning this same 11-cell section, the tune shifts created by the insert are $\Delta\mu_x = +0.108$, $\Delta\mu_y = -0.184$, which is within the compensation range of the Recycler phase trombone. Horizontally, the beam is largest at the gradient magnet entering the low-β insert, where β = 53.12m & η = 8.65m. For 25π emittance & Δp/p = 0.2% the full horizontal beam width is 48.5 mm. The sagitta through the gradient magnet is 11.7 mm & the interior beampipe width is 66.6 mm. The beam can therefore squeak through the magnet with 3.2 mm to spare on each side.

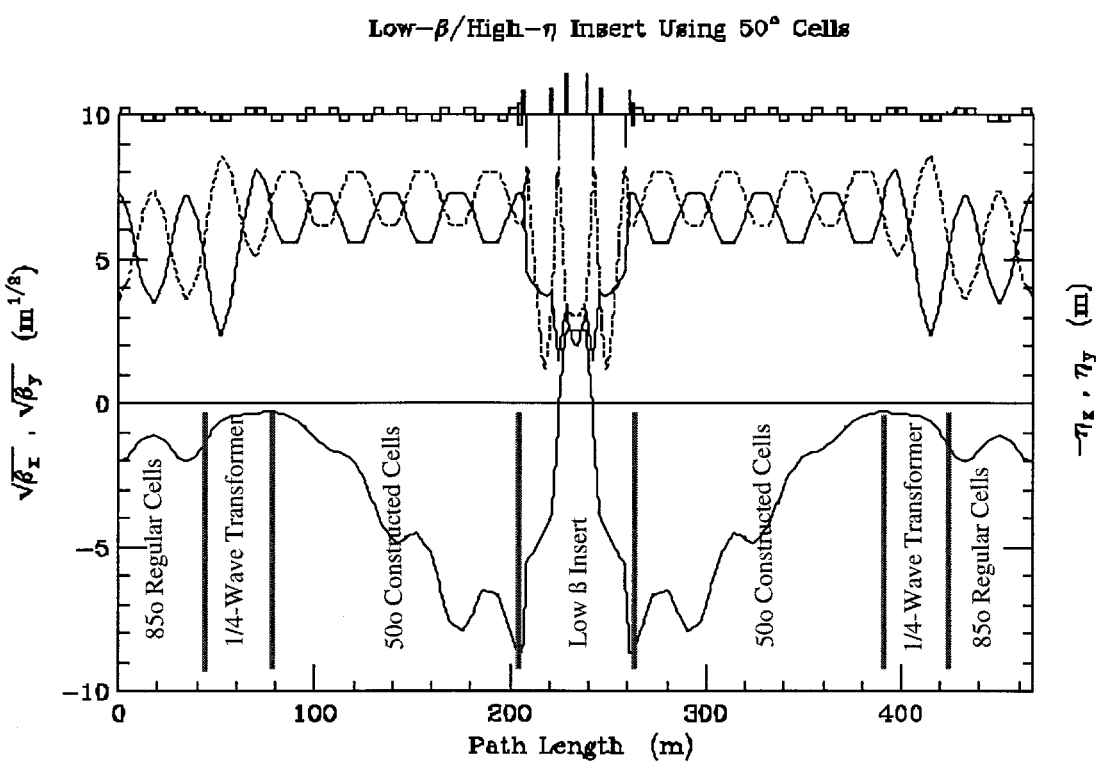
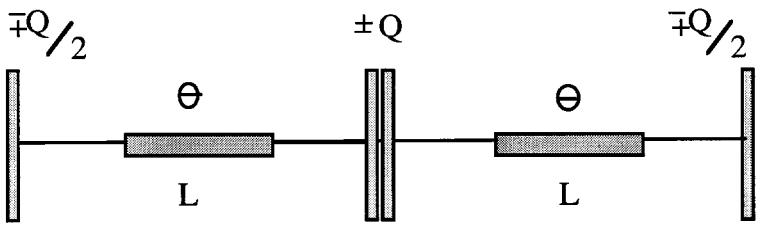


Fig. 1. General layout of the high $\eta/\sqrt{\beta}$ Recycler insert, showing the matching section between the regular 85° and modified 50° arc cells, growth of the dispersion wave, and the final low β

insert. In the example shown, $\beta_x^* = 4.0$ m, $\eta^* = -2.50$ m, and $\beta_y^* = 9.0$ m. The maximum β's occur in the $\lambda/4$ matching section, with $\beta_x(\max) = 66$ m and $\beta_y(\max) = 73$ m.

Modified Recycler Arc Cells



$$R_{x,y} = \begin{pmatrix} \cos(\mu) & 2L\cdot[1 \pm \sin(\mu/2)] & \theta_x L\cdot[2 \pm \sin(\mu/2)] \\ -\frac{2}{L}\cdot\sin^2(\mu/2)\cdot[1 \mp \sin(\mu/2)] & \cos(\mu) & \frac{\theta_x}{2}\cdot[3+\cos(\mu) \mp 2\sin(\mu/2)] \\ 0 & 0 & 1 \end{pmatrix}$$

From the thin-lens approximation to the cell optics, the maximum dispersion and amplitude are related to the phase advance per cell μ, and the half-cell's length L and bend angle θ by:

$$\eta_{max} = \frac{\theta L}{2}\cdot\frac{2+\sin(\mu/2)}{\sin^2(\mu/2)} \quad : \quad \beta_{max} = 2L\cdot\frac{1+\sin(\mu/2)}{\sin(\mu)}$$

from which the following relation between β and η can be obtained:

$$\frac{\eta}{\sqrt{\beta}} = \theta\sqrt{L}\cdot\frac{2+\sin(\mu/2)}{\sin(\mu/2)}\cdot\sqrt{\frac{1-\sin(\mu/2)}{2\sin(\mu)}}$$

Accepting the fact that $\theta\sqrt{L}$ can not be altered without extensive redesign of the arcs, $\eta/\sqrt{\beta}$ can only be enhanced by reducing the phase μ. Practically though, due to the 'untunable tune' nature of the Recycler, μ can only be lowered to ~45°. Even this alteration would require new, wider beampipe. Decreasing μ even further would also require new magnets for the entire insert plus at least one additional insert for tune compensation.

In the present design the phase advance per cell is reduced simply by rearranging the gradient magnets. As illustrated in Figs. 2a & 2b, by moving like-polarity magnets as far apart as possible the horizontal tune is reduced to 50° from the nominal 85°. Horizontally, $\beta(\max)$ is essentially unchanged but $\eta(\max)$ is increased by a factor of 2.25 to 4.50m.

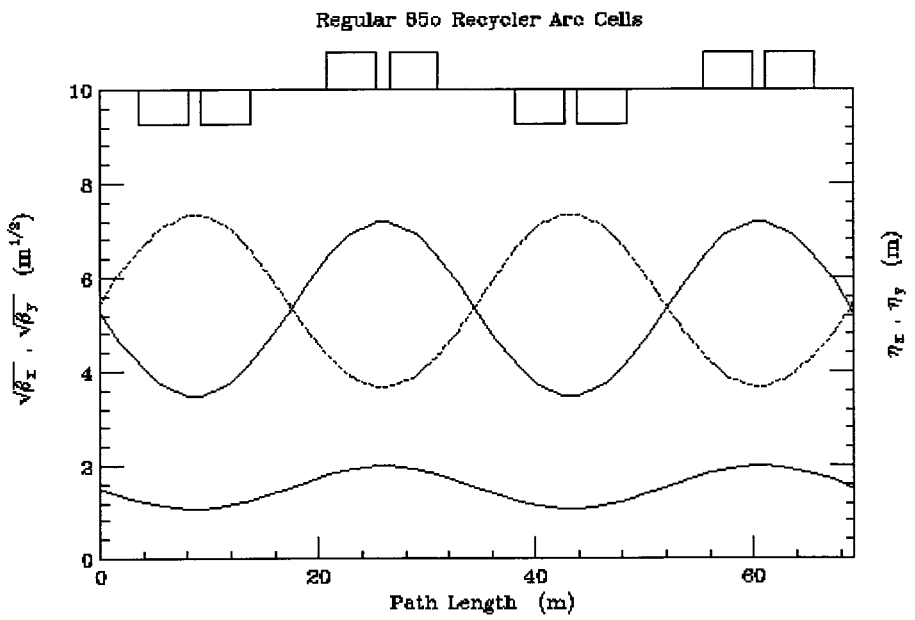


Fig.2a. Regular Recycler arc cells : [μ_x, μ_y] = [0.237, 0.220] ; [β_x , β_y] = [51.8, 54.1] m, and ; dispersion η = 1.99 m.

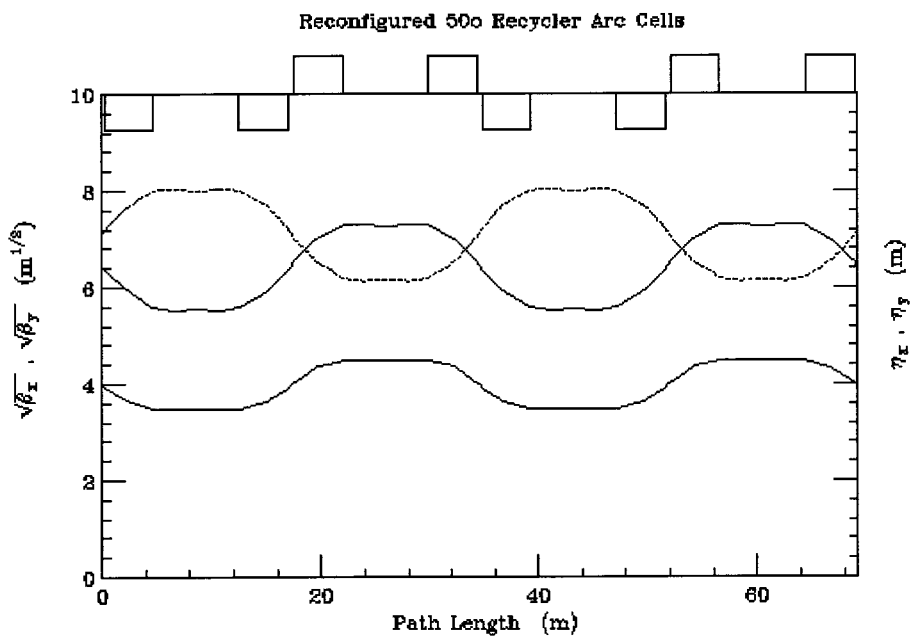
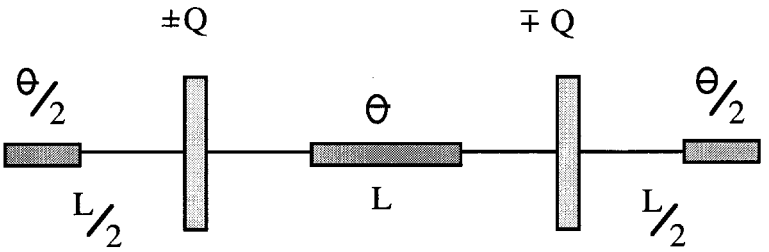


Fig.2b. Reconstructed 50° cells : [μ_x, μ_y] = [0.138, 0.113] ; [β_x , β_y] = [53.1, 64.3] m, and ; dispersion η = 4.50 m.

<u>Quarter-Wave Matching Transformer</u>



$$R_{x,y} = \begin{pmatrix} \cos(\mu) \pm 2\sin(\mu/2) & L\cdot[2-\sin^2(\mu/2)] & \theta_x L/4 \cdot [7+\cos(\mu) \mp 2\sin(\mu/2)] \\ -4/L \cdot \sin^2(\mu/2) & \cos(\mu) \mp 2\sin(\mu/2) & \theta_x/2 \cdot [3+\cos(\mu) \mp 4\sin(\mu/2)] \\ 0 & 0 & 1 \end{pmatrix}$$

It follows from the transfer matrix for this cell that, at the entrance & exit, $\beta_x = \beta_y$ and $\alpha_x = -\alpha_y$:

$$\beta_{x,y} = L\cdot\frac{2-\sin^2(\mu/2)}{\sin(\mu)}\ ;\ \alpha_{x,y} = \pm 1/\cos(\mu/2)\ ;\ \eta = \frac{L\theta}{8}\cdot\frac{7+\cos(\mu)}{\sin^2(\mu/2)}\ ,\ \text{and}\ ;\ \eta' = \mp\theta/\sin(\mu/2)\cdot$$

A simple module for matching $\beta_{x,y}$ and $\alpha_{x,y}$ between FODO cells uses this cell with $\mu = 90^o$ as its core. A single quad at the entrance to the cell adjusts the $\beta_x = \beta_y$ value at the exit. Another quad at the exit tunes the $\alpha_x = -\alpha_y$ value, thus completing the match. A byproduct of this α, β matching method, however, is that it nearly completely suppresses η and η' at the cell exit. While this would normally be viewed as a serious flaw, it is a veritable boon for the current application. This behavior is demonstrated with the example of matching from 90° to 45° cells (which are values 'close' to the 85° & 50° cells of true interest here):

μ	$\beta_{x,y}$	$\alpha_{x,y}$	η	η'
90°	$3/2\ L$	$\pm\sqrt{2}$	$7/4\ L\theta$	$-\sqrt{2}\ \theta$
45°	$L/2\cdot(1+3\sqrt{2})$	$\pm\sqrt{2\cdot(2-\sqrt{2})}$	$\frac{L\theta}{4}\cdot(15+8\sqrt{2})$	$-\theta\cdot\sqrt{2\cdot(2+\sqrt{2})}$

Matching to $\beta_{x,y} = L/2\cdot(1+3\sqrt{2})$ is accomplished by adding a focusing quad at the entrance to the 90° cell with strength $k_1\ell = \frac{2}{3L}\cdot\sqrt{\sqrt{2}-2/3}$ m^{-1}. At the exit of the 90° cell however dispersion is transformed to:

$$\eta = \frac{7}{4}\cdot\left[1-\sqrt{\sqrt{2}-2/3}\right]\cdot L\theta = 0.237\cdot L\theta\ ,\ \text{and}\ \eta' = -\sqrt{2}\cdot\left[1-\frac{7}{6}\cdot\sqrt{\sqrt{2}-2/3}\right]\cdot\theta = +0.0123\cdot\theta$$

These are a very small fraction of the matched 45° values, $\eta_{45} = 6.578 \cdot L\theta$ and $\eta'_{45} = -2.6131 \cdot \theta$, and therefore propagate a large dispersion wave downstream.

In the true lattice, where μ_x and μ_y are not quite equal in either the standard or modified cells, quadrupole doublets are required at the entrance & exit of a regular arc cell to match into the 50° lattice parameters : $\beta_x = 41.80$m, $\alpha_x = 2.163$, $\beta_y = 50.68$m, $\alpha_y = -2.586$. The dispersion and its slope, on the other hand, are nearly optimally mis-matched to the 50° cells, with $\eta = 0.292$ m and $\eta' = -0.006$. The matching section is illustrated in Fig.3.

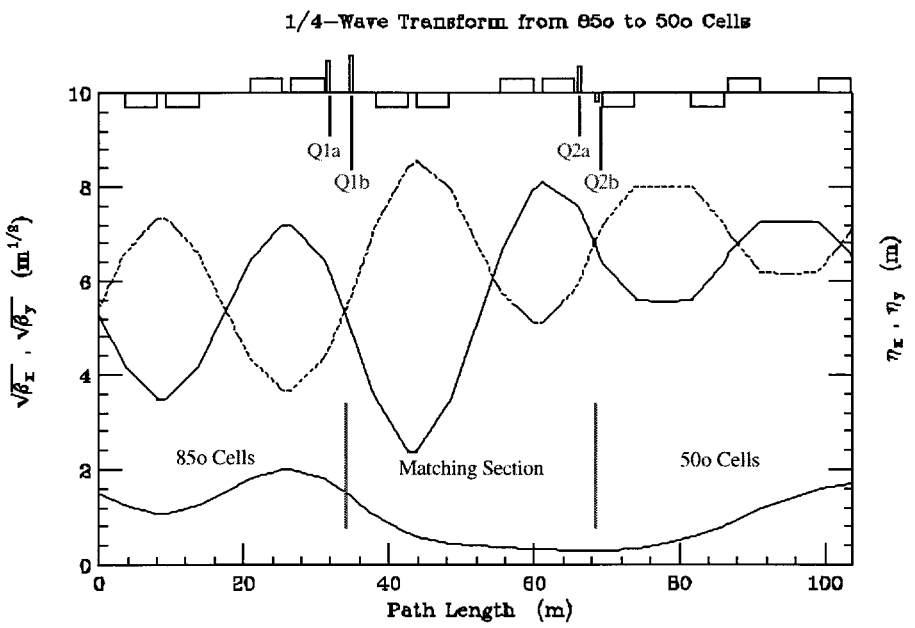


Fig. 3. Matching from the regular 85° cells into the high dispersion 50° cells. The maximum amplitudes in the matching section are $\beta_x = 65.6$ m and $\beta_y = 73.0$ m.

Matching Doublet Quad Parameters ($B_o \rho = 29.650$ T·m)			
Quad	Length (m)	$B'/B_o \rho$ (m^{-2})	B'L (T·m / m)
Q1 a	0.500	0.023565	0.349353
Q1 b	0.500	0.027217	0.403494
Q2 a	0.500	0.019274	0.285739
Q2 b	0.500	-0.0057314	0.084968

Low β - High η Final Focus:

The final focusing section is shown in Figs. 4a & 4b. This section is created by removing 6 of the 8 gradient magnets from two arc cells. A short dipole has been added at the ends of the new straight section, each bending the beam by 62.55mr to compensate for the missing gradient magnets. By concentrating the bend at the ends of the insert the beam trajectory becomes offset by 1.00m into the tunnel at the straight section center.

Immediately downstream of the dipole $\beta_x = 52.87$m, $\alpha_x = .028$, $\eta = 8.692$m, $\eta' = .048$, and $\beta_y = 37.89$m, $\alpha_y = .136$. With $\alpha_x \approx \alpha_y \approx 0$ at this point, a quadrupole doublet (Q5 & Q4) is situated here to provide focusing in both planes. The low β–high η final focusing is provided mainly by the inner triplet (Q1, Q2, & Q3). To a large extent β_x^* and β_y^* can be tuned independently, but the ratio $\eta/\sqrt{\beta_x} = 1.25\ m^{1/2}$ is fixed. The table below & Figs.4 show results for two possible choices of β_x^* and β_y^*, but the insert can be tuned over a much wider range.

		$\beta_x = 4$ m : $\beta_y = 9$ m $\eta = -2.50$ m		$\beta_x = 1$ m : $\beta_y = 4$ m $\eta = -1.25$ m	
Quad	Length (m)	B'/Bo ρ (m^{-2})	B'L (T·m / m)	B'/Bo ρ (m^{-2})	B'L (T·m / m)
Q1	0.500	0.653062	9.68164	0.698792	10.35959
Q2	0.500	-0.688260	10.20345	-0.681711	10.10637
Q3	0.500	0.418719	6.20751	0.259920	3.85331
Q4	0.500	-0.490913	7.27779	-0.527242	7.81636
Q5	0.500	0.369542	5.47846	0.403753	5.98564

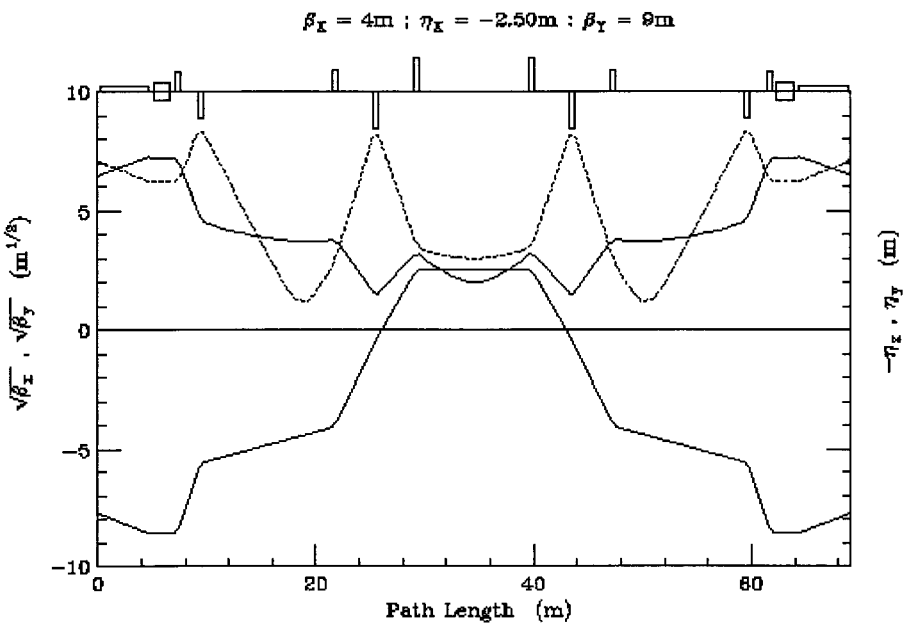


Fig.4a. High $\eta/\sqrt{\beta}$ insert with $\beta_x^* = 4.00\text{m}$, $\eta^* = -2.50\text{m}$, & $\beta_y^* = 9.00\text{m}$. $\beta_X(\text{max})$ = 53.1m, $\eta(\text{max})$ = 8.69m, & $\beta_Y(\text{max})$ = 67.1m.

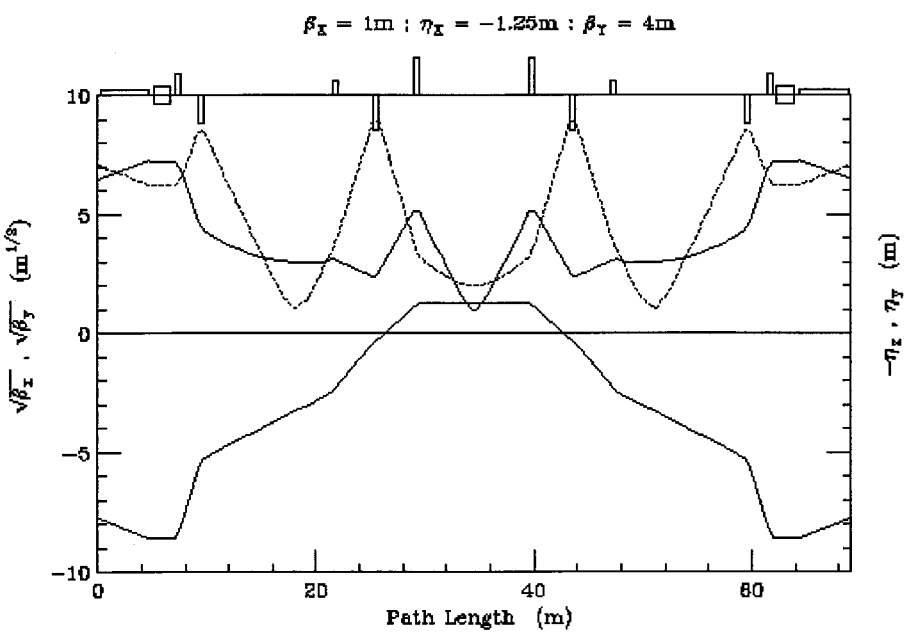


Fig.4b. High $\eta/\sqrt{\beta}$ insert with $\beta_x^* = 1.00\text{m}$, $\eta^* = -1.25\text{m}$, & $\beta_y^* = 4.00\text{m}$. $\beta_X(\text{max})$ = 53.1m, $\eta(\text{max})$ = 8.69m, & $\beta_Y(\text{max})$ = 79.7m.

SUMMARY:

By generating a dispersion wave with minimally invasive modifications to the Recycler arc the ratio of $\eta/\sqrt{\beta}$ can be enhanced to 1.25 $m^{1/2}$, which is a factor of $\approx$ 4.55 greater than obtainable from the standard lattice parameters. Even modest improvements to this result using the techniques discussed here would be difficult, requiring extensive rebuilding with new magnets & impacting ever larger sections of the ring. The most likely candidate for improvements would probably be a hybrid design, combining dispersion wave generation with a section of strong bending magnets.