MI-0274

Determining the Radius and Energy of the Recycler Ring

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Method:

1. Synchronize MI and RR
2. Center the beam in the RR

1. Synchronize MI and RR by changing the MI bend field

The synchronization was checked by turning the RR RF off and looking at the beam with a RR mountain range. The bend field in MI (I:MBOFF) had to be changed by –20 MeV (from –12 to –32 MeV) in order for the two machines to be synchronized.
In this case:

$$\frac{\Delta B}{B} = \frac{\Delta I}{I} = -\frac{1.135A}{506.5A} = -2.24 \times 10^{-3}$$

Since the MI is locked to the RR RF frequency the energy of the beam in MI will change by:

$$\frac{dp}{p} = \frac{\gamma^2}{\gamma^2 - \gamma_t^2}\frac{dB}{B} = 5.41 \times 10^{-4}$$

or dp=4.8 MeV.

The MI radius will also change with the higher momentum beam by:

$$\frac{dR_{MI}}{R_{MI}} = \frac{1}{\gamma^2}\frac{dp}{p} = 5.96 \times 10^{-6}$$

or dR_{MI}=3.15 mm.

The MI BPMs will show a momentum offset:

$$\left(\frac{dp}{p}\right)_{MI_{BPM}} = \gamma_t^2 \frac{dR_{MI}}{R_{MI}} = 2.78\times10^{-3}$$

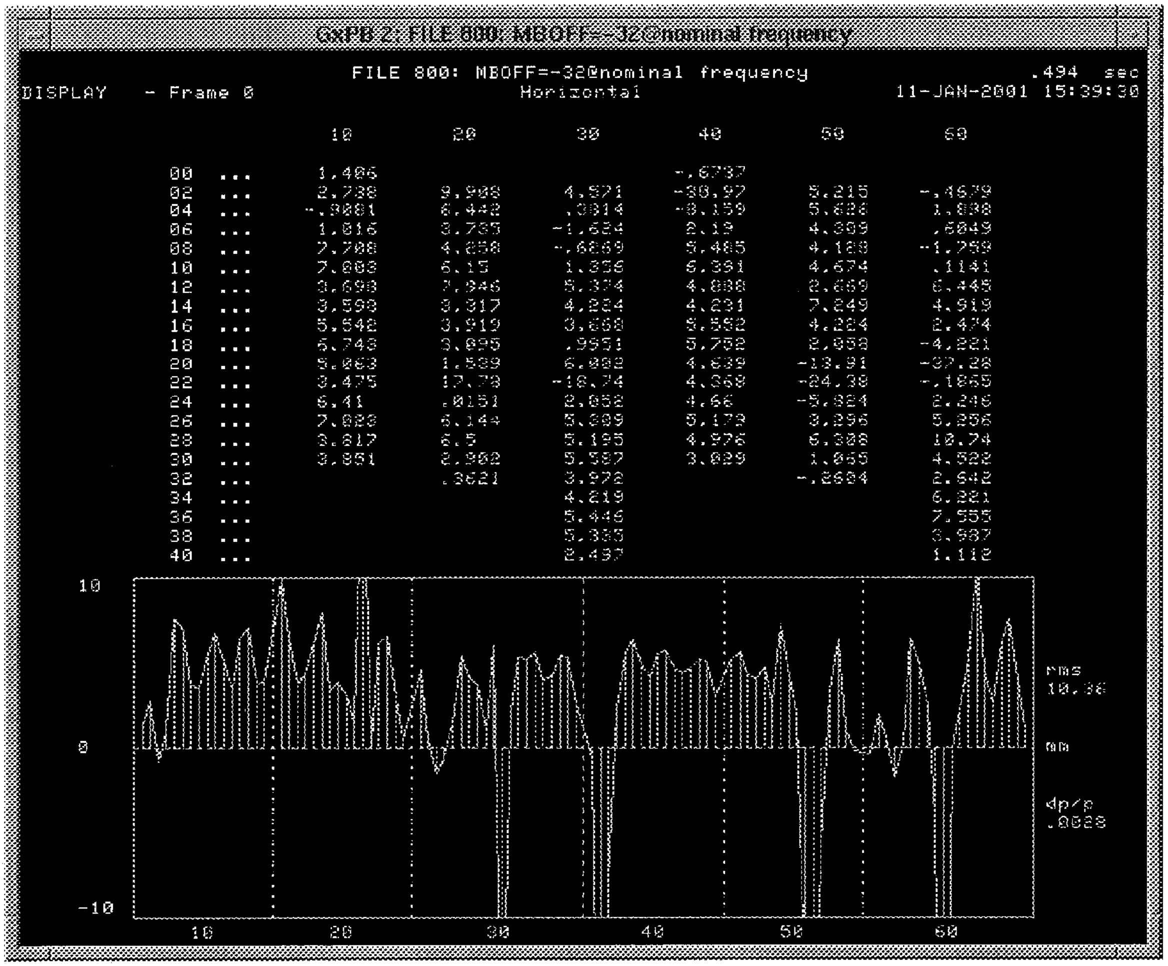

Fig. 1: Horizontal BPM display in MI with MBOFF=-32 MeV and nominal frequency.

2a) Change the RR RF frequency till the beam is centered at the RR aperture based on RR BPMs.

After the MBOFF change MI and RR are synchronized i.e. they have the same revolution frequency and since the beam momentum is the same in both Rings they have the same Radius and Bend field.

The BPMs in the MI show a dp/p=0.28% while the RR BPMs show a dp/p=0.11%. To center the beam in the RR we will have to change the RR RF frequency by:

$$\frac{df}{f} = -\eta\left(-\frac{dp}{p}\right)_{RR} = -9.75\times10^{-6}$$

or df_{rf}=-515.1 Hz and the new RR RF frequency would be 52,810,885 Hz.

The RR radius will change by:

$$\frac{dR_{RR}}{R_{RR}} = \frac{1}{\gamma_t^2}\left(-\frac{dp}{p}\right)_{RR} = -2.36\times10^{-6}$$

or dR_{RR}=-1.25 mm.

So finally based on RR BPMs the center Radius of the RR is 3.15mm-1.25mm=1.9 mm longer than the MI center Radius. Since the MI Radius was measured to be R_{MI}= 528.3013m, we conclude that based on RR BPMs the RR radius is $\mathbf{R_{RR}} = 528.3032\mathbf{m}$. This Radius is 1.96 mm shorter than the ideal (based on the Tevatron measured Radius).

Since we now know the RR Radius and frequency we can calculate the RR energy:

$$\beta_{RR} = \frac{2\pi R_{RR}}{c}\frac{f_{rf}}{h} = 0.994463$$

$$\gamma_{RR} = \frac{1}{\sqrt{1-\beta^2}} = 9.516$$

$$p_{RR} = \beta_{RR}\gamma_{RR} m_p = 8.879\,\text{GeV}$$

$$E_{RR} = \sqrt{p_{RR}^2 + m_p^2} = 8.928\,\text{GeV}$$

Note: The MI momentum at 8 GeV calculated from the MI measured Radius and RF frequency is 4.8 MeV larger than the RR momentum.

2b) Estimate the RR center from the Frequency scan

In this case we kept changing the RR RF frequency till the beam was lost.
Based on those scans the center RF frequency is 52811900 Hz. This means that in order to center the beam in the RR we will have to change the beam energy by:

$$\frac{dp}{p} = -\frac{1}{\eta}\frac{df}{f} = 1.065\times10^{-3}$$

or dp=9.46 MeV/c.

The RR Radius will change by:

$$\frac{dR_{RR}}{R_{RR}} = \frac{1}{\gamma_t^2}\left(\frac{dp}{p}\right)_{RR} = 2.28\times10^{-6}$$

or $dR_{RR} = 1.2\text{mm}$. The radius of the RR will then be:
$R_{RR} = R_{MI} + 1.2\text{mm} + 3.15\text{mm} = 528.30567\text{m}$. This is 0.5 mm larger than the ideal Radius.

The RR energy will then be:

$$\beta_{RR} = \frac{2\pi \mathbf{R_{RR}}}{\mathbf{c}} \frac{\mathbf{f_{rf}}}{\mathbf{h}} = 0.9944865$$

$$\gamma_{RR} = \frac{1}{\sqrt{1-\beta^2}} = 9.536$$

$$\mathbf{p_{RR}} = \beta_{\mathbf{RR}} \gamma_{\mathbf{RR}} \mathbf{m_p} = 8.898\mathbf{GeV}$$

$$\mathbf{E_{RR}} = \sqrt{\mathbf{p_{RR}^2} + \mathbf{m_p^2}} = 8.947\mathbf{GeV}$$

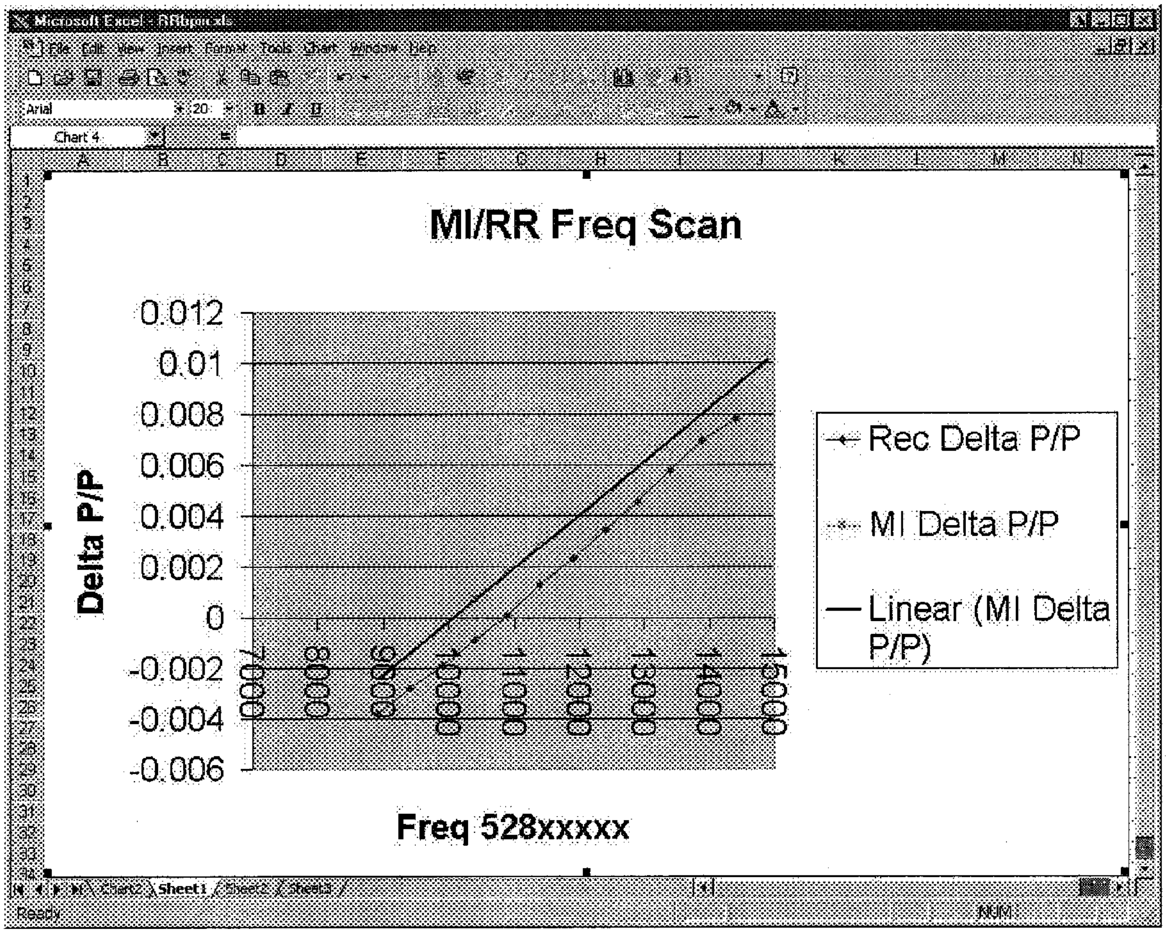

Fig. 2: Momentum dependence as a function of the rf frequency change as measured by the BPMs in both machines. The end frequency points in the RR correspond to the physical aperture of the machine.

Summary:

We measured the RR Radius and momentum using two different methods. From all methods the RR Radius is measured to be larger than the measured MI Radius i.e. is closer to the ideal based on the TeV measured Radius ($R_{RR}^{Ideal} = \frac{588}{1113} R_{TeV}^{Measured} = 528.30528\text{m}$).
The amount that the RR Radius is different from ideal depends on the method we used to measure it and it varies from 0.4 to 2.0 mm.

The RR central momentum also varies with the measurement method. The results are summarized in the table below.

Method	RR BPMs	RR Frequency Scan
Ideal RR Radius R_{RR}^{I} (m)	**528.3053**	**528.3053**
Measured RR Radius R_{RR}^{M} (m)	528.3032	528.3057
Measured MI Radius R_{MI}^{M} (m)	528.3013	528.3013
$R_{RR}^{M} - R_{MI}^{M}$ (mm)	1.9	4.4
RR Momentum p_{RR} (MeV)	8879	8898
MI Momentum p_{MI} (MeV)	8884	8884
$p_{RR} - p_{MI}$ (MeV)	-5	14

Table 1: RR Radius and momentum as determined by the two methods. The ideal RR Radius is determined by the Tevatron measured Radius.

References:

1. C. Kerns, Q. Kerns and H. Miller, "Measuring the Orbit length of the Tevatron" , FERMILAB TM-1330, June 1985.