# Induced Fields in the Recycler

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The Recycler is sensitive to induced magnetic and electric fields caused by ramping of the Main Injector which occupies the same tunnel. Although it is impossible to know the fields everywhere, we can attempt to estimate several effects of these time dependent fields. Some of the error field must be the field due to net current flowing in the cable trays. Other fields will be due to saturation of the main injector magnets, which will have a very different time dependence. In any case the fields can be described by a vector potential, with only an s component $Å_s(s,x,z,t,)$, so long as there are no currents linking the beam. (We use coordinates (s,x,z) where s is arc length along the Recycler central orbit, x is in the outward direction and z is in the vertical.) We can expand the potential in a power series in x and z where all coefficients depend on both s and t.

$$Å = A_0 + B_x z - B_z x - B'\left(\frac{x^2 - z^2}{2}\right) + B'_r xz + \ldots\ldots \qquad (1)$$

$A_0$ is the flux per meter linking the orbit, $B_x$ and $B_z$ are components of the magnetic field at the central orbit, while B' and $B'_r$ are the normal and skew (or rotated) magnetic field gradients. For now we will ignore higher order multipoles, but they can be added in, if necessary.

## Orbit Lengths:

We will assume that the time scales of all variations are long compared to the periods of betatron oscillations. Then the principal effects of the new fields can be inferred by looking at the new terms one by one.

The first term gives a local accelerating electric field. The s integral of this field gives the energy gain per turn. As the field pulses to its maximum, the beam energy changes by an amount

$$\Delta E = -q\beta\Delta\Phi = -q\beta\oint \Delta A_0 ds \quad \text{or} \quad \frac{\Delta P}{P} = -\frac{\delta Bw}{B\rho} \qquad (2)$$

where $\Delta A_0$ is the change in $A_0$, which is equivalent (flux per meter) to the product of a field $\delta B$ and a width w. $\Phi$ is the total change in flux linking the orbit. This changes the radius and circumference of the orbit by

$$\frac{\Delta C}{C} = \alpha \frac{\Delta P}{P} = -\frac{\delta B w}{B\rho} \frac{1}{\gamma_t^2} \qquad (3)$$

The second term changes the net bending of the beam. Even without detailed knowledge of the dependence of the error field on s, we can make some observations about the effects on the beam. The fields $B_x$ and $B_z$ will cause a change in the equilibrium orbit in the x and z direction. Ignoring for a moment the change in energy (2) then there will be a mean shift of the orbit in the x direction which depends on the mean field. Other changes will depend mostly on harmonics of the field near $\nu_x$ . This mean shift changes the length of the orbit of particles of momentum $P_0$, the momentum of the unperturbed central orbit. Then using Courant & Snyder's method of harmonic analysis, we find

$$x = -\sqrt{\beta_x} \frac{R}{\nu} \left\langle \sqrt{\beta_x} \frac{\Delta B}{B\rho} \right\rangle \qquad (4)$$

to be the shift due to a uniform field. The change in orbit length due to x is given by

$$\Delta C = \oint \Omega x ds, \quad \Omega = \frac{B_0}{B\rho} \qquad (5)$$

where $B_0$ is the bending field pattern is the unperturbed ring. There are several cases worth mentioning. If $\Delta B$ is uniform then

$$\frac{\Delta C}{C} \cong -\frac{R}{\nu_x^2} \frac{\Delta B}{B\rho} = -\frac{R}{\rho} \frac{\Delta B}{B_d} \frac{1}{\nu_x^2} \qquad (6)$$

where $B_d$ and $\rho$ are the field and orbit curvature in the dipoles. Another case of interest is the situation in which the error field is only in the dipoles. Since the s dependence is the same as for the main bending field, the change can be easily calculated

$$\frac{\Delta C}{C} \cong -\frac{\Delta B}{B_d} \frac{1}{\gamma_t^2} \qquad (7)$$

Cases of differing field changes in dipoles and elsewhere can be concocted as linear combinations of (5) and (6)

The radial field $B_x$ will cause closed orbit deviations in the vertical closed orbit, but these will not change the orbit length in first order since the undisturbed orbit has no curvature (see (4)).

Then the major problem seems to be the orbit length change. We can compare the shifts by acceleration and bending as the ratio of (3) to (6) or (7)

$$\text{Ratio} \cong \frac{\delta B}{\Delta B} \frac{w}{\rho}$$

since $\nu_x^2$ is about equal to $\gamma_t^2$. Now the fields must be of the same order, but w must be at most about 1 meter, much smaller that the radius of curvature, so the acceleration shift must be negligible.

Tune Shifts:

There are two sources of tune shifts. The B' terms in (1) will give tune shifts. If the gradient magnets have a field reduction, they will also have a gradient reduction so the tune is shifted

$$\frac{\Delta\nu}{\nu} = \frac{\Delta B'}{B'} = \frac{\Delta B}{B_d} \qquad (8)$$

Presumably gradients in the straight sections will give a result equivalent to (6), that is, a factor of R/ρ.

The orbit length shift moves the orbit in the sextupoles without changing the momentum, so the beam has an effective momentum shift

$$\frac{\delta P}{P} = \frac{\Delta C}{C}\gamma_t^2 \cong -\frac{\Delta B}{B_d}, \text{ so } \delta\nu = -\xi_c \frac{\Delta B}{B_d} \qquad (9)$$

where $\xi_c$ is the chromaticity correction.

Effect of Magnetic Shields

Bill Foster has raised the issue of enhanced betatron acceleration caused by the shielding around the beam pipe. I do not believe this is important. To estimate this, consider a transient current flowing in a conductor near and parallel to the beam pipe. The field from the conductor will cause some flux to link the beam path. If a ferromagnetic core is placed around the beam and the conductor then the flux will increase by about μ times the flux in the region the core replaces. If an air gap is introduced in the core, then the core field and flux will be reduced. If the gap is increased to almost the length of the original flux line L, the fractional increase in the field over the field without the iron is of order t/L where t is the thickness of the iron. Since the divergence of B is zero, the number of flux lines exiting any region equals the number entering it, so adding a thin shell of ferromagnetic material cannot greatly increase the flux linkage.

What to do about it

Assuming that the mean orbit shift and the rms orbit shift are small, so that there is no scraping then the principal effect of the fields is the dilution of longitudinal emittance because of the frequency shift of the beam. There are several ways to correct this, both depending on a measure of the mean orbit position shift. In the first place, one can consider using dipole correction elements to cancel the mean orbit shift. This would be a feedback arrangement, and has the usual problems of loop gain and stability. Alternatively, a learning mode program could be used to generate a ramp to be used. Instead, one might use the measured mean orbit shift to shift the frequency driving the barrier bucket pulses. If it is determined that the tune shifts are a problem, then ramped quad correctors could be used to remove the tune shift.