Fermilab-TM-2145

Measured Longitudinal Beam Impedance of Booster Gradient Magnets

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INTRODUCTION

The Booster gradient magnets have no vacuum pipe which forces the beam image current to flow along the laminated pole tips. Both D and F style magnets were measured with a stretched wire to determine the longitudinal beam impedance caused by these laminations. Results are compared to calculations done 30 years ago. The inductive part of the magnet impedance is interesting because it partially compensates for the negative inductance effects of space charge on the beam. An R/L circuit consisting of 37KΩ in parallel with between 40 and 100uH is a reasonable approximation to the total impedance of Booster magnet laminations.

THE MEASUREMENT

A 30 gauge (.0100") tin plated copper wire was stretched through the magnet and an HP 8753E network analyzer was used to measure S21, the attenuation through the magnet. The characteristic impedance of the wire is estimated below for wire diameter d and pole tip spacing h:

$$Z_0 \approx \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\,\ln\left(\frac{4h}{\pi d}\right)$$

F magnet: Zo=321Ω for h=1.656"
D magnet: Zo=338Ω for h=2.188"

The characteristic impedance was matched to 50Ω with a resistive L pad at each end of the wire. The pad was made with four 220Ω resistors to ground and one 300Ω resistor in series. All resistors were 1/4 watt carbon. The measured attenuation through both pads agrees with calculation at -27.57 db. The small amount of frequency structure related to the length of the wire (half wavelength at 50MHz) demonstrates the 326Ω L pads are well matched to the characteristic impedance of the wire for both F and D magnets.

Losses from the matching L pads and skin effect losses along the wire were used to correct the measurement. As an example, the center wire contributes only .29 db from skin effect losses to the 4.9 db total loss through the magnet at 400 MHz.

Magnet sagitta causes the stretched wire to deviate from the center by ±.50" in an F magnet and ±.43" in a D magnet. The characteristic impedance along the wire changes by less than 1% from sagitta. The wire is 126" long and sags by about 1/8" vertically. The effect of these errors are considered small and ignored in this analysis.

The cutoff frequency for the propagation of microwave modes is 870 MHz in both the F and D magnets. This was determined by replacing the stretched wire with 1" stubs at each end and measuring the coupling between them. The stretched wire technique cannot be used to measure the impedance of Booster Magnets above 870 MHz.

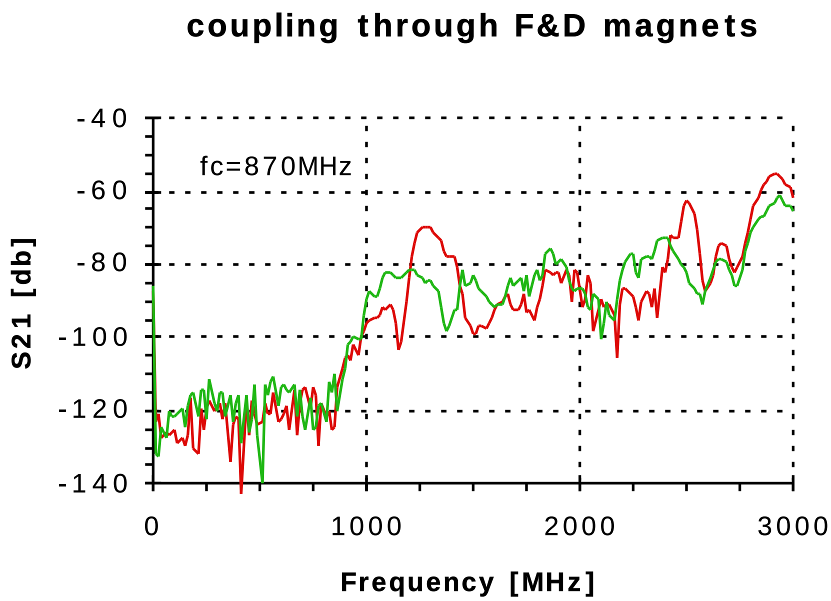


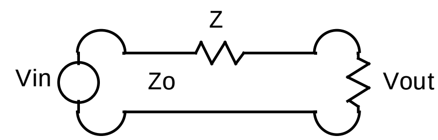
Figure 1. Coupling through Booster gradient magnets between 1" stubs at each end. Data from both the F and D magnets is shown. Microwave modes propagate through the magnet above 870MHz. The stretched wire technique will not work above this frequency.

CALCULATING IMPEDANCE

Lumped Element model

The first approach treats the magnet as a simple lumped impedance and applies ohm's law. The real and imaginary parts of the result seem well coordinated but the magnitude is about 30% higher than the more reliable high frequency approximation.

$$S_{21} = \left(\frac{Z_o}{Z + Z_o}\right)\left(1 + \frac{(Z + Z_o) - Z_o}{(Z + Z_o) + Z_o}\right) = \frac{2Z_o}{Z + 2Z_o}$$



$$Z = 2Z_o\left(\frac{1}{S_{21}} - 1\right)$$

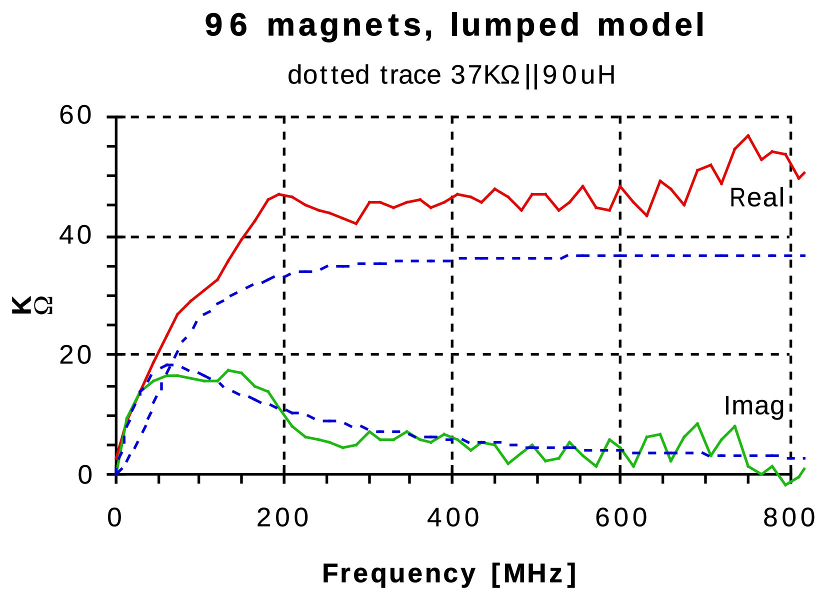


Figure 2. The total magnet impedance using Ohm's law. The dotted traces indicate the impedance of a 37KΩ resistor in parallel with a 90uH inductor. These values were chosen to match the imaginary impedance.

High Frequency approximation
A single F or D magnet has a total longitudinal resistance of about 120Ω/m. The 330Ω characteristic impedance of our stretched wire has 1.1 uH/m. Above 17MHz (120Ω/2π 1.1uH), the high frequency approximation provides a good estimate of the real part of the lamination impedance. This approach does not provide the imaginary part of the impedance. The conductance, G, between the center and outer conductors is negligible.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

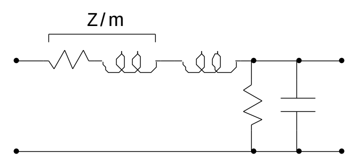
$$\textit{high frequency approximation} \quad R \ll \omega L \textit{ and } G \ll \omega C$$

$$Z_0 \approx \sqrt{\frac{L}{C}} \qquad \alpha \approx \frac{R}{2Z_0} + \frac{G}{2}Z_0 \qquad \beta \approx \frac{\omega}{\sqrt{LC}} = \frac{\omega}{\upsilon_p}$$

$$R = 2Z_0 \frac{S_{21}[db]}{8.686} \qquad \left(8.686 = \frac{\ln_e 10}{20}\right)$$

Distributed transmission line model

The derivation below assumes that L and C are independent of frequency and can be estimated from the characteristic impedance and velocity of the stretched wire in absence of the lamination impedance. The real part is the same as found with the high frequency approximation. In order to obtain the correct imaginary part of the impedance, the flight time delay through the magnet must be removed from beta.



$$\gamma = \sqrt{(R + j\omega L) j\omega C} = \alpha + j\beta$$

$$\gamma^2 = (R + j\omega L) j\omega C$$

$$R = \frac{\gamma^2}{j\omega C} - j\omega L = \frac{\alpha^2 - \beta^2 + j2\alpha\beta}{j\omega C} - j\omega L$$

Letting R be a complex impedance Z:

$$\frac{Z}{m} = 2\alpha \frac{\beta}{\omega C} - j\left(\omega L + \frac{\alpha^2 - \beta^2}{\omega C}\right)$$

$$assume \quad L = \frac{Z_o}{\upsilon} \quad C = \frac{1}{Z_o \upsilon} \quad \beta = \frac{\omega}{\upsilon}$$

$$\frac{Z}{m} = 2\alpha Z_o - j\frac{\alpha^2}{\beta} Z_o$$

$$Z_{tot} = 2Z_o \alpha l \left(1 - j\frac{\alpha l}{2\beta l}\right)$$

The above arguments require Z / m << wL / m of the stretched wire. The inductance of the magnet laminations is .17uH / m or .10uH / m for the F and D magnets respectively compared to 1.1uH / m for the wire.

$$\frac{V_{out}}{V_{in}} = e^{-\gamma x} = e^{-\alpha x - j\beta x}$$

$$\alpha x = \ln\left[mag\left(\frac{V_{out}}{V_{in}}\right)\right] \qquad \beta x = ang\left(\frac{V_{out}}{V_{in}}\right)$$

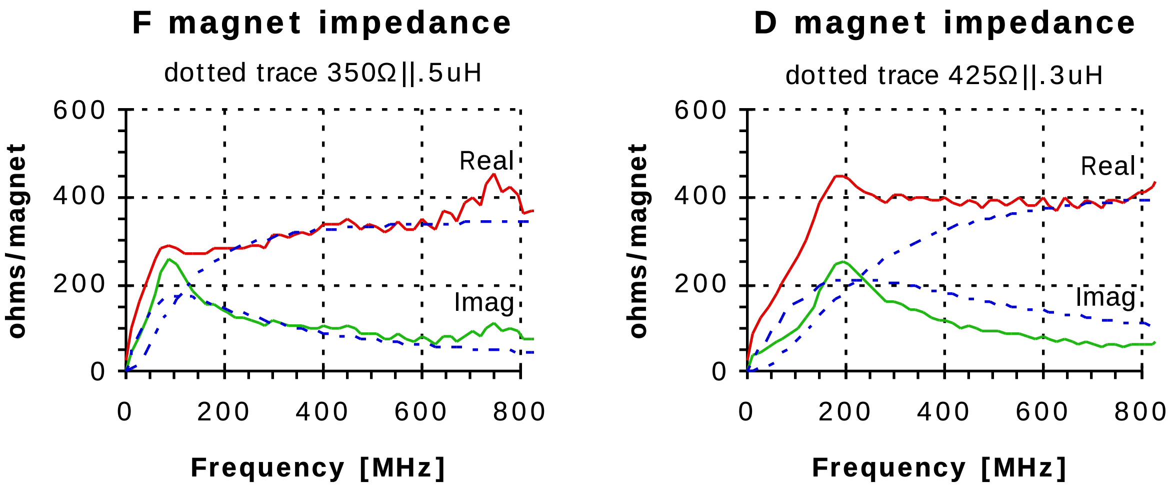


Figure 3. Comparison of lamination impedance for Booster F and D gradient magnets using the transmission line model. Dotted lines indicate the impedance of a parallel R/L circuit with values indicated.

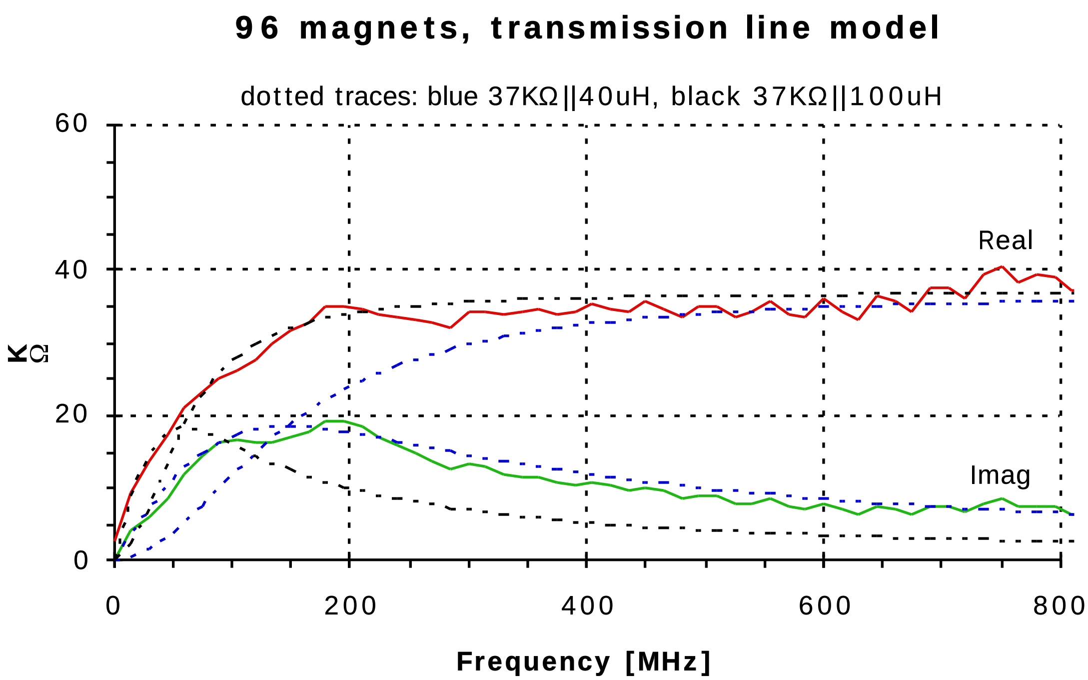


Figure 4. Total Booster magnet impedance using the transmission line model. Dotted lines indicate the impedance of parallel R/L circuits with values indicated. The 40uH model matches the imaginary impedance and the 100uH model matches the real impedance.

BEAM MEASUREMENTS

The beam looses energy to the real part of the lamination impedance which must be compensated with the accelerating rf system. Several attempts have been made in the past to estimate the resistive part of the magnet impedance by measuring it's effect on the beam.

> The phase between the beam and the rf fan back near extraction was measured at several intensities. The effect was consistent with 25.5KΩ.
>
> Measuring the transition phase jump at two intensities suggested 16.7KΩ
>
> Measuring the synchrotron frequency at several intensities suggested 54KΩ.

These measurements are difficult and have significant uncertainties but suggest alternate ways of verifying the real part of the magnet impedance. Bunch shape and frequency dependence of the impedance affect the result. None of these measurements have been published.

IMPEDANCE REDUCING STRIPS

Ten titanium strips .001" thick and .750" wide are stretched through the magnet and connected to ground at one end and through a 1KΩ resistor to ground at the other. Five strips lay against each pole tip. The 1KΩ resistor reduces the currents driven by the normal 15Hz magnetic program of the Booster. The DC resistance of each strip is about 10Ω. A 6" wide ribbon of kapton is used to insulate the titanium strips from the magnet laminations. Periodically along the length a small piece of kapton is attached to the larger ribbon to trap the titanium strips and prevent them from vibrating or moving when the magnets are driven at 15Hz. Each strip is tensioned to 16 pounds with a "fish" scale. Under tension, the strips stretch by 1/8 to 1/4".

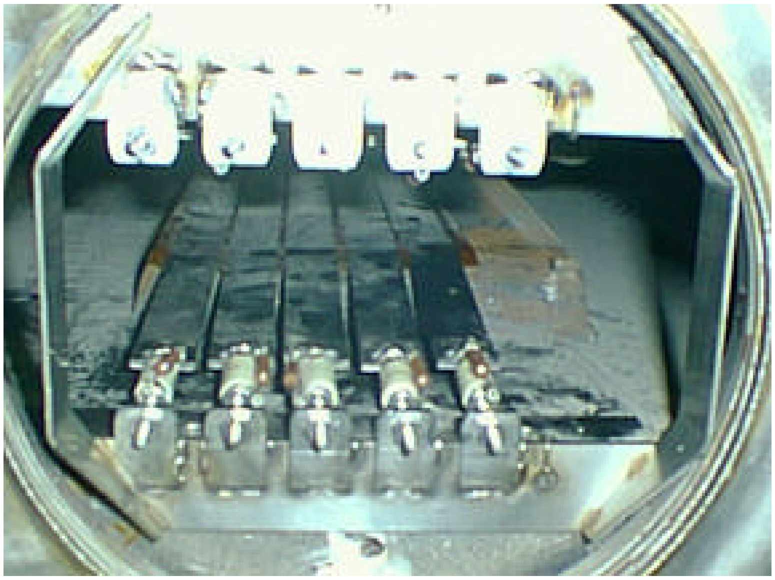


Figure 5. Titanium strips installed in a D style Booster magnet. Note the ceramic standoffs and 1KΩ resistors used at one end. The opposite ends of the strips are electrically connected to ground.

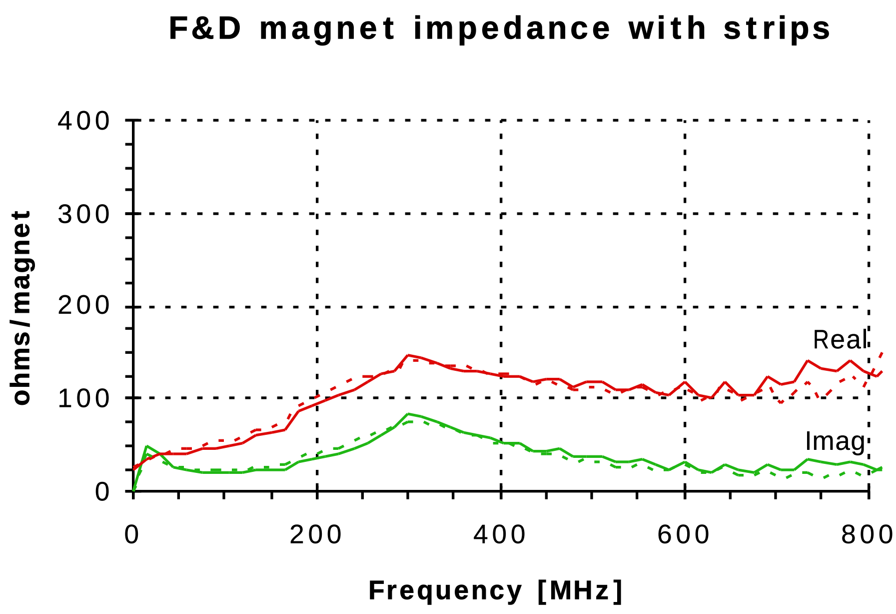


Figure 6. Impedance of one Booster magnet with ten titanium strips installed. The strips are connected to ground through 1KΩ resistors at one end. The 100Ω impedance is from the ten 1KΩ resistors in parallel. The solid and dotted curves are for an F and D magnet respectively.

The 100Ω impedance measured with the strips in place reflects the parallel combination of the ten 1KΩ resistors at one end of the strips. This agreement provides some validity for the stretched wire measurement.

The effectiveness of two 18 gauge insulated wires, one on each pole tip, was also evaluated. Both ends were tied directly to ground. The titanium strips were significantly better.

LOW FREQUENCY INDUCTANCE

The laminations make a complete magnetic path around the beam. At low frequency, the inductance can be substantial.

$$L = N\frac{\phi}{I} \qquad \phi = \frac{mmf}{Reluctance} \qquad mmf = NI \qquad Reluctance = \frac{l}{\mu A}$$

$$L = N^2 \mu \frac{A}{l} \qquad N = 1, \quad u = 100\,\mu_0, \quad A = 2.5\text{"} \times 113.75\text{"}, \quad l = 52\text{"}$$

$$L \approx 17.5 \; {}^{\mu H}\!/_{magnet} \qquad (at\ low\ frequency)$$

Eddy currents in the laminations will reduce this inductance above 10KHz.

DISCUSSION

The current measurement differs slightly from previous estimates. One cause is the higher characteristic impedance of the wire. The high frequency approximation works best when Z << jwL where L is proportional to the characteristic impedance squared. Previous measurements attempted to follow the sagitta of the magnet with a 1.125" diameter copper pipe and a 1/16" diameter copper plated welding rod. The smaller wire (larger characteristic impedance) and better test equipment used in the present measurements should provide more accurate results.

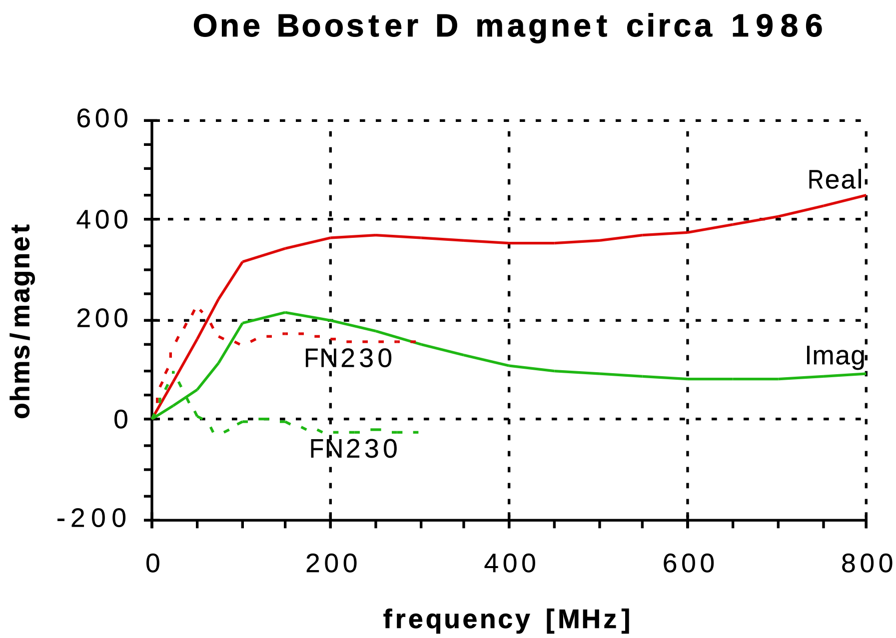


Figure 7. Measurements of one Booster D magnet done in 1986 using a 1/16" wire compared with calculated values from FN230, [3].

The lamination impedance was calculated by treating the magnet poles as a shorted transmission line [1,2,3,4,5]. Laminations are electrically connected through #8 copper wires pinched between the 1x1.5" key and the key way at the top and bottom of the magnet laminations, drawing numbers 2538 an 2539. The keys run along the entire length of the magnet and are welded to the end packs. The parameters used to estimate the impedance are:

laminations:	
laminations/mag	4483
thickness	.0250" (24 gauge)
resistivity	20 uΩ-cm
permitivity	100
dielectric	
gap	.000375"
resistivity	100 KΩ-cm
permitivity	4.7

REFERENCES:

1. Snowdon, S.C.; "Wave Propagation Between Booster Laminations Induced by Longitudinal Motion of Beam"; Fermilab TM-277; November 3, 1970.

2. Ruggiero, A.G.; "Longitudinal Space Charge Forces Within a Bunched Beam in the Presence of Magnetic Lamination"; Fermilab FN-220; January 4, 1971.

3. Ruggiero, A.G.; "Energy Loss due to the Resistive Magnet Lamination in the NAL Booster"; Fermilab FN-230; May 15, 1971.

4. Gluckstern, R.; "Coupling Impedance and Energy Loss with Magnet Laminations"; Fermilab TM-1374; November 1985.

5. Shafer, R.E.; "Coupling Impedance of Laminated Magnets in the Booster"; Fermilab TM-1408; July 11, 1986

DRAWINGS:

D Magnet Assembly	0321-ME-2538
F Magnet Assembly	0321-ME-2539
D Magnet Lamination	0321-MD-2127
F Magnet Lamination	0321-MD-2126
D Magnet Lamination Profile	0321-MD-2190
F Magnet Lamination Profile	0321-MD-2188

MISCELLANEOUS DATA:

lamination thickness	.0250"	24 gauge
gap	.000375"	
laminations/m	1552	
F magnet		
height at center	1.640"	
height at 1" from center	1.74"	
sagitta	±.503"	
magnet length	125.828"	
lamination length	113.741"	
D magnet		
height at center	2.250"	
height at 1" from center	2.42"	
sagitta	±.428"	
magnet length	126.328"	
lamination length	113.741"	

TABULATED IMPEDANCE FOR PRESENT MEASUREMENT

	s/n F47		s/n D10		48xF+48xD	
MHz	Re [Ω]	Im [Ω]	Re [Ω]	Im [Ω]	Re [KΩ]	Im [KΩ]
0	25	0	30	0	2.6	.0
15	101	45	90	39	9.2	4.1
30	161	81	122	48	13.6	6.2
45	212	125	151	58	17.4	8.8
60	257	181	180	67	20.9	11.9
75	282	227	205	74	23.4	14.5
90	289	257	235	86	25.1	16.4
105	282	248	267	102	26.3	16.8
120	272	218	303	121	27.6	16.3
135	272	188	348	150	29.8	16.2
150	270	171	387	183	31.6	17.0
165	269	157	418	217	33.0	17.9
180	283	153	445	248	34.9	19.2
195	284	144	446	254	35.0	19.1
210	284	137	439	249	34.7	18.5
225	282	127	424	229	33.9	17.1
240	285	122	411	208	33.4	15.8
255	287	117	403	190	33.1	14.8
270	287	112	393	174	32.7	13.8
285	285	104	384	158	32.1	12.6
300	311	116	407	164	34.5	13.4
315	312	113	404	157	34.4	13.0
330	309	105	396	145	33.8	12.0
345	316	104	399	140	34.3	11.7
360	321	106	402	138	34.7	11.7
375	316	98	391	126	33.9	10.8
390	324	99	390	120	34.3	10.5
405	338	106	397	120	35.3	10.8
420	335	102	390	113	34.8	10.3
435	335	98	381	103	34.3	9.6
450	349	105	393	106	35.6	10.1
465	339	99	387	101	34.8	9.6
480	324	87	376	91	33.6	8.5
495	338	90	392	95	35.0	8.9
510	335	88	395	96	35.0	8.8
525	317	76	383	87	33.6	7.9
540	328	77	388	86	34.4	7.9
555	346	85	400	90	35.8	8.4
570	325	75	381	81	33.9	7.5
585	324	70	378	76	33.7	7.0
600	353	81	397	82	36.0	7.9
615	336	74	379	74	34.3	7.1
630	323	64	368	67	33.2	6.3
645	366	80	396	75	36.6	7.5
660	360	79	383	70	35.7	7.2

675	341	67	373	64	34.3	6.3
690	388	84	395	70	37.6	7.4
705	401	92	386	66	37.8	7.6
720	379	80	375	60	36.2	6.7
735	430	99	391	64	39.4	7.8
750	451	115	394	64	40.6	8.6
765	411	96	389	60	38.4	7.5
780	421	98	399	62	39.4	7.7
795	407	94	410	64	39.2	7.6
810	364	74	410	62	37.2	6.5