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Notes on Skew Quadrupole Fields in the Tevatron

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Introduction

Recently there has been interest in understanding the coupling of the Tevatron. In particular it was noticed that the strength of the skew quad correction circuit, T:SQ, was operating at a much larger current at flattop energies than when the Tevatron was first commissioned.[1] This memo contains a collection of notes, formulas, and calculations regarding the quadrupole fields in the Tevatron and in particular is meant to document the present settings of the skew quadrupole circuits in the Tevatron at 150 GeV and relate these to the amount of coupling in the Tevatron. We also comment on the magnitude of the skew quadrupole component in the Tevatron dipoles (the a_1 multipole component) that would be needed to explain the present strength of the skew quad correctors.

Based on the present settings (on 3/6/03) of the skew quad circuits, calculations using the design Tevatron injection lattice, and the hypothesis that there is a significant amount of a_1 component in the Tevatron dipoles we find:

- Using the present settings of skew quadrupole circuits at 150 GeV we calculate that the skew quadrupole circuits are compensating for about 0.3 units of minimum tune split.
- 1 unit of a_1 (in units of 10^{-4} at 1 inch) results in a calculated minimum tune split of 0.209 tune units and this can be compensated with the combination of –2.11 Amps in T:SQ and +2.2 Amps in T:SQA0.
- Using the present settings of skew quadrupole circuits at 150 GeV the calculated minimum tune split is only 0.011 tune units if 1.37 units of a_1 is added to all of the Tevatron dipole magnets.
- The skew quad circuit T:SQ is 40% stronger at 980 GeV than at 150 GeV after scaling for the energy difference.

[1] ***This observation was recently pointed out to me by D. Edwards and M. Syphers.***

Trim Quadrupole Magnet Strengths

First we define the quadrupole magnetic field as

$$B_x(x,y) = B_1 y$$
$$B_y(x,y) = B_1 x$$

and want to relate this to the magnetic field in a Tevatron quadrupole trim magnet. From "The Tevatron Energy Doubler: A superconducting Accelerator", H. Edwards, (*Ann. Rev. Nucl. Part Sci.* 1985, 35:605-60) we have that the integrated strength of a trim quadrupole magnet is 75 kG-in at 1 inch at 50 Amps.

Converting the integrated strength into a gradient we get

$$\int B_y(at\ 1\ inch) \bullet dl = 75\,kG - in@50Amp$$

or

$$B_1 L(1\ inch)/I = 7.5\ T - inch/50A\ .$$

This gives a field strength of

$$B_1 L/I = 7.5\ T/50A = 0.15T/A$$

where L is the length of the quadrupole and I is the current in the magnet.

If the quadrupoles are treated as thin magnets, then in the MAD convention the thin quadrupole strength is given by

$$\mathrm{K_1 L} = (B_1 L/I)\frac{1}{|B\rho|}I$$

where I is the current in the quadrupole, and $|B\rho|$ is the magnetic rigidity. At 150 Gev $B\rho$ = 500.34 T-m and then the gradient strength at 150 Gev is

$$\mathrm{K_1 L} = (B_1 L/I)\frac{1}{|B\rho|}I = (0.15T/A)/(500.34\ \mathrm{Tesla\ \text{-}\ m})$$
$$\mathrm{K_1 L} = 0.00030\ \mathrm{m^{-1}\ /\ Amp}$$

If this gradient field is a normal field, then if produces a change in the tune of

$$\Delta\nu_x = \frac{1}{4\pi}\sum \beta_{\mathrm{x,i}}\ (\mathrm{K_1L})_{\mathrm{no,i}}$$
$$\Delta\nu_y = -\frac{1}{4\pi}\sum \beta_{y,i}\ (\mathrm{K_1L})_{\mathrm{no,i}}$$

If the gradient field is a skew field, then it produces a change in the coupling. To quantify this we calculate a sine and cosine term of the coupling as

$$\Delta C_{coup} = \frac{1}{2\pi}\sum (K_1L)_{skew,i} \sqrt{\beta_{x,i}\beta_{y,i}} \cos(\phi_y - \phi_x)$$
$$\Delta S_{coup} = \frac{1}{2\pi}\sum (K_1L)_{skew,i} \sqrt{\beta_{x,i}\beta_{y,i}} \sin(\phi_y - \phi_x)$$

where $(K_1L)_{skew,i}$ is the skew component of the quadrupole field, $(\phi_y - \phi_x)$ is the difference between the vertical and horizontal betatron phase advances, and the sum is over all elements in the skew quadrupole circuit.

To better understand the coupling constants we express the minimum tune split in terms of $\Delta C_{coup}, \Delta S_{coup}$. If the Tevatron is adjusted such that there is no coupling then in principle the horizontal and vertical tunes could be set equal to one another. However, if we add a small amount of coupling then the measured minimum tune split would be

$$\Delta\nu_{min} = \sqrt{\Delta C_{coup}^2 + \Delta S_{coup}^2} \ .$$

We can also relate the amount of skew quad strength $(K_1L)_{skew,i}$ to the amount of a_1 multipole in the Tevatron dipole. With

$$K_{1,skew} = \frac{B_0}{B\rho\, r_0} a_1 \times 10^{-4}$$

and the length of the dipole as L = 6.1214 meters we get

$$(K_1L)_{skew,i} = \frac{4.44 Tesla}{3335.64 Tesla - m} \cdot \frac{1 inch}{0.0254 m} \cdot a_1 \times 10^{-4} \cdot 6.1214 m$$

or

$$(K_1L)_{skew,i} = 3.208 \times 10^{-5} \times a_1$$

where $(K_1L)_{skew,i}$ is in units of meter $^{-1}$ and a_1 is in units of 10^{-4} at 1 inch.

Calculated Coupling Strength and Phase from the Skew Quadrupole Circuits at 150 GeV.

These tables list some calculations done using the design Tevatron injection lattice. (This is the lattice used at 150 GeV and at 980 GeV flattop before the low beta squeeze.) For the skew quadrupole circuits in the Tevatron and for the Tevatron dipole magnets we calculate sums related to the coupling equations listed in the previous section. These are analytical results based on lattice functions from the design Tevatron injection lattice, the design strength of the skew quadrupole magnets, and simple perturbation theory for the effect of skew quadrupole fields on the coupling.

		$\Sigma\sqrt{\beta_x}\sqrt{\beta_y}\,\mathrm{Cos}(\phi_y-\phi_x)$	$\Sigma\sqrt{\beta_x}\sqrt{\beta_y}\,\mathrm{Sin}(\phi_y-\phi_x)$	Phase of coupling
Circuit	Number	(meters)	(meters)	(degrees)
"TSQ"	42	2160	-466.7	-12.2
"TSQA0"	2	155	71.82	24.8
"TSQA4"	1	102	-4.097	-2.3
"TSQB1"	1	99.9	3.350	1.9
"TSQD0"	2	193	-15.46	-4.6
"TSQE0"	2	159	31.00	11.0
Center of Dipoloes	772	39529	-10728	-15.2

Table 1 Sum of lattice functions over the elements in the various skew quadrupole circuits for the Tevatron injection lattice. We also include the sum of these lattice functions at the locations of the center of the Tevatron full-length dipoles.

	$\Delta\nu_{min}$/Amp	$\Delta\nu_{min}$/Amp		$\Delta\nu_{min}$/Amp	Coupling Phase
Circuit	Cosine term	Sine term		Sine term	Relative to Sq (in degrees)
"TSQ"	0.103	-0.0223		0.1055	0
"TSQA0"	0.00741	0.00343		0.008161	37
"TSQA4"	0.00486	-0.0002		0.004865	9.9
"TSQB1"	0.00477	0.00016		0.004773	14
"TSQD0"	0.00919	-0.00074		0.009220	7.6
"TSQE0"	0.0076	0.00148		0.007742	23

Table 2 Magnitude of the minimum tune split introduced per amp in the various skew quad circuits at 150 Gev in the Tevatron. For instance, adding 1 Amp in T:SQ to an uncoupled lattice would introduce a minimum tune split of 0.105. Note the T:SQA0 circuit is the most orthogonal to the T:SQ circuit. These calculations use the design values of the Tevatron injection lattice and the design magnetic strengths of the trim skew quadrupole magnets.

	$\Delta\nu_{min}/a_1$	$\Delta\nu_{min}/a_1$		$\Delta\nu_{min}/a_1$	Coupling Phase
	Cosine term	Sine term		Sine term	Relative to Sq (in degrees)
Center of Dipole	0.202	-0.0548		0.209	-3.0

Table 3 The amount of coupling added for a skew quadrupole of a_1 = 1 unit in the Tevatron dipoles. For this calculation all of the skew quadrupole component in the dipoles were treated as a thin multipole magnet at the center of the dipole magnets.

In Table 4 we list the present setting of the skew quad circuits in the Tevatron at 150 GeV and at 980 GeV flattop (before the low beta squeeze.) We note that the T:SQ circuit operates with about 40% more strength at 980 GeV than it does at 150 GeV even after considering the scale factor of 980/150 for the higher energy.

Circuit	Current at 150 GeV (Amps)	Current at 980 GeV (Amps)	Scaled current at 980 GeV (amps @ 150 Gev equivalent)
"TSQ"	**-2.89**	-25.98	**-3.98**
"TSQA0"	+6.29	36.55	5.59
"TSQA4"	-5.18	-33.81	-5.18
"TSQB1"	0.56	3.92	0.60
"TSQD0"	0.00	0.72	0.11
"TSQE0"	0.00	0.00	0.00

Table 4 Present (3/6/03) skew quad circuit settings at 150 GeV and at 980 GeV on the injection lattice. Note that the T:SQ circuit is about 40% stronger at 980 GeV than at 150 GeV after factoring in the change of energy.

Estimate of a_1 component in the Tevatron dipoles.

Using the above information and calculations we can get an estimate of the amount of a_1 component in the Tevatron dipoles. As an example calculation we add 1 unit of a_1 to all of the Tevatron full-length dipoles. For this calculation all of the skew quadrupole is treated as if it were a thin multipole at the center of the Tevatron dipoles. With 1 unit of a_1 the minimum tune split in the Tevatron at 150 GeV would be 0.209 tune units. From the above calculations we find that we can compensate this with –2.11 Amps in the T:SQ circuit and +2.2 Amps in the T:SQA0 circuit. The coupling components from these sources of skew quadrupole fields are plotted in Figure 1.

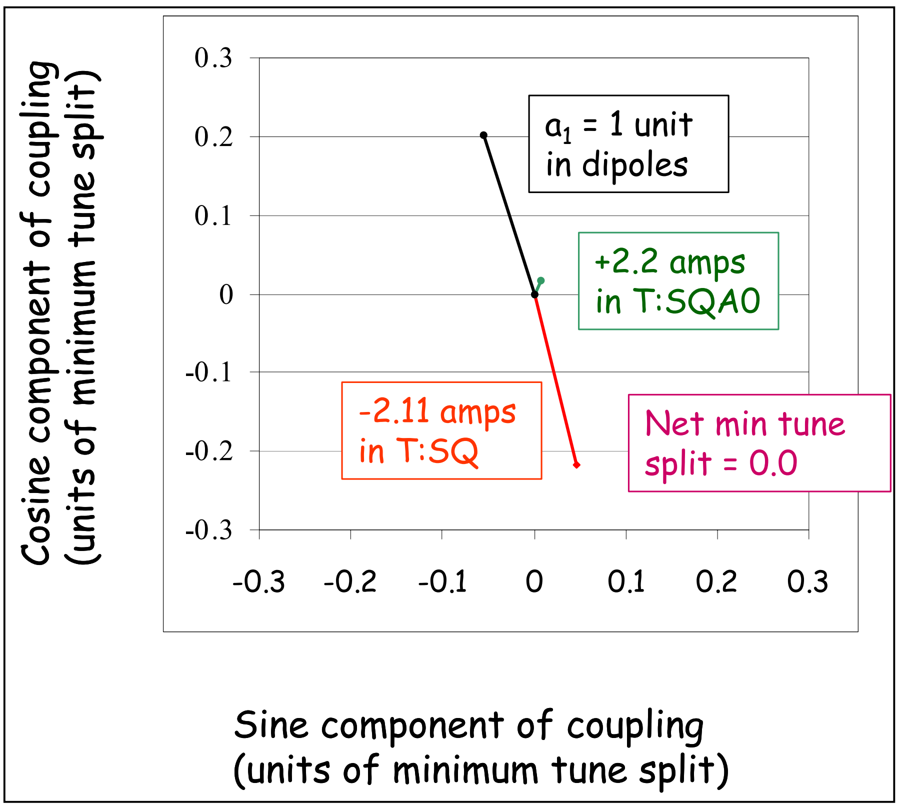

Figure 1 Plot of cosine and sine part of the coupling from 1 unit of a_1 in the Tevatron dipoles, -2.11 Amps in the T:SQ circuit, and +2.2 Amps in the T:SQA0 circuit. These are calculated values based on the design Tevatron injection lattice at 150 GeV.

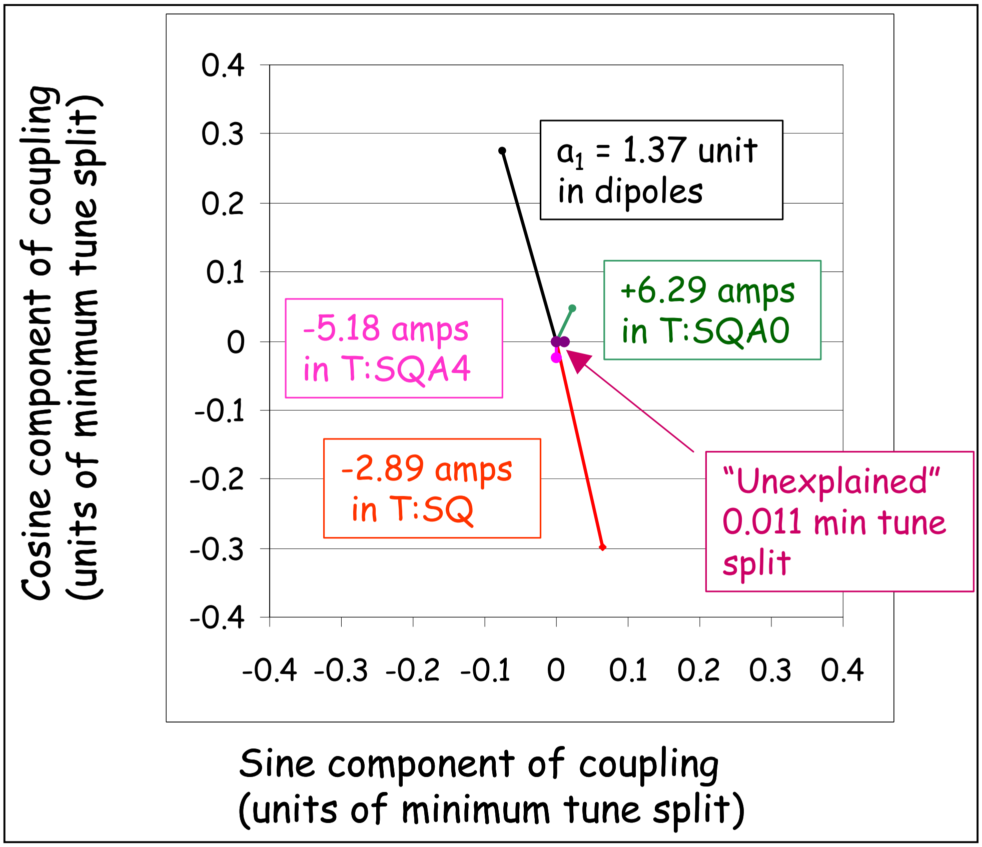

Figure 2 Plot of cosine and sine part of the coupling with the present settings (on 3/6/03) of the skew quad circuits. 1.37 units of a_1 is added in order to get the minimum tune split as small as possible at 0.011 tune units. There are –2.89 Amps in the T:SQ circuit, +6.29 Amps in the T:SQA0 circuit, and -5.18 Amps in the T:SQA4 circuit. These are calculated values based on the design Tevatron injection lattice at 150 GeV.

Some Examples of MAD Calculations

As a separate check of the above calculations, several calculations were done with MAD and the design Tevatron injection lattice. These are noted below.

Start with the design Tevatron injection lattice, file (v3h01i11v2.lat + tevgen.lat), but with the separators off. Set the tunes to 0.585 horizontal and 0.575 vertical and then perform a virtual tune scan to verify that the minimum tune split is nil.

Split the dipole magnets into two halves and add a skew multipole in the center of each magnet. In the skew multipole add some amount of skew quadrupole field by setting K1L to some value and make it a skew field by including “T1” in the multipole definition.

Define

$$K1L = a_1 / \rho / 0.0254\,\text{meter} / 10^4 \times 6.1214\,\text{meters}$$

where a_1 is the skew quadrupole multipole (in units of 10^{-4} at 1 inch), $\rho = B\rho / B_0$, and 6.1214 meters is the length of the dipole.

With a_1 = 0.01 (K1L = 3.2079×10^{-5} m^{-1}) perform a virtual tune scan and find that the minimum tune split is 0.0209 units. Scaling up to 1 unit of a_1 would give a minimum tune split of 0.209 units.

With a_1 = 0, add K1L = -0.01×0.002844 m^{-1} to the SQ trim skew quad circuit (~42 magnets) and perform a virtual tune scan and find that the minimum tune split is 0.01 tune units. Note that the strength of the skew quadrupole magnets translates to $K_1L = 0.00030\ \text{m}^{-1} / \text{Amp}$ at 150 GeV. (See below.) Scaling up to a minimum tune split of 0.2 units gives a value of K1L = $-0.2\times0.002844 = -5.688\times10^{-4}$ m^{-1} which corresponds to $I_{Sq} = -5.688\times10^{-4}\ m^{-1} / (3.00\times10^{-4}\ m^{-1}/\text{Amp}) = -1.896$ Amps.

Next, try to add 1 unit of a_1 and decouple with the T:SQ and T:SQA0 circuit. The lowest value for the minimum tune split using just T:SQ is 0.011 when K1L in SQ is -0.209×0.002844 m^{-1}. To get the minimum tune split as low as 0.0021 tune units we need to use both SQ and SQA0. The best values I found were K1L = -0.22×0.002844 m^{-1} in SQ and K1L = 0.015×0.03658 m^{-1} in SQA0. This corresponds to –2.08 Amps in T:SQ and +1.83 Amps in T:SQA0.