Estimate of Skew Quadrupole Field in Tevatron Dipoles due to Creep

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March 24, 2003

1 Introduction

The strong systematic transverse coupling observed in the Tevatron is believed to have developed over time as fiberglass spacers used in the magnet design have crept into a compressed state during the lifetime of the accelerator. Below we present a simple model of the Tevatron dipole magnet using the method of magnetic images to calculate the skew quadrupole coefficient, a_1, which would develop due to a displacement of the superconducting coil with respect to the iron yoke of the magnet. A numerical estimate based on the Tevatron dipole magnet parameters along with the Smart Bolt measurements made of Tevatron magnets in the tunnel produce a result consistent with beam observations. The influence on a_1 of the coil placement within the iron yoke is well known[1] – during construction of the Tevatron magnets, the coil was positioned in order to zero this variable while on the magnet test stand. The purpose of this document is to advance the understanding of this effect in a straight-forward way.

2 Smart Bolt Measurements and Coil Displacement

A (very) schematic diagram of the Tevatron dipole magnet suspension is shown in Figure 1. The coil is held in place by an array of four bolts spaced along the magnet, the top two of which are spring-loaded (Smart Bolts). Fiberglass spacers, made of G11 material, are used at the interfaces of the warm and cold regions to minimize heat leak. The conjecture is that these spacers have slowly compressed over time. If the spacers at each of the four points shown in the figure have had their total lengths compressed by an amount δ, then the springs in the Smart Bolts will relax by an amount 2δ. In the recent tunnel measurements, the average displacement of the Smart Bolts for 18 magnets was observed to be $2\delta \approx 6$ mil (0.15 mm). As indicated in Figure 2, this translates into a vertical coil movement relative to the iron yoke of the magnet of an amount $\delta\sqrt{2} = 4.2$ mil (0.11 mm).

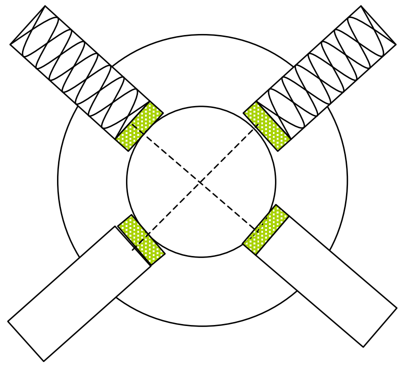

Figure 1: Schematic of cold mass support system in Tevatron dipole magnet.

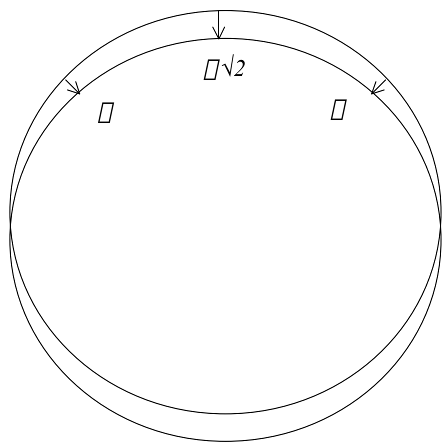

Figure 2: Coil displacement deduced from Smart Bolt measurement.

3 Simple Model of Tevatron Dipole with Iron Yoke

To estimate the effects of a vertical coil displacement we begin with a simple model of a superconducting magnet with a coil centered inside an infinitely large iron yoke. The geometry of the coil can be approximated by the intersection of two cylindrical regions of uniform current, each with radius c, as shown in Figure 3 – the result of a famous textbook problem in electromagnetism.[2] The current in the cylinder on the left moves out of the page, current on the right moves into the page. By superposition, there is no current in the central overlap region (best for particle propagation!) and the magnetic field in this region is a pure dipole field pointing vertically upward of strength

$$B_y = \frac{\mu_0 j d}{2}$$

where j is the current density, and d is the distance of separation between the centers of the two circles (also the maximum thickness of the "coil"). We note for future reference that the total current in each circle is $\pm I = \pm \pi c^2 j$.

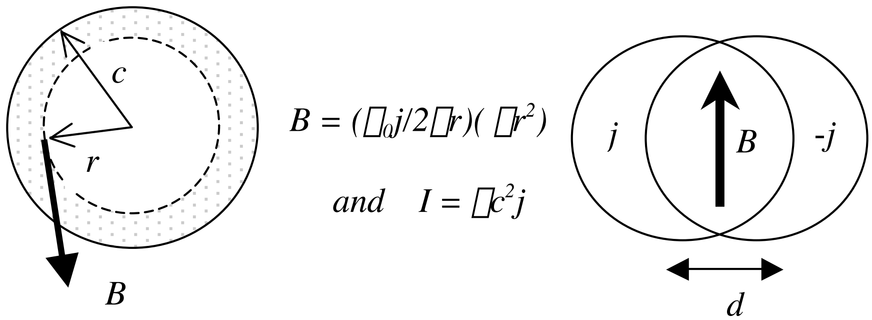

Figure 3: Dipole field generated by two overlapping current regions.

We next must add in the fact that the coil is placed inside an iron yoke of cylindrical cross section with radius R. Since the field within the magnet is independent of longitudinal position, the magnetic field is equal to the gradient of a scalar potential. The iron boundary is a magnetic equipotential surface and as such the method of images can be applied to compute the contribution of the magnetic field due to the iron onto the central field. For example, it is known that a line charge inside a grounded cylinder of radius R located a distance a from the center will have an image line charge of opposite sign located a distance R^2/a from the center of the cylinder. The field within the cylinder can be computed by adding the fields due to the interior line charge with the field due to its image. Since the 2-D magnetostatic problem relies on solutions to Poisson's equation just as in the electrostatic case, the same principle can be applied. Here, however, the magnetic image currents are of the same sign as the interior current. (The iron enhances the field.) And so our situation is that shown in Figure 4. In this figure, the large circle is the boundary of the iron yoke, and the two small circles inside indicate the centers of the two overlapping current regions described earlier. The open circles indicate the locations of the images.

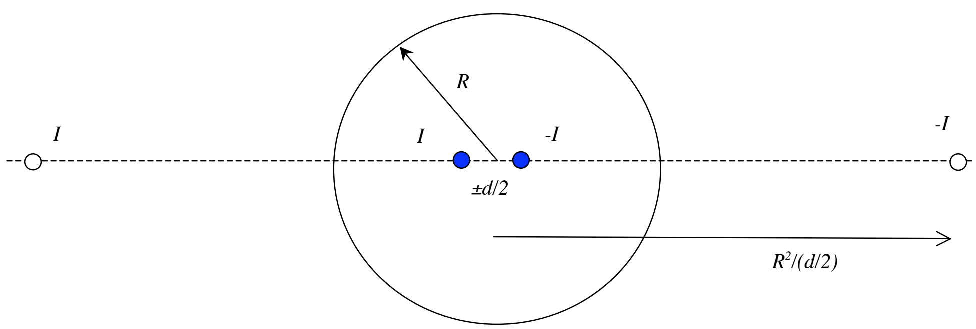

Figure 4: Image currents in the iron yoke.

The images will be very far away. Since we are talking about units of a measured in mils, and R measured in inches, the images are about 1000 inches distant. Thus, the image of each of the two intersecting cylinders making up the coil can be treated as a line current of value $I = \pm\pi c^2 j$. The field at the center of of the coil due to the images will be vertical and is given by

$$B_y^{(i)} = 2\,\frac{\mu_0 I}{2\pi[R^2/(d/2)]} = \frac{\mu_0 j d}{2}\,\left(\frac{c}{R}\right)^2 \; .$$

Therefore, the total field due to the coil and the iron yoke will be

$$B_y(x=0, y=0) \equiv B_0 = \frac{\mu_0 j d}{2}\left[1 + \left(\frac{c}{R}\right)^2\right].$$

For the dimensions of the Tevatron we note that the ratio of the average coil radius to the yoke radius is approximately 50%. Thus, the iron enhances the central field by about 25%. (According to the Tevatron Design Report[3], this value is "a little more than 20%." For the final magnet design, the value is reported as $\sim$ 18%.[4] Here, the details of the coil and the finite extent of the iron likely come into play.) For Tevatron magnet parameters we see that

$$B_y \approx \frac{(4\pi \times 10^{-7}\mathrm{T}\cdot\mathrm{m/A})(500\mathrm{A/mm}^2)(10\mathrm{mm})(10^3\mathrm{mm/m})}{2} \cdot (1.25) \approx 4\ \mathrm{T}.$$

4 Development of a_1 due to Coil Displacment

We next look at the situation in which the center of the coil is displaced vertically relative to the center of the iron yoke. Since we want to expand the resulting magnetic field about the coil center it is easiest to consider the iron yoke raised by a distance Δ relative to the coil. The geometry is presented in Figure 5. From the figure one can deduce the following:

$$\begin{aligned}
B_y^{(i)}(r) &= 2\ \frac{\mu_0 I}{2\pi r}\ \sin\alpha \\
a^2 &= \frac{d^2}{4} + \Delta^2 \\
r^2 &= \frac{R^2}{a}\cos\theta + \left[\frac{R^2}{a}\sin\theta + (y-\Delta)\right]^2
\end{aligned}$$

Therefore, along $x = 0$, the magnetic field due to the images will be vertical and have strength

$$\begin{aligned}
B_y^{(i)} &= 2\ \frac{\mu_0 I}{2\pi r}\ \frac{R^2}{ar}\cos\theta \\
&= 2\ \frac{\mu_0 I}{2\pi r^2}\ \frac{R^2}{a}\frac{d}{2a} \\
&= \frac{\mu_0(\pi c^2 j)d}{2\pi r^2}\ \frac{R^2}{a^2} = \frac{\mu_0 j d}{2}\ \left(\frac{c}{R}\right)^2 \frac{R^4}{a^2}\frac{1}{r^2}
\end{aligned}$$

Next, we wish to write the $1/r^2$ factor in terms of the vertical coordinate y to be able to look at the gradient of the field $\partial B_y/\partial y$. We note that

$$r^2 = (R^2/a)^2 + 2(R^2/a)(y-\Delta)\sin\theta + (y-\Delta)^2$$

from which we write

$$\begin{aligned}
r^2/(R^2/a)^2 &= 1 + 2(a/R^2)(y-\Delta)(\Delta/a) + (y^2 - 2y\Delta + \Delta^2)(a/R^2)^2 \\
&= 1 + 2\frac{\Delta}{R^2}\ y - 2\Delta\frac{a^2}{R^4}y + \ldots
\end{aligned}$$

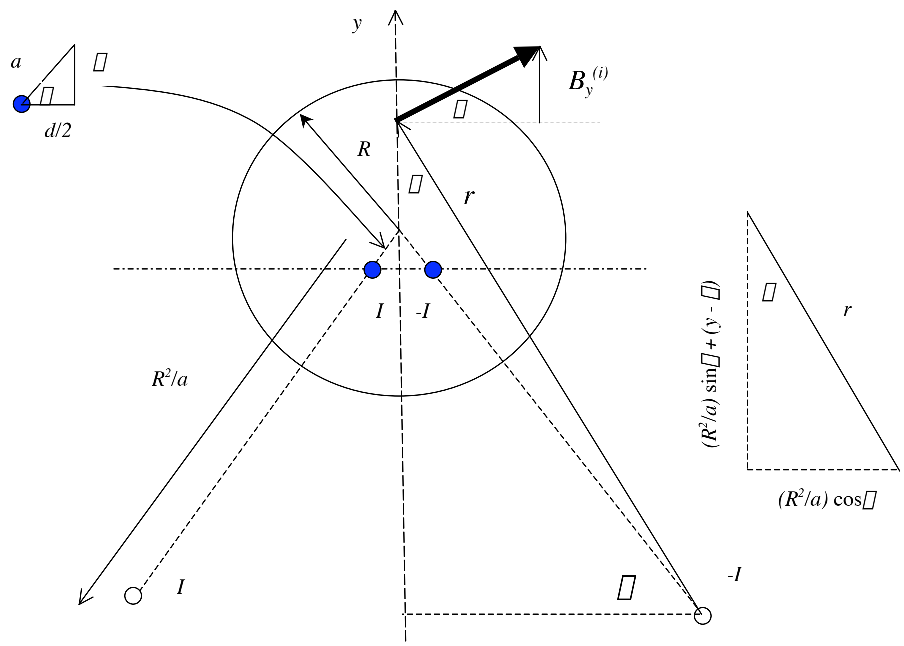

Figure 5: Geometry of the iron yoke being displaced a distance Δ from the center of the coil. The magnetic images are now below the midplane, generating a top-down asymmetry (and a "skew" field).

So, to close approximation, since $R/a \gg 1$,

$$\frac{R^4}{a^2}\frac{1}{r^2} = 1 - 2\frac{\Delta}{R^2}\left[1 - \left(\frac{a}{R}\right)^2\right] \cdot y \approx 1 - 2\frac{\Delta}{R^2}y.$$

The total field due to the coil plus the off-centered iron yoke is thus

$$\begin{aligned} B_y(x=0) &= \underset{(coil)}{\frac{\mu_0 jd}{2}} + \underset{(images)}{\frac{\mu_0 jd}{2}\left(\frac{c}{R}\right)^2\left(1 - 2\frac{\Delta}{R^2}y\right)} \\ &= \frac{\mu_0 jd}{2}\left[1 + \left(\frac{c}{R}\right)^2\right] - 2\frac{\mu_0 jd}{2}\left(\frac{c}{R}\right)^2\frac{\Delta}{R^2}y \\ &= B_0 - 2B_0\frac{(c/R)^2}{1+(c/R)^2}\frac{\Delta}{R^2}\,y \end{aligned}$$

The usual field expansion in terms of normal and skew multipole coefficients is

$$B_y + iB_x = B_0 + B_0\sum_n (b_n + ia_n)(x+iy)^n$$

and so for our case the skew quadrupole moment, a_1, is found from

$$a_1 = -\frac{1}{B_0}\left(\frac{\partial B_y}{\partial y}\right)_{(x=0)}.$$

From the result of our derivation above, this gives

$$a_1 = 2\frac{(c/R)^2}{1+(c/R)^2}\frac{\Delta}{R^2}$$

as the value of a_1 due to the coil being displaced vertically downward by a distance Δ. For a positive value of Δ, the coil is closer to the bottom of the iron yoke, making the field stronger below the coil center than above the coil center. Thus, the gradient $\partial B_y/\partial y$ will be negative, or a_1 will be positive.

The measurements described in Section 2 implied that the centers of the coils of the Tevatron dipoles may have crept vertically by an average distance of 0.0042 in. From the Tevatron Design Report the average coil radius of a Tevatron dipole magnet is approximately 1.84 in. The yoke radius is 3.77 in. Using the above result, the change in a_1 generated by a 1 mil displacement would be

$$a_1/\text{mil} = 2\frac{(1.84/3.77)^2}{1+(1.84/3.77)^2}\frac{10^{-3}\text{in}}{(3.77\text{ in})^2}\frac{1}{\text{mil}} = (0.27\times 10^{-4}/\text{in})/\text{mil}$$

which is consistent with the value quoted in [1]. For 4.2 mil, the induced $a_1 = 1.1\times 10^{-4}/\text{in}$, which is consistent with the beam measurements taken last month.[5]

Special thanks goes to Don Edwards for discussions of the calculation and making helpful suggestions to the document.

References

[1] A. V. Tollestrup, "The Amateur Magnet Builder's Handbook," Fermilab note UPC No. 86, February 22, 1979.

[2] While this problem certainly exists in many texts, the author first came to know it well from P. Lorrain and D Corson, "Electromagnetic Fields and Waves," 2nd Ed., W. H. Freeman and Co., San Francisco (1970).

[3] "A Report on the Design of the Fermi National Accelerator Laboratory Superconducting Accelerator," Fermilab Report, May 1979.

[4] H. T. Edwards, "The Tevatron Energy Doubler: A Superconducting Accelerator," *Ann. Rev. Nucl. Part. Sci.*, **35**, 605-60 (1985).

[5] D. Edwards and M. Syphers, "StrongTransverse Coupling in the Tevatron," Beams-doc-501, March 2003.