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Recycler Beam Life Time: Beam-Gas Interactions

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Abstract

In this note, we study the Recycler beam life time due to beam-gas interactions such as single coloumb scattering, inelastic scattering, multiple coloumb scattering, and nuclear inelastic scattering etc.

1 Beam-Gas Scattering

The Recycler vacuum is modelled with residual gas composed of primarily two component gases H_2, and $CO + N_2$ as shown in the following table [2]:

Gas	Pressure [nTorr]	Gas Density [$1/m^3$]	Mean Free Path [m]
H_2	0.1	3.5×10^{12}	2.3×10^6
$CO + N_2$	0.002	7.1×10^{10}	1.1×10^8

Table 1: Residual gases in the Recyler Vacuum as given in the Recyler Ring Technical Design Report.

But the gas densities shown above may not reflect the present situation. The ratio $R_1 = H_2/(CO + N_2) = 49.3$ may be a more close to 10 in reality. Moreover, there is no information available about the actual proportion of N_2 relative to CO [3]. Preparations are being done to measure them in the near future. Therefore, we compute the life time in the range where N_2 is assumed to be 0 - 100 % for the gas density shown for the combination $CO + N_2$. We also compute this for both cases: $R_1 = 49.3$, and $R_1 = 9.9$ The four different scenerios are shown in the following table:

Scenerios	H_2 Density [$1/m^3$]	N_2 Density [$1/m^3$]	CO Density [$1/m^3$]
Case 1a	3.5×10^{12}	7.1×10^{10}	0.0
Case 1b	3.5×10^{12}	0.0	7.1×10^{10}
Case 2a	3.5×10^{12}	35.5×10^{10}	0.0
Case 2b	3.5×10^{12}	0.0	35.5×10^{10}

Table 2: Life times are evaulated for the above four different gas density combinations

1.1 Single Coloumb Scattering:

It is possible by a single coloumb scattering off a residual gas nucleus, a proton from the beam can be lost. To estimate the rate of loss by this mechanism, we use the classic Rutherford scattering formula [4].

$$\frac{1}{\tau_{sc}} = \frac{-1}{N}\frac{dN}{dt} = \beta c \sum \sigma_j n_j$$

where the sum is over the species of residual gas molecules with density n_j and σ_j is given by:

$$\sigma_j = \frac{4\pi Z_j^2 r_p^2}{\beta^4 \gamma^2 \theta_{max}^2}$$

with Z_j, the atomic number of jth gas and θ_{max} denoting the maximum angle deflection needed for knocking of the proton. The angle θ_{max} is estimated by:

$$\theta_{max} = \sqrt{\frac{Acceptance}{\pi \beta_{avg}}}$$

Therefore, the beam life time due to single coloumb scattering can be cast as:

$$\frac{1}{\tau_{sc}} = \frac{4\pi r_p^2 c}{\beta^3 \gamma^2 \theta_{max}^2} \sum Z_j^2 n_j$$

Using Table I and the relevant Recyler parameters listed above, we obtain $\theta_{max} = 0.325$ mr and $\tau_{sc} = 7.59 - 7.66 \times 10^7$ s $(2.11 - 2.13 \times 10^4$ hours) for case 1. For case 2, the $\tau_{sc} = 2.35 - 2.56 \times 10^7$ s $(6.53 - 7.11 \times 10^3$ hours). The contribution from single coloumb scattering for the Recycler beam life appears small as one might expect.

1.2 Inelastic Scattering:

There are two important processes in the inelastic scattering: (a) Bremsstrahlung scattering where the proton emits a photon and the nucleus of the gas atom is left unexcited (b) Inelastic scattering (excitation) of the electrons of the atoms from momentum transfer. We examine each case below.

The total cross section for the bremstrahlung can be written for a given gas as [5]:

$$\sigma_{br} = \int_{\epsilon_m}^{E} \{\frac{d\sigma}{d\epsilon}\} d\epsilon$$

where E denotes the energy of the proton, ϵ_m the photon energy and

$$(\frac{d\sigma}{d\epsilon})_{br} = \frac{4\alpha Z^2 r_p^2}{\epsilon} \{[\frac{4}{3}(1 - \frac{\epsilon}{E}) + \frac{\epsilon^2}{E^2}][\frac{\phi_1(0)}{4} - \frac{1}{3}\text{InZ}] + [\frac{1}{9}(1 - \frac{\epsilon}{E})]\}$$

with $\phi_1(0)$ representing the screening function. A similar expression applies to the case of atomic/molecular electron excitations:

$$(\frac{d\sigma}{d\epsilon})_{ee} = \frac{4\alpha Z r_e^2}{\epsilon} \{[\frac{4}{3}(1 - \frac{\epsilon}{E}) + \frac{\epsilon^2}{E^2}][\frac{\psi_1(0)}{4} - \frac{2}{3}\text{InZ}] + [\frac{1}{9}(1 - \frac{\epsilon}{E})]\}$$

Here $\psi_1(0)$ denotes the screening function as $\phi_1(0)$. They can be approximated by $\psi_1(0) \simeq 28.34$ and $\phi_1(0) \simeq 20.836$.

For $\epsilon_m \ll$ E, the above expressions can be evaluated and can be cast as:

$$\sigma_{br} = 4\alpha\{\frac{4}{3}Z^2 r_p^2 \text{In}\frac{183}{Z^{1/3}}[\text{In}(E/\epsilon_m) - (5/8)] + \frac{1}{9}(Z^2 r_p^2)[\text{In}(E/\epsilon_m) - 1]\}$$

$$\sigma_{ee} = 4\alpha\{\frac{4}{3}Z r_e^2 \text{In}\frac{1194}{Z^{2/3}}[\text{In}(E/\epsilon_m) - (5/8)] + \frac{1}{9}(Z r_e^2)[\text{In}(E/\epsilon_m) - 1]\}$$

The total cross section is then:

$$\sigma_{br+ee} = \sigma_{br} + \sigma_{ee}$$

The σ_{br} is quite negligible compared to σ_{ee} and therefore we drop it from further consideration. The life time due to inelastic scattering τ_{in} becomes as before:

$$\frac{1}{\tau_{in}} = \frac{-1}{N}\frac{dN}{dt} = \beta c \sum \sigma_{eej} n_j$$

Now for $\epsilon_m = 89$ MeV, using the relevant Recycler parameters and Table I, $\tau_{in} = 4.845 - 4.845 \times 10^7$ s ($1.35 - 1.35 \times 10^4$ hours) for case 1. For case 2, the $\tau_{in} = 3.420 - 3.420 \times 10^7$ s ($0.95 - 0.95 \times 10^4$ hours).

1.3 Multiple Coloumb Scattering:

Unlike the previous two cases, the mutiple coloumb scattering causes emittance growth of the beam. As a result, protons are lost via diffusion across the boundary of the allowed particle distribution in the beam pipe. Therefore we should approach this problem by solving the diffusion equation [6] for a particle distribution f:

$$\frac{\partial f}{\partial \tau} = \frac{\partial}{\partial Z}(Z\frac{\partial f}{\partial Z})$$

subject to the boundary conditions:

$$f(Z,0) = f_0(Z)$$

$$f(1,\tau) = 0$$

where Z = ϵ/ϵ_a = emittance/acceptance, and τ = tR/ϵ_a with R, the diffusion coefficient. The diffusion coefficient R is given in terms of scattering angle θ by:

$$R = \beta_{avg}\langle\dot{\theta}^2\rangle$$

The general solution of the above equation can be written as:

$$f(Z,\tau) = \sum_n C_n J_0(\lambda_n \sqrt{Z}) e^{-\lambda_n^2 \tau/4}$$

with coefficients C_n:

$$C_n = \frac{1}{J_1(\lambda_n)^2} \int_0^1 f_0(Z) J_0(\lambda_n \sqrt{Z}) dZ$$

where λ_n is nth root of the Bessel function $J_0(Z)$. Now we can obtain the total beam particles as a function of time:

$$N(\tau) = \int_0^1 f(Z,\tau) dZ = 2 \sum_n \frac{C_n}{\lambda_n} J_1(\lambda_n) e^{-\lambda_n^2 \tau/4}$$

The life time due to multiple coloumb can be now computed using the standard expression:

$$\tau_{mc} = -\frac{N(\tau)}{dN(\tau)/d\tau}$$

The beam life time varies with time and normally reaches an asymptotic value:

$$\tau_a = \frac{4\epsilon_a}{\lambda_1^2 R}$$

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To compute $< \dot{\theta}^2 >$, we use the small angle limit of the Rutherford scattering cross section, parametrization of atomic and nuclear radii:

$$< \dot{\theta}^2 > = \frac{8\pi r_p^2 c}{\gamma^2 \beta^3} \sum_j n_j Z_j^2 \mathrm{In}[\frac{38360}{(A_j Z_j)^{1/3}}]$$

with A_j denoting the atomic weight of jth gas component. Using Table I and the relevant Recycler parameters, we obtain $< \dot{\theta}^2 > = 2.770 - 2.772 \times 10^{-14}$ radians The asymptotic life time due to multiple coloumb scattering is given by $\tau_{mc} = 8.143 - 8.481 \times 10^6$ s ($2.26 - 2.36 \times 10^3$ hours) for case 1. For case 2, the $\tau_{mc} = 2.945 - 2.989 \times 10^6$ s ($8.18 - 8.30 \times 10^2$ hours).

1.4 Nuclear Scattering:

Here we estimate the beam life time due to the loss of protons from interaction with nucleus of residual gas molecules via strong force. As there is no simple expression for the interaction cross section for the strong force, we use the general formula:

$$\frac{1}{\tau_{nu}} = \frac{-1}{N}\frac{dN}{dt} = \beta c \sum \sigma_j n_j$$

where σ_j denotes the total (elastic + inelastic + quasi-elastic) proton-nucleus cross for each gas in the proton relevant energy range. The total cross sections are [7]: $\sigma_{nc}(H) = 40$mb, $\sigma_{nc}(N) = 387$mb, $\sigma_{nc}(C) = 344$ mb, and $\sigma_{nc}(O) = 429$mb.

Now using the above cross sections and the gas densities in Table 1, we obtain $\tau_{nu} = 9.975 - 9.972 \times 10^7$ s ($2.77 - 2.77 \times 10^4$ hours) for case 1. For case 2, the $\tau_{nu} = 6.021 - 6.026 \times 10^7$ s ($1.67 - 1.67 \times 10^4$ hours).

1.5 Final Beam-Gas Life Time:

Now we can combine the contributions from various beam-gas interactions to obtain the life time as:

$$\frac{1}{\tau_{bg}} = \frac{1}{\tau_{sc}} + \frac{1}{\tau_{in}} + \frac{1}{\tau_{mc}} + \frac{1}{\tau_{nu}}$$

Using the asymptotic life time for the multiple coloumb scattering, direct evaluation gives the total beam life time due to beam-gas interactions. The results are tabulated for all cases. The cases 3 and 4 are for the over-all gas densities higher by an order of magnitude 1 and 2 respectively (obtained by scaling as all life times are inversely proportional to the gas density).

Physical Process	Case 1 [hours]	Case 2 [hours]	Case3 [hours]	Case 4 [hours]
Single Coloumb	$2.11 - 2.13 \times 10^4$	$6.53 - 7.11 \times 10^3$	$6.53 - 7.11 \times 10^2$	$6.53 - 7.11 \times 10$
Inelastic Scatt.	$1.35 - 1.35 \times 10^4$	$0.95 - 0.95 \times 10^4$	$0.95 - 0.95 \times 10^3$	$0.95 - 0.95 \times 10^2$
Mult. Coloumb	$2.26 - 2.36 \times 10^3$	$8.18 - 8.30 \times 10^2$	$8.18 - 8.30 \times 10$	$8.18 - 8.30$
Nuclear Scatt.	$2.77 - 2.77 \times 10^4$	$1.67 - 1.67 \times 10^4$	$1.67 - 1.67 \times 10^3$	$1.67 - 1.67 \times 10^2$
Total life time	$1.66 - 1.67 \times 10^3$	$6.49 - 6.62 \times 10^2$	$6.49 - 6.62 \times 10$	$6.49 - 6.62$

Table 3: The total life time summary for all four cases considered. The range is obtained by assuming either 100% N_2 or 0% N_2 for the gas component $CO + N_2$. See section 2.1 for details. The case 3 and case 4 are same as case 2 except the gas densities are assumed to be higher by a factor of 10 and 100 correspondingly.

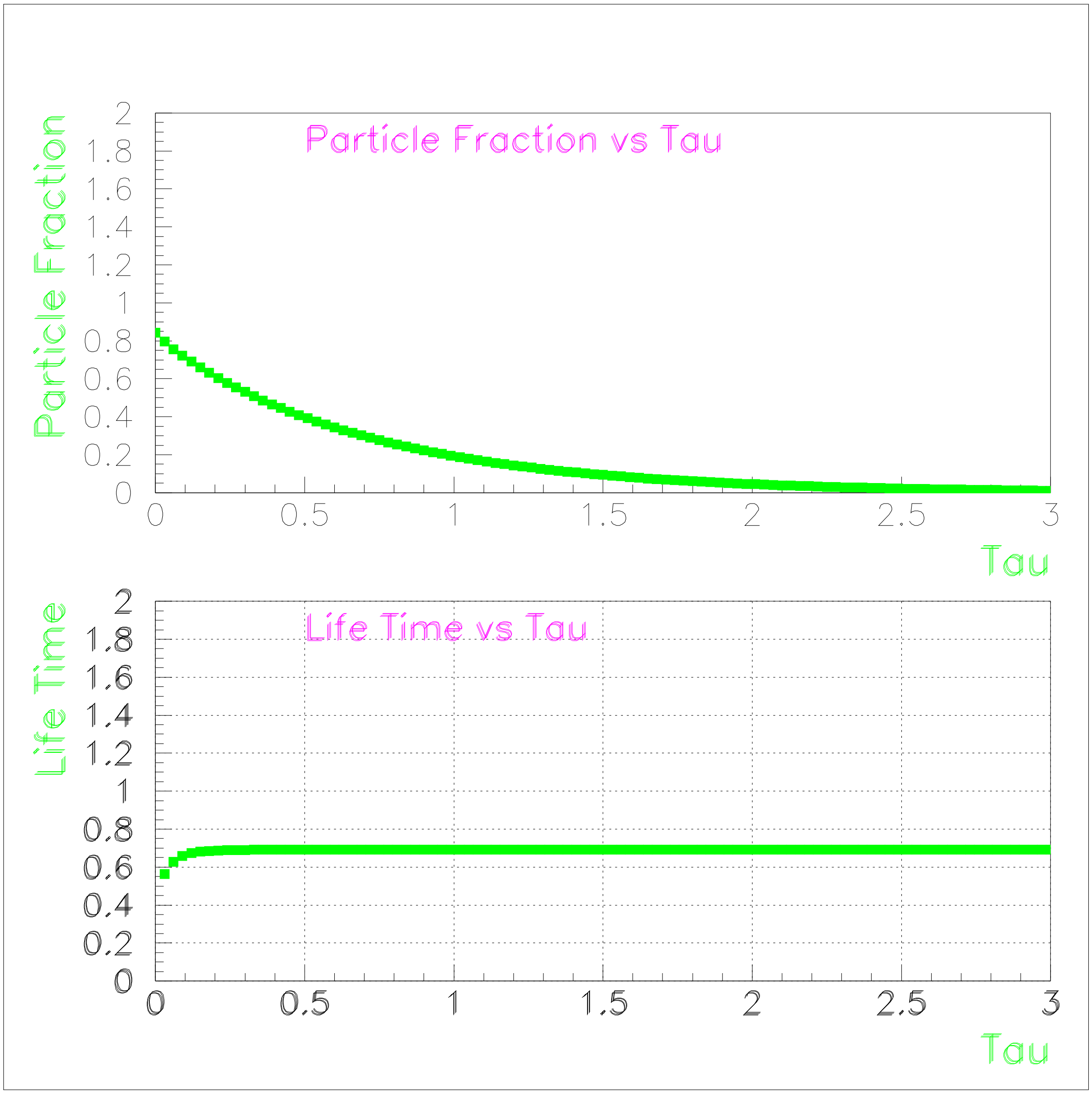

Figure 1: The picture above shows the particle fraction and life time as a function of $\tau = tR/\epsilon_a$ for gas components in Table I, initial beam and the Recyler parameters assumed in the case of multiple coloumb scattering. The half life time is $\tau_{1/2} = 4.023 - 4.056 \times 10^6 s (1.12 - 1.13 \times 10^3 hours)$ for case 1.

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References

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3 The Ratio τ_{sc}/τ_{mc}

From the expressions shown above, we could compute the ratio τ_{sc}/τ_{mc} as:

$$\tau_{sc}/\tau_{mc} = \frac{\lambda_1^2}{2\pi} \frac{\sum n_j Z_j^2 \ln[38360/(A_j Z_j)^{1/3}]}{\sum n_j Z_j^2}$$

Now with $\lambda = 2.405$ anf gases ranging from Hydroegn to Argon, the ratio τ_{sc}/τ_{mc} shound be around 8 or so! Using a pencil beam, we can measure the single coloumb scattering rate from the initial slope of the beam loss curve. Similary we can measure the multiple scattering rate from the intial emittance growth. The multiple scattering life time can also be measured from the beam loss curve after the beam is grown and filled the whole available aperture provided other beam loss mechanisms are negligible.

4 Single Coloumb Scattering: θ_{max} Improvement

In the above calculations we have used:

$$\theta_{max} = \sqrt{\frac{Acceptance}{\pi \beta_{avg}}}$$