CDF/MEMO/TRACKING/CDFR/4189

Measuring beam width and SVX impact parameter resolution

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Introduction

The observed impact parameter distribution of SVX tracks is the convolution of the primary vertex distribution with the impact parameter resolution and its width ($\sigma_d \sim 46\ \mu$m) is given by:

$$\sigma_d^2 = \sigma_F^2 + \sigma_r^2 \tag{1}$$

where σ_F is the actual beam width and σ_r is the impact parameter resolution. We propose a method to measure σ_F and σ_r separately from data.

The idea is to extract σ_F from the measurement of the correlation between impact parameters of track pairs exiting from the same primary vertex. Since σ_d is easily measured, impact parameter resolution (σ_r) is then determined by subtraction (Eq. (1)).

This method can also be used to determine σ_F and σ_r dependence on z and p_t.

1 Measurement of beam width

If we neglect measurement errors for a moment, impact parameter has the following dependence on vertex coordinates (x_v,y_v) and track azimuthal angle (ϕ) (Figure 1):

$$d = y_v \cdot \cos(\phi) - x_v \cdot \sin(\phi) \tag{2}$$

Using Eq. (2) we can evaluate the correlation between impact parameters of track pairs exiting from the same vertex:

$$< d_1 d_2 > = < (y_v \cdot \cos(\phi_1) - x_v \cdot \sin(\phi_1)) \cdot (y_v \cdot \cos(\phi_2) - x_v \cdot \sin(\phi_2)) > \tag{3}$$

(angle brackets indicate averaging over all track pairs). We define two new angular variables:

Figure 1: Dependence of impact parameter on vertex coordinates (x_v,y_v) and azimuthal angle (ϕ)

$$\Phi = \tfrac{1}{2}(\phi_1 + \phi_2)$$
$$\Delta\Phi = \phi_1 - \phi_2$$

that we substitute in Eq. (3):

$$< d_1 d_2 >= \frac{1}{2}(< y_v^2 > - < x_v^2 >)\cos(2\Phi) - < x_v y_v > \sin(2\Phi) + \frac{1}{2}(< y_v^2 > + < x_v^2 >)\cos(\Delta\Phi) \tag{4}$$

We now define a new reference (ζ_1,ζ_2), the *principal axes* reference, in which the symmetric matrix:

$$\begin{pmatrix} < \zeta_{1v}^2 > & < \zeta_{1v}\zeta_{2v} > \\ < \zeta_{2v}\zeta_{1v} > & < \zeta_{2v}^2 > \end{pmatrix}$$

that gives the correlation between primary vertex coordinates, is diagonal ($< \zeta_{1v}\zeta_{2v} >= 0$). In this new reference we define $< \zeta_{1v}^2 >= \sigma_1^2$ and $< \zeta_{2v}^2 >= \sigma_2^2$ and we find that Eq. (4) is thus simplified:

$$< d_1 d_2 >= \frac{1}{2}(\sigma_1^2 + \sigma_2^2)\cos(\Delta\Phi) - \frac{1}{2}(\sigma_1^2 - \sigma_2^2)\cos(2\Phi - 2\alpha) \tag{5}$$

with α being the angle of rotation between the new and the old reference. If the beam is circular Eq. (5) is further simplified:

$$< d_1 d_2 >= \sigma_F^2 \cdot \cos(\Delta\Phi) \tag{6}$$

where $\sigma_F^2 = \frac{1}{2}(\sigma_1^2 + \sigma_2^2)$.

Now we will show that this result does not change when including the effect of measurement errors (finite impact parameter resolution). Adding random errors ϵ_1 and ϵ_2 to d_1 and d_2 the correlation changes as follows:

$$< d_1 d_2 > \rightarrow < (d_1 + \epsilon_1)(d_2 + \epsilon_2) > = < d_1 d_2 > + < d_1 \epsilon_2 > + < \epsilon_1 d_2 > + < \epsilon_1 \epsilon_2 > \quad (7)$$

This reduces to the old expression if the terms $< d_1 \epsilon_2 >$, $< \epsilon_1 d_2 >$ and $< \epsilon_1 \epsilon_2 >$ can be neglected. Now:

- $< d_i \epsilon_j >$: we can safely assume the error on one d does not depend on the value of the other d. Therefore d_i and ϵ_j are uncorrelated (for $i \neq j$):

  $< d_i \epsilon_j > = < d_i >< \epsilon_j >$. Since impact parameter is measured with respect to the beam axis $< d_i > = 0$ and $< d_1 \epsilon_j >$=0.

- $< \epsilon_1 \epsilon_2 >$: the term $< \epsilon_1 \epsilon_2 >$ is zero assuming that the measurement errors ϵ_1 and ϵ_2 on the two tracks are not correlated ($< \epsilon_1 \epsilon_2 > = < \epsilon_1 >< \epsilon_2 >$) and that the impact parameter measurement in not biased $< \epsilon_i > = 0$.

Therefore, we conclude that the correlation between d_1 and d_2 does not depend on σ_d, but only on σ_F as discussed above.

All of the above rests on the assumption that all tracks come from the primary vertex. This should be a good approximation as long as the sample used is not particularly rich in long-lived particles (e.g. Minbias or low-pt jet samples).

2 Application to CDF data

As an example, we applied this method to a sample of about 90,000 track pairs taken from run 40494 (run IA). Figure 2 shows the impact parameter distribution on this sample, and Figure 3 shows $< d_1 d_2 >$ vs $\cos(\Delta\Phi)$. The linear relationship predicted by Eq. (6) appears to be well verified by the data. From a linear fit we get:

- $\sigma_F = 28.2 \pm 0.2\ \mu$m
- $\sigma_r = 35.9 \pm 0.2\ \mu$m

Note the smallness of the resulting statistical uncertainty. The obtained value of σ_F appers somewhat smaller than the usually quoted values.

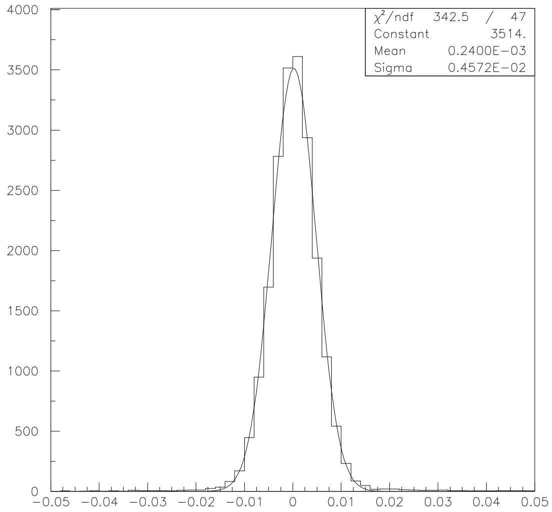

Figure 2: Distribution of impact parameter [cm] measured with respect to beam axis for a sample of Run Ia tracks (run 40494)

2.1 Measure of $\sigma_F(z)$ and $\sigma_r(p_t)$

The above results represent averages over z_0 and p_t. The z dependence of σ_F can be easily determined selecting pairs with both tracks in the same z_0 bin. Figure 4 shows σ_d, σ_F and σ_r as z_0 functions, with a choice of 5 cm as z_0 bin size.

In Figure 5 σ_d, σ_F and σ_r are reported as p_t functions: bin size is chosen to have approximately the same statistics in all bins. The expected decreasing behavior of σ_d with p_T is observed. There is also some evidence of a σ_F increase at high p_t. Further studies are needed in order to understand the reason for this effect: it could be caused by contamination of higher p_t track pairs coming from a common secondary vertex, or by correlated errors on close tracks found in jet cores for which impact parameters correlation is expected to be artificially increased. In fact, our sample included all triggers for that run, with no selection.

For this reason we think the beam width is best estimated by limiting our sample to low-p_t tracks. The average of the three lower p_t bins gives:

- $\sigma_F = 26.5 \pm 0.5\ \mu$m

- $\sigma_r = 37.2 \pm 0.4\ \mu$m

Figure 6 shows σ_d, σ_F and σ_r for pairs with both tracks in the same wedge. Since no dependence on wedge number is observed the hypothesis that the beam spot is circular is verified.

Conclusion

We have demonstrated a method to disentangle impact parameter resolution and finite beam size contributions to the width of the raw impact parameter distribution obtained from SVX tracks.

Though further investigation may be needed in order to understand the systematics, we believe the method is simple and sound and can be extensively applied to present and future SVX data in order to monitor detector performance and beam geometry.

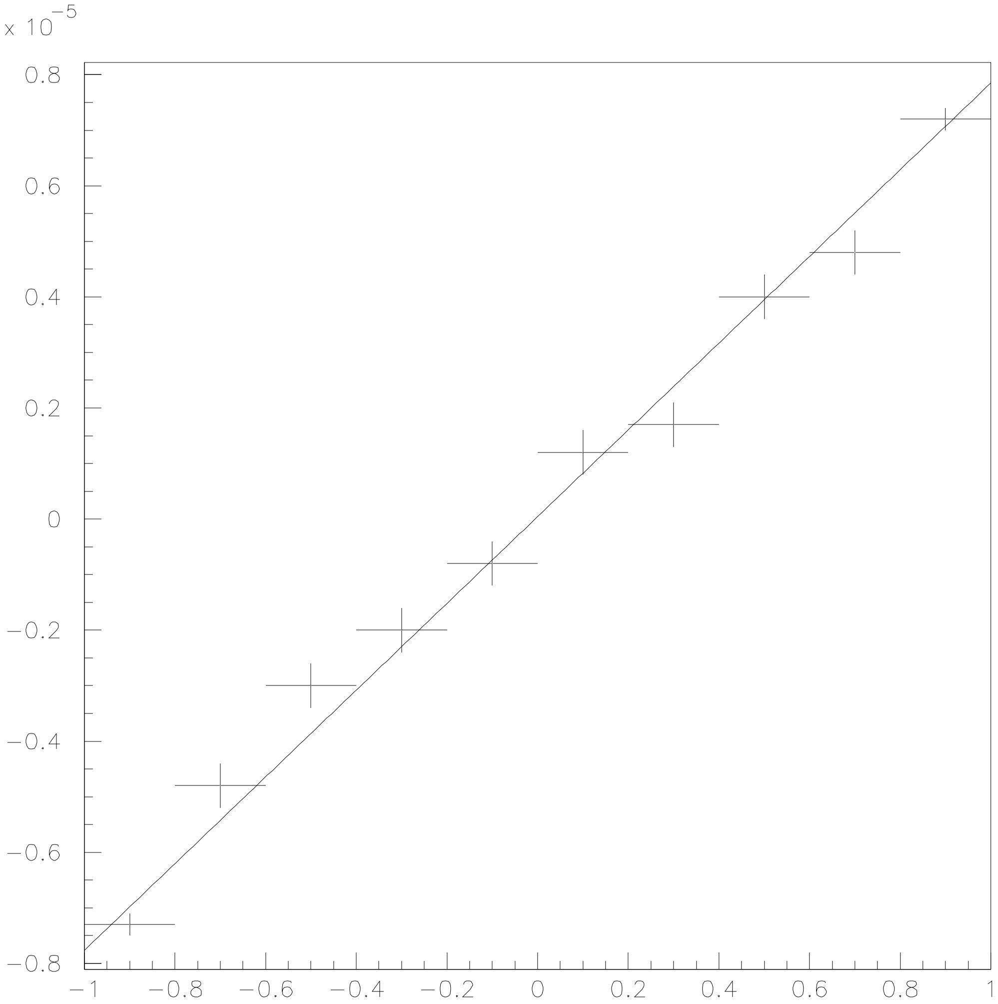

Figure 3: $< d_1 d_2 >$ [cm^2] vs $\cos(\Delta\Phi)$ for a sample of Run Ia tracks (run 40494). Statistical errors are shown.

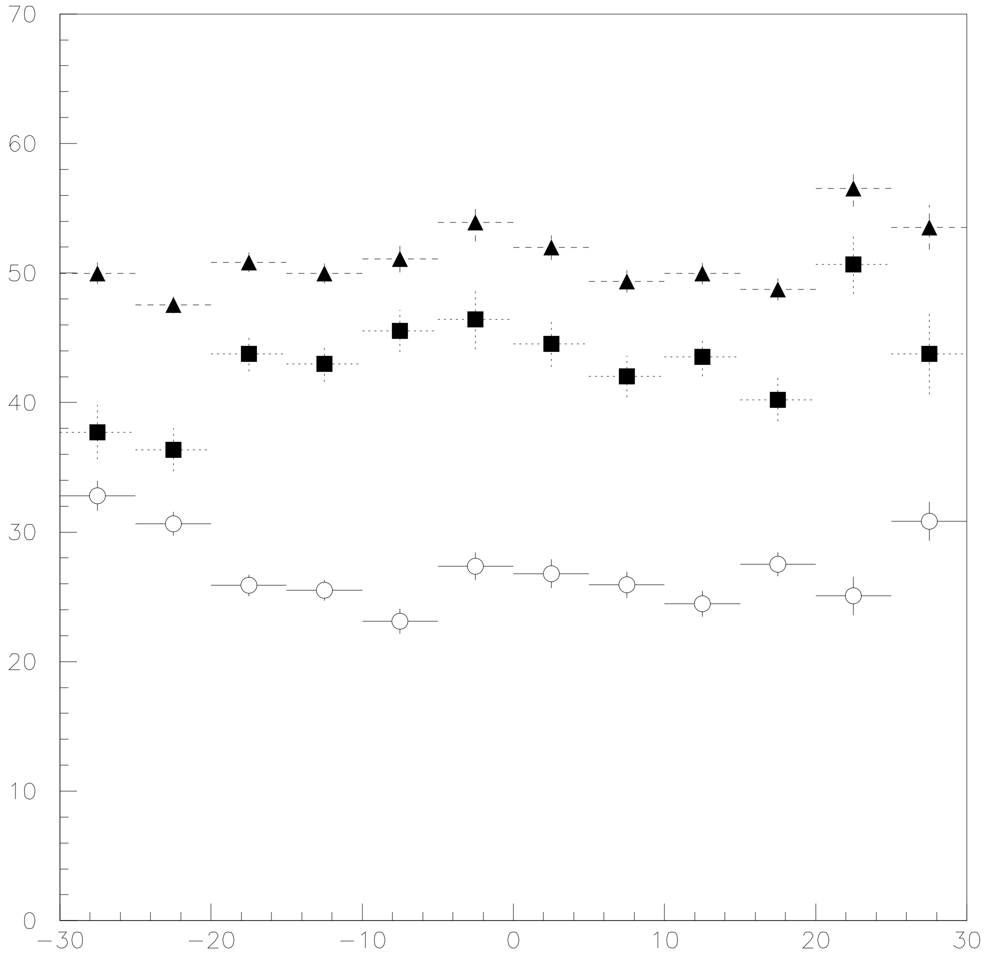

Figure 4: Total Width (σ_d, triangles), Beam Width (σ_F, circles) and Impact Parameter Resolution (σ_r, squares) dependence on z for a sample of Run Ia events (run 40494). Units are [cm] on horizontal axis and [μm] on vertical axis. Statistical errors are shown.

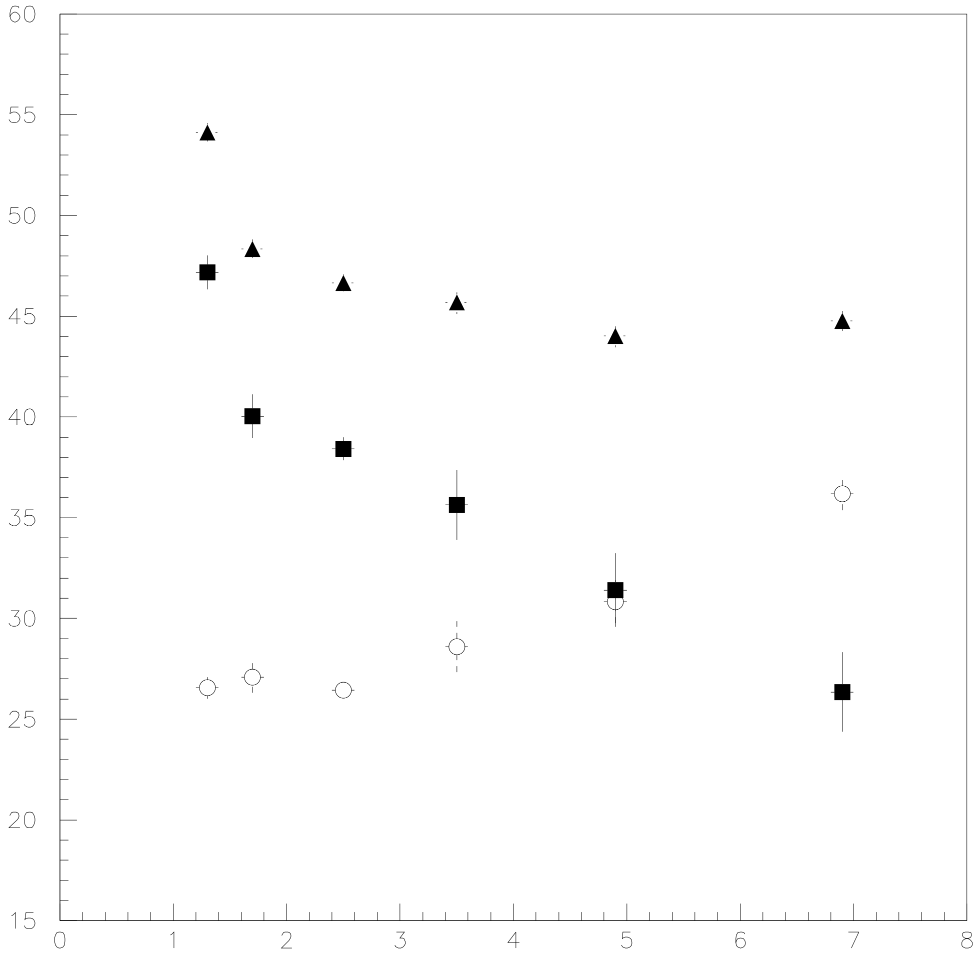

Figure 5: Total Width (σ_d, triangles), Beam Width (σ_F, circles) and Impact Parameter Resolution (σ_r, squares) dependence on p_t for a sample of Run Ia events (run 40494). Units are [GeV/c] on horizontal axis and [μm] on vertical axis. Statistical errors are shown.

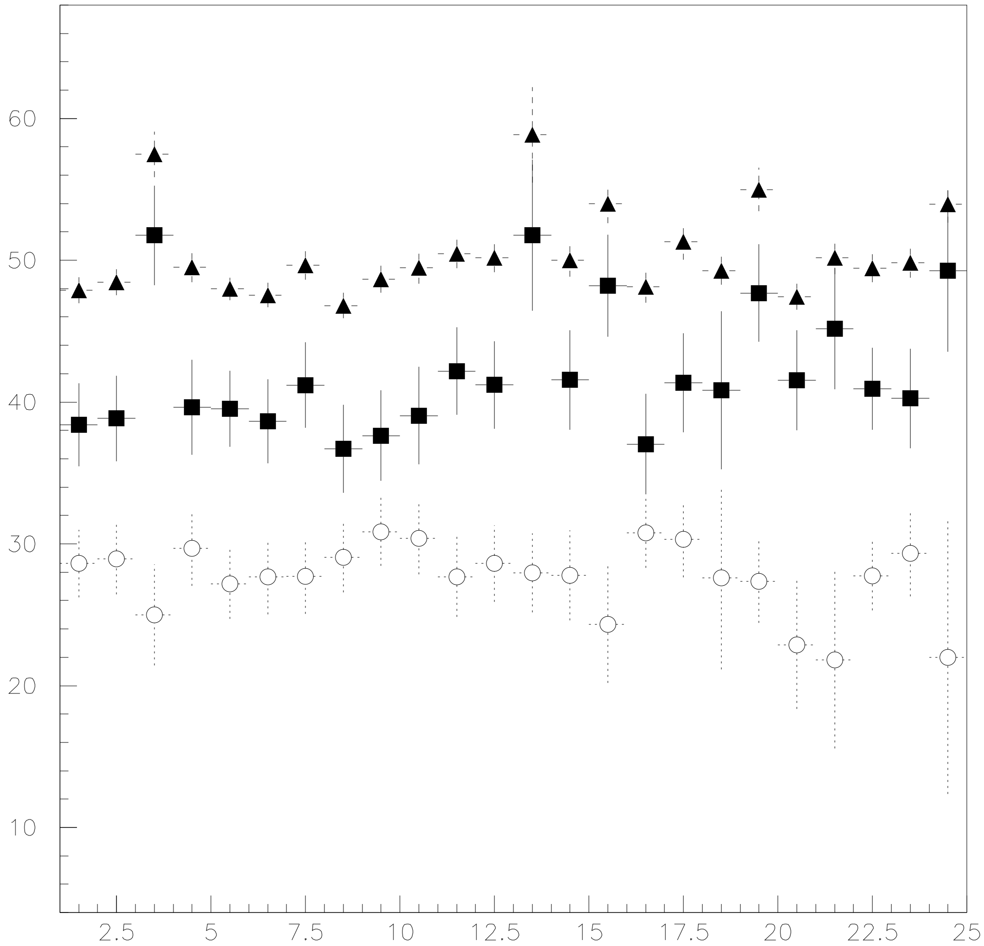

Figure 6: Total Width (σ_d, triangles), Beam Width (σ_F, circles) and Impact Parameter Resolution (σ_r, squares) dependence on wedge for a sample of Run Ia events (run 40494). Units are "wedge number" on horizontal axis and [μm] on vertical axis. Statistical errors are shown.