Proton Single Coalesced-Bunch Intensities in the Main Injector

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The document cited as reference 1 discusses many of the accelerator physics issues involved in the utilization of the Main Injector for colliding-beam physics in the Tevatron. In section 3.2.4, estimates are made for the single coalesced-bunch proton intensities and transverse emittances to be expected in the Tevatron in the era of the Main Injector. This brief report discusses the details of how these estimates were made for the single coalesced-bunch proton intensities.

1. Booster Bunch Intensities and Longitudinal Emittances

Figures 1[2] and 2[3] present experimental data on Booster beam longitudinal emittance (ε_l, 95%, in ev-sec) and rms momentum spread (σ_p/p, in units of 10^{-3}), respectively, as a function of beam intensity. The momentum spread data has been converted to longitudinal emittance data using the relation

$$\varepsilon_l = 0.3\text{x}((\sqrt{6}\sigma_p/p)/0.1\%)^2 \text{ ev-sec} \quad (1)$$

and both sets of data are plotted in fig. 3. The increase of the longitudinal emittance with intensity is believed to be due to the longitudinal coupled bunch instability, which is driven primarily by an impedance (Z) associated with higher-order modes in the Booster RF cavities. In this case (see Appendix, which was written by Yu Chao), the dependence of the longitudinal emittance on the Booster bunch intensity N and the impedance Z is exponential, and can be parameterized by the following relation:

$$\varepsilon_l = b\exp(aN) \quad (2)$$

where the factor in the exponent $a \propto Z$.

The solid line in fig. 3 corresponds to a fit to the data to the above assumed exponential form. The intersection of this fit with the double solid line labelled "Main Injector bucket area limit" (at about 0.5 ev-sec) determines the maximum bunch intensity at injection into the Main Injector (just above $4\text{x}10^{10}$/bunch).

When the Linac Upgrade is complete, it is expected that 1/3 of the RF cavities will be able to be removed from the Booster. Since the impedance Z is due to these cavities, it will be reduced by a factor of 1/3. As a result, the dependence of longitudinal emittance on intensity after the Linac Upgrade will be

$$\varepsilon_l = b \exp(2aN/3) \tag{3}$$

since a is proportional to Z. This relation is plotted in fig. 3 as the dotted line: the maximum bunch intensity at Main Injector injection is now about 6×10^{10}/bunch.

2. Single Coalesced-Bunch Intensities from the Main Injector

To form coalesced bunches for injection into the Tevatron, a number B of single bunches from the Booster, each having intensity N and longitudinal emittance per bunch ε_l, are injected into the Main Injector, accelerated to 150 GeV, and coalesced into a single bunch. The total intensity in this bunch is N_c, where

$$N_c = B\, N\, \eta_c\, (B, \varepsilon_l) \tag{4}$$

where ε_l is given by equation (3), and $\eta_c\,(B, \varepsilon_l)$ is the coalescing efficiency. This latter quantity is a function of ε_l and B; the numbers shown in Tables 1 and 2 are from an ESME simulation of the process, as calculated by Dave Wildman. The assumptions about the coalescing system RF are shown in the tables. The higher efficiencies for the Main Injector are due primarily to the fact that the coalescing bucket height is larger by a factor of about 1.35 for the same RF voltage, since the harmonic number is smaller.

Table 1: Main Ring simulation
Bunch Coalescing efficiency

22 kV of h = 53
3.7 kV of h = 106
Efficiency = $\eta_c\,(B, \varepsilon_l)$ = Charge in coalesced bunch/charge in machine

B	5	7	9	11	13
ε_l (eV-sec)					
.08	100%	100	100	-	-
.16	96	96	95	-	-
.26	83	81	80	76	73
.30	81	78	77	75	72
.48	59	57	56	55	52

Table 2: Main Injector Simulation
Bunch Coalescing efficiency

23 kV of h = 28
4 kV of h = 56
Efficiency = η_c (B, ϵ_l) = Charge in coalesced bunch/charge in machine

B	5	7	9	11	13
ϵ_l (eV-sec)					
.1	100%	100	99	-	-
.16	99	99	97	-	-
.20	99	98	96	-	-
.26	98	96	93	-	-
.48	83	82	75	75	71

Based on the coalescing efficiencies given in Table 2, figure 4 shows the calculated relation between N_c and ϵ_l for various values of B. Figure 5 shows the calculated relation between N and N_c for various values of B.

Figures 6 and 7 display the same quantities as figs 4 and 5, but with ϵ_l given by equation 2. This would correspond to the situation if the improvement in longitudinal emittance expressed in equation (3) is not realized. It should be noted that a total single coalesced-bunch intensity in excess of about $3x10^{11}$ (the Main Injector specification) can still be realized, in principle.

It is interesting to repeat the same calculations for the Main Ring. This can be done calculating N_c as presented in equations (3) and (4) above, but using the numbers given in Table 1 for the dependence of the Main Ring coalescing efficiency on B and ϵ_l . In addition, a factor of 0.7 is applied to N_c to account for Main Ring inefficiency; in reality, this number depends on N, but we neglect this here. The results are shown in figs. 8 and 9; they imply that single coalesced-bunch intensities as large as almost $3x10^{11}$ would be possible in the Main Ring after the Linac Upgrade. Actually, this would not be expected to be the case, because the coalescing efficiency in the Main Ring is not good as the simulations presented in Table 1 predict; in particular, the actual coalescing efficiency appears to depend strongly upon N_c .[4]

3. Beam-beam tune shift and luminosity

At the highest single coalesced-bunch intensity, the beam-beam interaction will limit collider luminosity, even with only two crossings per revolution. This is illustrated in fig. 11, which shows the beam-beam tune shift δ_v vs N_c. δ_v is computed from

$$\delta_v = 0.00733\, n_c N_c / \varepsilon_t \qquad (5)$$

where n_c = number of crossings = 2, N_c is computed from eqs. (3) and (4) (in units of 10^{10}), and ε_t, the beam invariant transverse emittance, is given in π mm-mrad as a function of N by the dotted line in fig. 10:

$$\varepsilon_t = \alpha\,\beta + \Delta\varepsilon \qquad N<\beta,$$
$$\varepsilon_t = \alpha\,N + \Delta\varepsilon \qquad N\geq\beta \qquad (6)$$

Here α is the slope of the dotted line in Fig. 10 ($\alpha = 2.83\ \pi$ mm-mrad/10^{10}); β is the Booster bunch intensity corresponding to the intersection of the dotted line in fig. 10 with the "Linac emittance" line ($\beta = 2.5\times10^{10}$); and $\Delta\varepsilon$ (=15 π mm-mrad) is an assumed emittance dilution deliberately added to make the proton transverse emittance greater than the antiproton transverse emittance at the highest intensities, a technique used to minimize the non-linearities in the beam-beam interaction. The limit on the beam-beam tune shift which can be tolerated under such circumstances is known experimentally to be about .024; this is indicated by the double solid line in fig. 11. This will limit the total useful single coalesced-bunch intensities to about 4.5×10^{11}.

The corresponding luminosity of the collider may be estimated by scaling. Based on operation at a luminosity of 1.6×10^{30}/cm^2/sec in 1989, the scaled luminosity, in units of 10^{32}/cm^2/sec, is given roughly by

$$L = .016*(N_c/7)*(\overline{N}/15)/\{(\varepsilon_t+\bar{\varepsilon}_t)/(25+18)\},$$

where $\bar{\varepsilon}_t$ =antiproton transverse emittance at low-β in the Tevatron (in π mm-mrad), and $\overline{N}$ is the total number of antiprotons in the Tevatron. Here Npbar and N_c are in units of 10^{10}. For the Main Injector, the parameters as given in ref. 1 are $\overline{N}$ =130 and $\bar{\varepsilon}_t$ = 22 π mm-mrad; then we have

$$L \cong 0.8\, N_c / (\varepsilon_t + 22) * 1.6 \qquad (7)$$

The extra factor of 1.6 comes from the assumption of collider operation at β^* = 25 cm. The luminosity from eq. (7) (with N_c from equations (3) and (4), and ε_t from equation (6)) is plotted against the beam-beam tune shift from equation (5) in fig. 12. It can be seen that the beam-beam tune shift limit is reached at a luminosity of about 1.2×10^{32}/cm^2/sec.

References:

1. "Fermilab Upgrade Phase II: The Main Injector"(version dated 1/10/90)
2. J. Crisp, private communication
3. S. Childress et. al. "1987 DOE Review-First Collider Run Operation" Fermilab TM-1454,
4. P. S. Martin, K. G. Meisner, D. W. Wildman, "Improvements in Bunch Coalescing in the Fermilab Main Ring", in *Proceedings of the 1989 Particle Accelerator Conference,* Chicago, IL (1989)

Appendix
(by Yu Chao)

The dependence of the impedance inside the exponent is more reasonable than in the overall factor. Let me restate the rationale for an exponential growth:

The total growth over a time period τ can be expressed as

$$\text{Final size/initial size} = T = \exp\left[\int_0^{\tau} \text{Im}[\omega(t)]\, dt\right] \tag{1}$$

where the relation between $\text{Im}(\omega)$, the total number of particles N, and impedance Z can be conceptually described by the dispersion relation:

$$1 = A\, N\, Z\, I(\omega) \tag{2}$$

where A is a multiplicative factor, and $I(\omega)$ is the frequency response of the beam:

$$I(\omega) \propto \int \frac{\Psi'(r)\, r^2\, dr}{\omega - \omega_0(r)} \tag{3}$$

where $\omega_0(r)$ is the unperturbed tune as a function of the action variable r and $\Psi(r)$ is the beam distribution function. It's not clear how $\text{Im}(\omega)$ is related to N & Z except when $|\omega| >> |\omega_0(r)|$, namely, near the resonance peak, in which case ω can be taken outside the integral in (3) and thus is temporarily proportional to NZ (from (2)),

$$\text{Im}(\omega) \propto NZ \tag{4}$$

In general (4) does not necessarily hold. Now for a sharp enough resonance (4) holds most of the time and we can roughly say that the integral in (1) is also proportional to NZ and write

$$T \propto \exp(kNZ) \tag{5}$$

where k is another multiplicative factor. This would imply that the exponent of the total growth formula depends linearly on Z (and N).