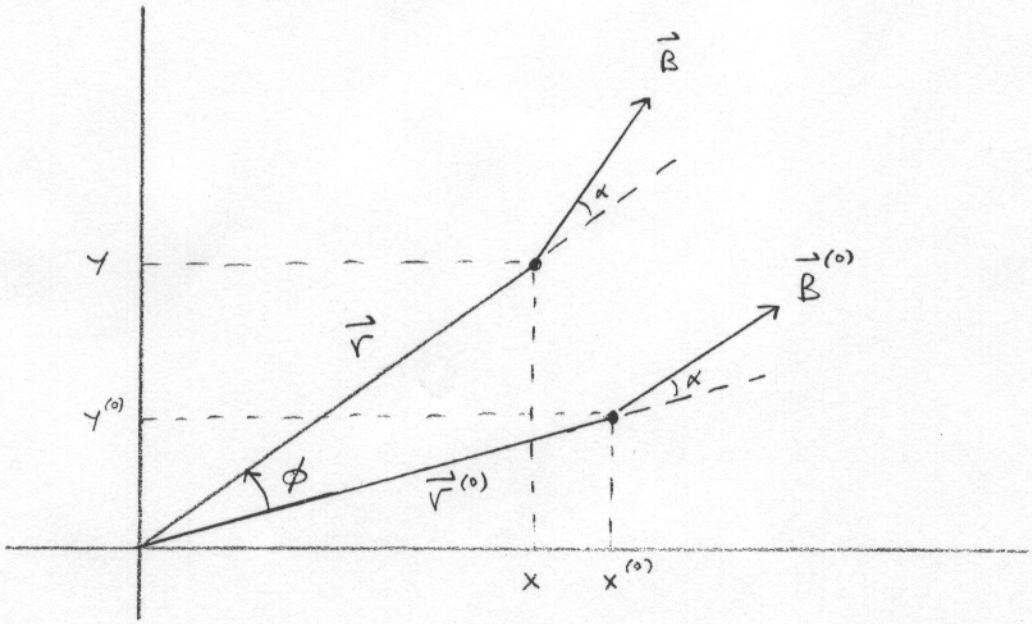
Rotated Quadrupole Field...



$$\vec{B} = \begin{pmatrix} B_X \\ B_Y \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} B_X^{(0)} \\ B_Y^{(0)} \end{pmatrix} = R_\phi^{-1} \vec{B}^{(0)}$$

$\vec{B}^{(0)}$ due to quad magnet:

$$\vec{B}^{(0)} = B' \begin{pmatrix} Y^{(0)} \\ X^{(0)} \end{pmatrix} = B' \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} X^{(0)} \\ Y^{(0)} \end{pmatrix}$$

$$= B' \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \vec{r}^{(0)}$$

note: $\vec{r} = R_\phi^{-1} \vec{r}^{(0)}$

so,

$$\vec{B} = R_\phi^{-1} \vec{B}^{(0)} = B' R_\phi^{-1} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \vec{r}^{(0)}$$

$$= B' R_\phi^{-1} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} R_\phi \vec{r}$$

$$= B' \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \vec{r}$$

$$= B' \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} -s & c \\ c & s \end{pmatrix} \vec{r}$$

$$\vec{B} = B' \begin{pmatrix} -2cs & c^2-s^2 \\ c^2-s^2 & 2cs \end{pmatrix} \vec{r} = B' \begin{pmatrix} -\sin 2\phi & \cos 2\phi \\ \cos 2\phi & \sin 2\phi \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

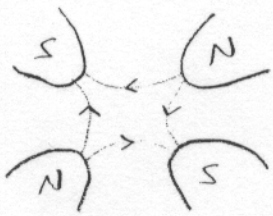
$$\Rightarrow \quad B_y = B' \cos 2\phi \; x + B' \sin 2\phi \; y$$

$$B_x = -B' \sin 2\phi \; x + B' \cos 2\phi \; y$$

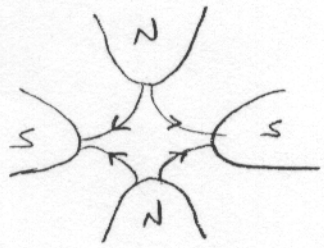
So,

"normal" quad: $\phi = 0 \longrightarrow B_y = B'x \;, \quad B_x = B'y$

"skew" quad: $\phi = 45^\circ \longrightarrow B_y = B'y \;, \quad B_x = B'x$



NORMAL



SKEW

general: along $y=0$: "normal": $\vec{B} \perp$ midplane

"skew": $\vec{B} \parallel$ midplane

Take a normal quad, and rotate by a small angle, ϕ ...

then
$$B_y = B'x + 2\phi B'y$$
$$B_x = \underbrace{B'y}_{\text{normal}} - \underbrace{2\phi B'x}_{\text{skew quad, of strength}}$$

$$\frac{\Delta B' l}{B\rho} = k = \frac{1}{f} = 2\phi \frac{B'l}{(B\rho)}$$

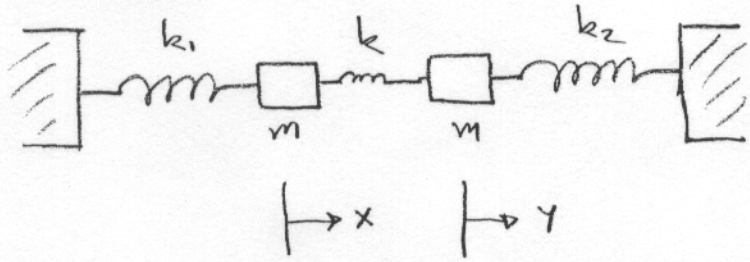
Clearly, skew quad couples horizontal + vertical motion...

$$\Delta x' = \frac{B_y \Delta s}{(B\rho)} = \frac{\Delta B' \Delta s}{(B\rho)} y$$

vertical displacement

causes a horizontal "kick"

Coupled Oscillator...



$$m\ddot{x} + (k_1 + k)x - ky = 0$$
$$m\ddot{y} + (k_2 + k)y - kx = 0$$

$$\Rightarrow \quad \ddot{x} + \omega_1^2 x - g^2 y = 0 \qquad g^2 = k/m$$
$$\ddot{y} + \omega_2^2 y - g^2 x = 0 \qquad \omega_1^2, \omega_2^2 = (k_1, k_2 + k)/m$$

trial solutions: $x = e^{i\omega t}$
$y = a e^{i\omega t}$

$$\Rightarrow \quad -\omega^2 + \omega_1^2 = a g^2$$
$$-a\omega^2 + a\omega_2^2 = g^2$$

$$\Rightarrow \quad a(\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2) = a g^4$$

solve for ω^2... $\quad (\omega^2)^2 - (\omega_1^2 + \omega_2^2)\omega^2 + \omega_1^2\omega_2^2 - g^4 = 0$

$$\Rightarrow \quad \omega^2 = \frac{\omega_1^2 + \omega_2^2}{2} \pm \frac{\sqrt{(\omega_1^2 + \omega_2^2)^2 + 4\omega_1^2\omega_2^2 g^4}}{2}$$

$$\omega^2 = \frac{\omega_1^2 + \omega_2^2 \pm \sqrt{(\omega_2^2 - \omega_1^2)^2 + 4g^4}}{2}$$

let's use the positive frequencies $\omega_1, \omega_2 > 0$ for our system.

then $$\omega_\pm = \left[\frac{\omega_1^2 + \omega_2^2 \pm \sqrt{(\omega_2^2 - \omega_1^2)^2 + 4g^4}}{2}\right]^{1/2}$$

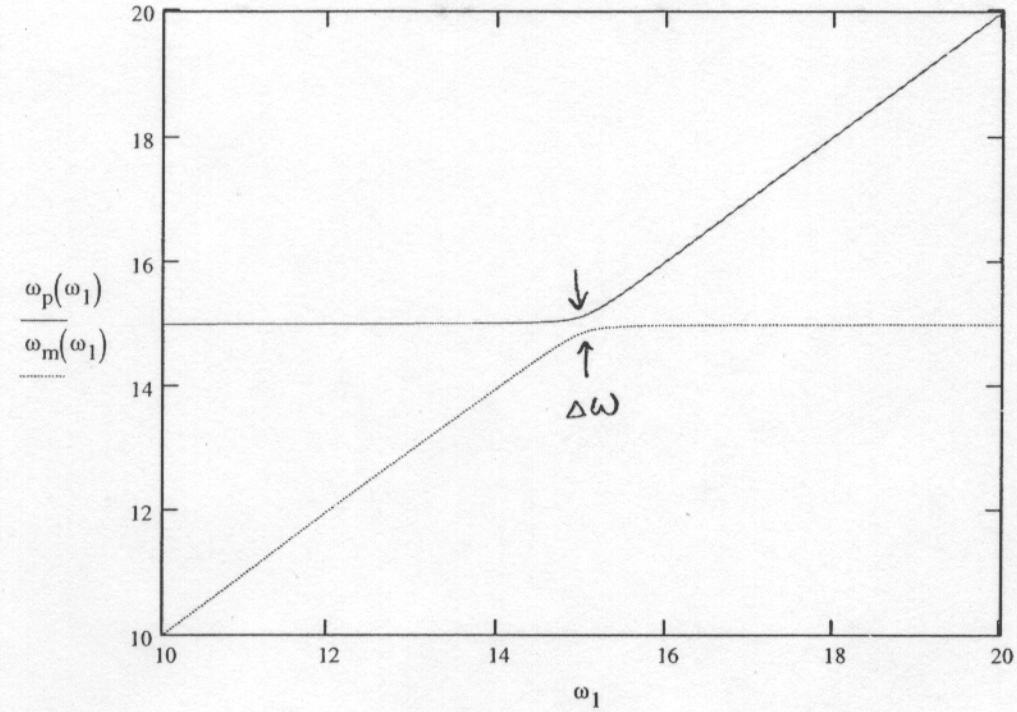
keep ω_2 (k_2) constant, and vary ω_1 (k_1) ; plot ω_+, ω_- ...

$q := 2$ $\qquad$ $\omega_2 := 15$

$$\omega_m(\omega_1) := \sqrt{\frac{\omega_1^{\,2} + \omega_2^{\,2} - \sqrt{\left(\omega_2^{\,2} - \omega_1^{\,2}\right)^2 + 4\cdot q^4}}{2}}$$

$\omega_1 := 10, 10.01 .. 20$

$$\omega_p(\omega_1) := \sqrt{\frac{\omega_1^{\,2} + \omega_2^{\,2} + \sqrt{\left(\omega_2^{\,2} - \omega_1^{\,2}\right)^2 + 4\cdot q^4}}{2}}$$



Look at where $\omega_1 = \omega_2 = \omega_0$ $\qquad$ (i.e., $k_1 = k_2 = k_0$)

$$\Rightarrow \quad \omega_\pm = \sqrt{\omega_0^2 \pm g^2} = \sqrt{k_0/m}\ ,\ \sqrt{(k_0 + 2k)/m}$$

$$\left.\begin{aligned} \omega_+^2 &= \omega_0^2 + g^2 \\ \omega_-^2 &= \omega_0^2 - g^2 \end{aligned}\right\} \qquad \omega_+^2 - \omega_-^2 = (\omega_+ + \omega_-)(\omega_+ - \omega_-) = 2g^2$$

$$\approx 2\omega_0\,\Delta\omega = 2g^2$$

$$\Rightarrow \quad \underbrace{\Delta\omega \approx g^2/\omega_0}_{\text{minimum frequency difference}} = k/m\omega_0$$

in accelerator...

$$x'' + K_x(s)\, x - \underbrace{\kappa y}_{\text{due to skew quad}} = 0$$
$$y'' + K_y(s)\, y - \kappa x = 0$$

$\Rightarrow$ will <u>not</u> be able to put the "x" + "y" tunes to any value desired.

$$\Delta\nu_{min} = \frac{1}{2\pi}\,\frac{\Delta B' l}{B\rho}\sqrt{\beta_x \beta_y}$$

$$\Delta B' = \frac{\partial B_x}{\partial x}$$

To analyze motion in general, must resort to 4x4 matrices

$$M = \begin{pmatrix} A & E \\ F & B \end{pmatrix} \qquad A, B, E, F \quad 2\times 2\text{'s}$$

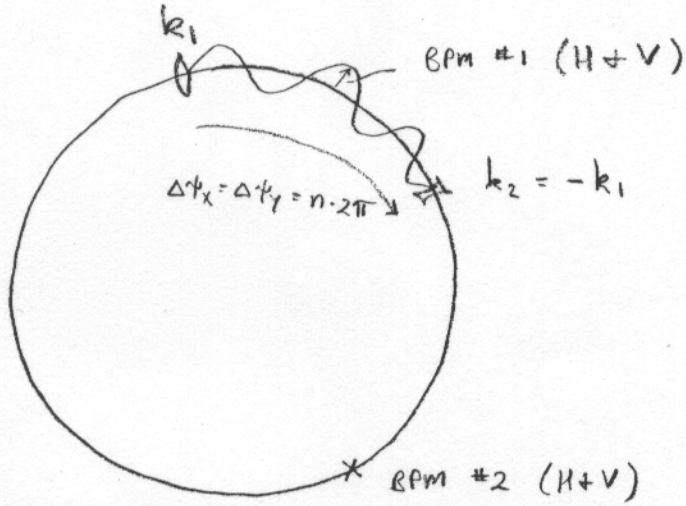
... lots of algebra ...

$$2\cos 2\pi\nu = \frac{\mathrm{Tr}A + \mathrm{Tr}B}{2} \pm \frac{\sqrt{(\mathrm{Tr}A - \mathrm{Tr}B)^2 + 4\det(E + F)}}{2}$$

$$\boxed{\text{if } m = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \text{ then } \bar{m} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}$$

also, β, ψ etc. as discussed previously lose meaning!

LOCAL
vs.
GLOBAL
Coupling



(a simple example)

Define the Standard 90deg Cell: $\mu := 90 \cdot deg$ $\quad \nu := \frac{\mu \cdot 27}{2 \cdot \pi}$ $\quad \nu = 6.75$

$F := 25$

$s := \sin(\mu) \quad c := \cos(\mu)$

$$\alpha := \sqrt{\frac{1 + \sin\left(\frac{\mu}{2}\right)}{1 - \sin\left(\frac{\mu}{2}\right)}} \qquad \beta_F := 2 \cdot F \cdot \alpha \qquad \beta_D := 2 \cdot F \cdot \frac{1}{\alpha}$$

$$\gamma_F := \frac{1 + \alpha^2}{\beta_F} \qquad \gamma_D := \frac{1 + \left(\frac{1}{\alpha}\right)^2}{\beta_D}$$

$$M_0 := \begin{pmatrix} c + \alpha \cdot s & \beta_F \cdot s & 0 & 0 \\ -\gamma_F \cdot s & c - \alpha \cdot s & 0 & 0 \\ 0 & 0 & c - \frac{1}{\alpha} \cdot s & \beta_D \cdot s \\ 0 & 0 & -\gamma_D \cdot s & c + \frac{1}{\alpha} \cdot s \end{pmatrix} \qquad M_0 = \begin{pmatrix} 2.414 & 120.711 & 0 & 0 \\ -0.057 & -2.414 & 0 & 0 \\ 0 & 0 & -0.414 & 20.711 \\ 0 & 0 & -0.057 & 0.414 \end{pmatrix}$$

Define the Skew Quad Matrices:

$$\boxed{k_1 := .02 \qquad k_2 := -.02}$$

$$M_{k1} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k_1 & 0 \\ 0 & 0 & 1 & 0 \\ k_1 & 0 & 0 & 1 \end{pmatrix} \qquad M_{k2} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k_2 & 0 \\ 0 & 0 & 1 & 0 \\ k_2 & 0 & 0 & 1 \end{pmatrix}$$

Define the Initial Conditions:

$$X0_0 := \begin{pmatrix} 5 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{matrix} x \\ x' \\ y \\ y' \end{matrix} \qquad X1_0 := M_0^{\,5} \cdot M_{k1} \cdot X0_0 \qquad X2_0 := M_0^{\,5} \cdot M_{k2} \cdot M_0^{\,7} \cdot X1_0$$

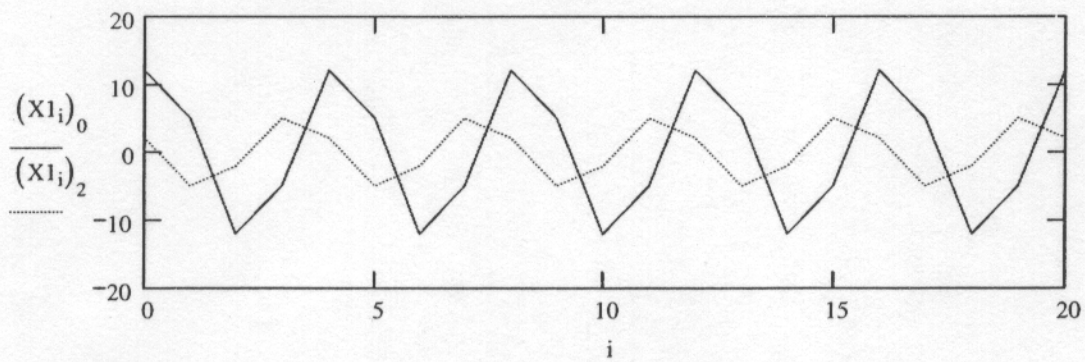
Number of turns to track: $i := 0 .. 20$

Iterate...

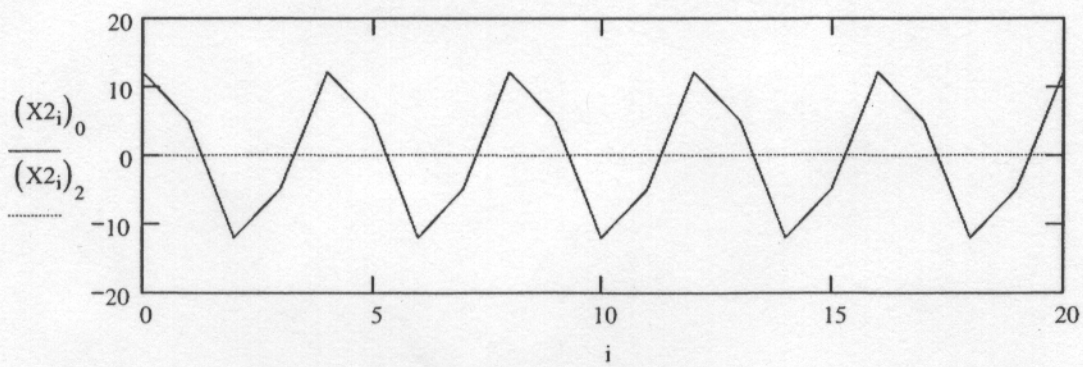
$$\begin{pmatrix} X0_{i+1} \\ X1_{i+1} \\ X2_{i+1} \end{pmatrix} := \begin{pmatrix} M_0^{\,15} \cdot M_{k2} \cdot M_0^{\,12} \cdot M_{k1} \cdot X0_i \\ M_0^{\,5} \cdot M_{k1} \cdot M_0^{\,15} \cdot M_{k2} \cdot M_0^{\,7} \cdot X1_i \\ M_0^{\,5} \cdot M_{k2} \cdot M_0^{\,12} \cdot M_{k1} \cdot M_0^{\,10} \cdot X2_i \end{pmatrix}$$

Results:

BPM 1: red (solid) = horizontal, blue (dash) = vertical



BPM 2: red (solid) = horizontal, blue (dash) = vertical



Symptoms:

a) Main Skew Quad Correction Circuit runs strong

b) Anomolous vertical dispersion

Expectations...

Suppose quads aligned w/ rms roll angle $\phi_{rms} \approx 1$ mrad

Then, would expect:

a) $$\Delta\nu_{min}^{(rms)} \approx \frac{1}{2\pi}\sqrt{\beta_x\beta_y}\; 2\phi_{rms}\cdot\frac{1}{F}\sqrt{N}$$

$$= \frac{1}{2\pi}(2F)\,\phi_{rms}\,\frac{1}{F}\sqrt{N}$$

$$\approx \frac{\sqrt{200}}{\pi}\,10^{-3} = 0.005$$ (out of an ensemble of Tevatrons)

b) $$\Delta D_y' = k\,D_x = 2\phi\cdot\frac{1}{F}\cdot D_x$$

$$\Rightarrow D_y^{(rms)} \approx \frac{\sqrt{\beta_0\langle\beta D_x^2\rangle}}{2|\sin\pi\nu|}\,\frac{2}{F}\,\phi_{rms}\sqrt{\frac{N}{2}}$$ @ $\hat{\beta}$ location

$$\approx \frac{\sqrt{(100\text{m})\left[\frac{100\cdot 2^2 + 25\cdot 4^2}{2}\right]\text{m}^2}}{25\text{ m}}\cdot 10^{-3}\cdot\sqrt{\frac{200}{2}}$$

$$= 8\text{ cm}$$

$\therefore$ expect random pattern of D_y @ BPM locations

w/ rms ~ 8 cm, peaks ~ 25 cm

Observations

a) Main Skew Quad circuit runs strong ...

tuned to make $\Delta\nu_{min} \approx 0.000\,(1)$

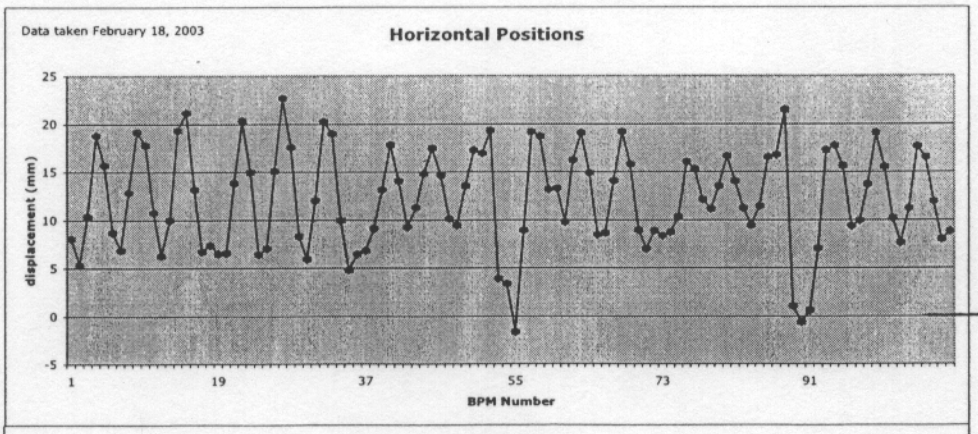
if turned off, would expect to see:

$$\Delta\nu_{min}^{(off)} = \frac{1}{2\pi} \sum_i \sqrt{\beta_x \beta_y}\; k_i \qquad k_i = \left(\frac{B'l}{B\rho}\right)_{50A} \cdot \frac{I}{50A}$$

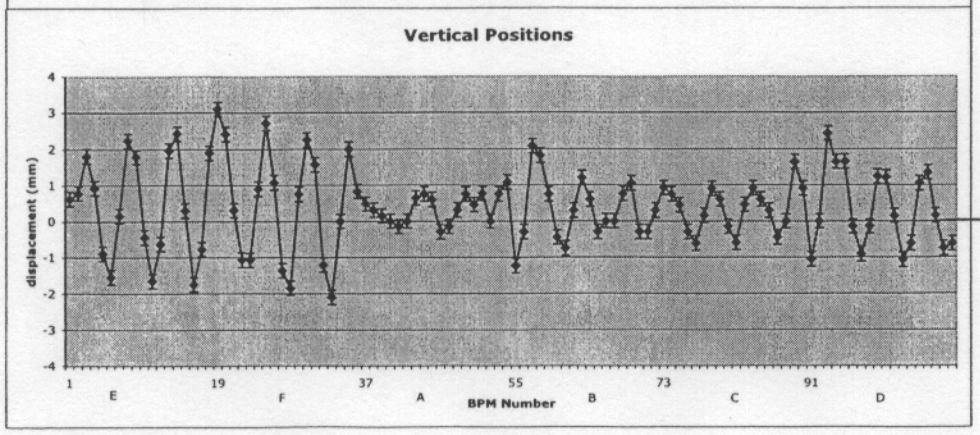
$$= \frac{1}{2\pi} \cdot 2F \cdot \underset{\uparrow \atop \#\text{ correctors}}{(42)} \cdot \left(\frac{6\,T}{1000 \cdot \frac{10}{3}\,T\cdot m}\right) \cdot \frac{30A}{50A}$$

$$\approx 0.3\,! \qquad (\text{not } 0.005, \text{ or so!})$$

b) Vertical Dispersion has coherent pattern, not random, with $\hat{D}_y \approx 60 - 80$ cm (not 25 cm)



$D_x \approx 6$ m



$D_y \approx 0.8$ m

	A	B	C	D	E	F
	11(A0)	11(B1)		11(D0)	11(E0)	
—	13	o	13	o	13	13
|	17	o	17	o	17	17
|	22	22	22	22	22	22
orig.	26	26	26	26	26	26
T:SQ	32	32	32	32	32	32
|	36	36	36	36	36	36
|	42	42	42	42	42	42
—	o	46	o	46	46	46
	49(A4)		49(D0)	49(E0)		49(A0)

Table 1: Table of skew quadrupole locations (station numbers) in the Tevatron. A "o" indicates locations of missing T:SQ elements.

Acnet Device T:SQ (42 skew quads, in series) is used to tune $\Delta\nu_{min} \rightarrow 0$

Effective, if all sources of coupling are in phase ($\Delta\psi_y - \Delta\psi_x = 0$, mod 2π)

Circuits added to skew quads @ "zero" locations...

Bϕ - T:SQA4, T:SQB1 (1 magnet each)
Dϕ - T:SQDϕ (2 magnets)
} after Run Ib low-β quad rolled

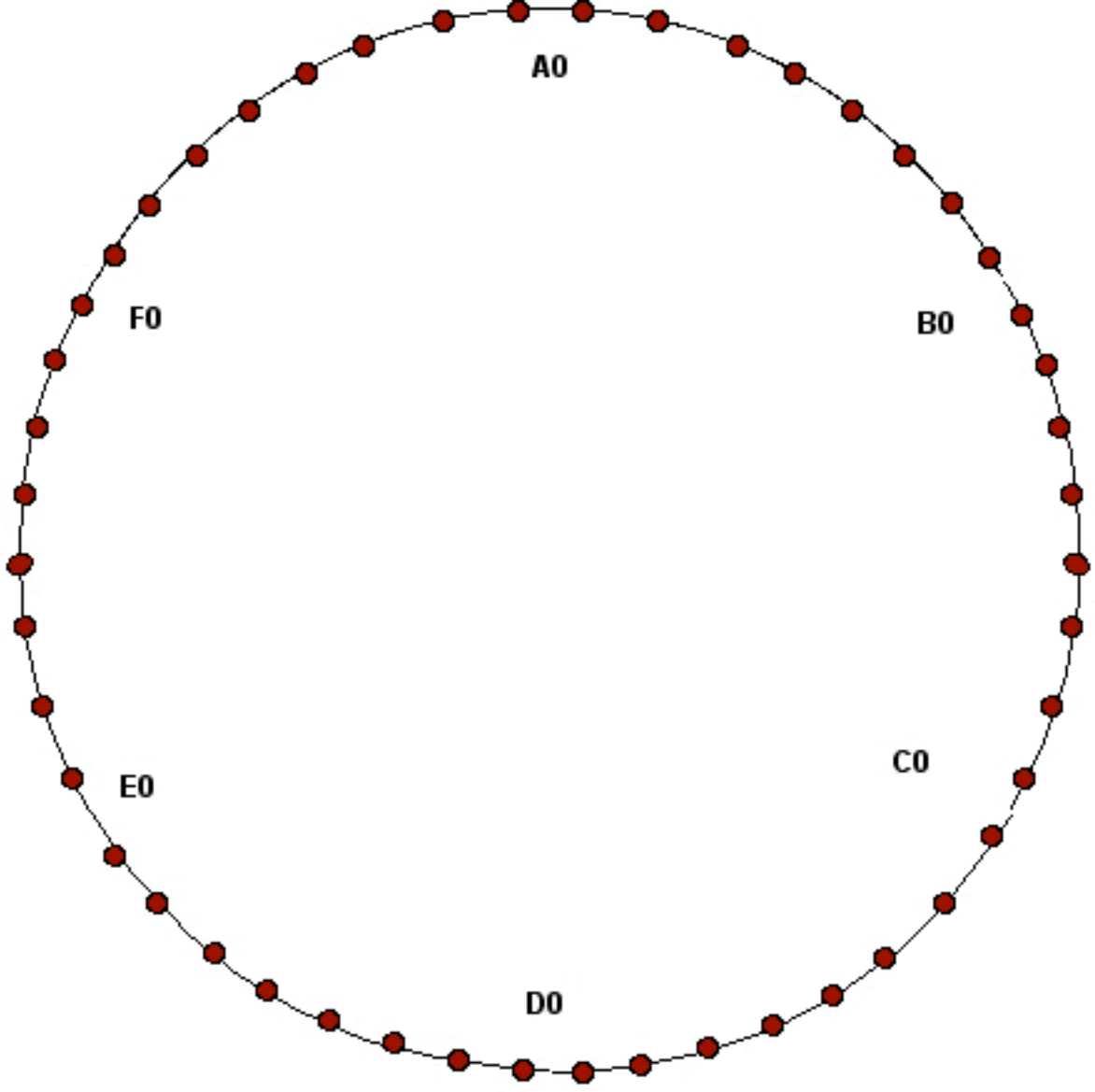
Aϕ - T:SQAϕ
Eϕ - T:SQEϕ
} out of phase (~30°) wrt other circuits

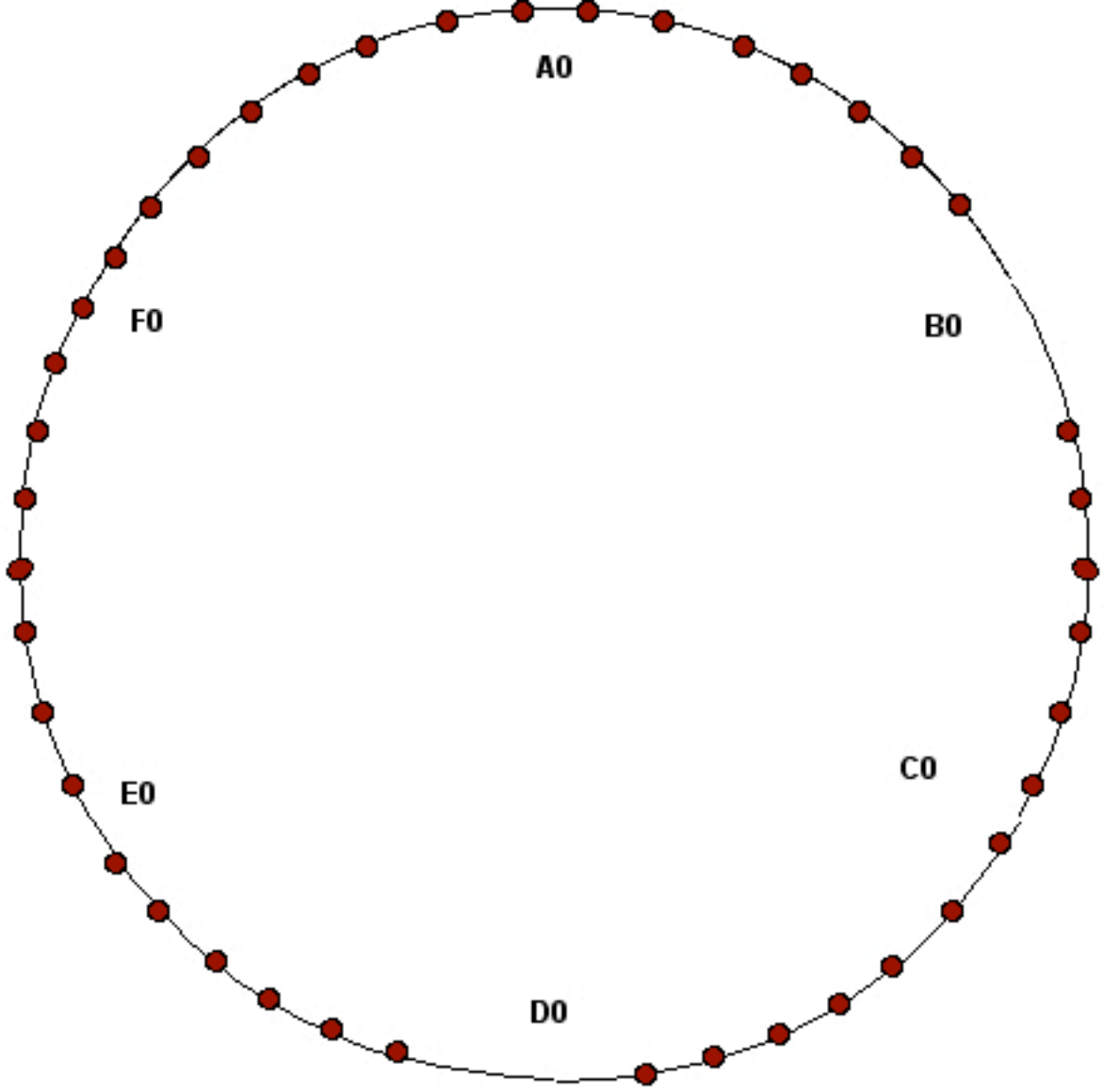
$\therefore$ allowed $\Delta\nu_{min} \rightarrow$ closer to zero

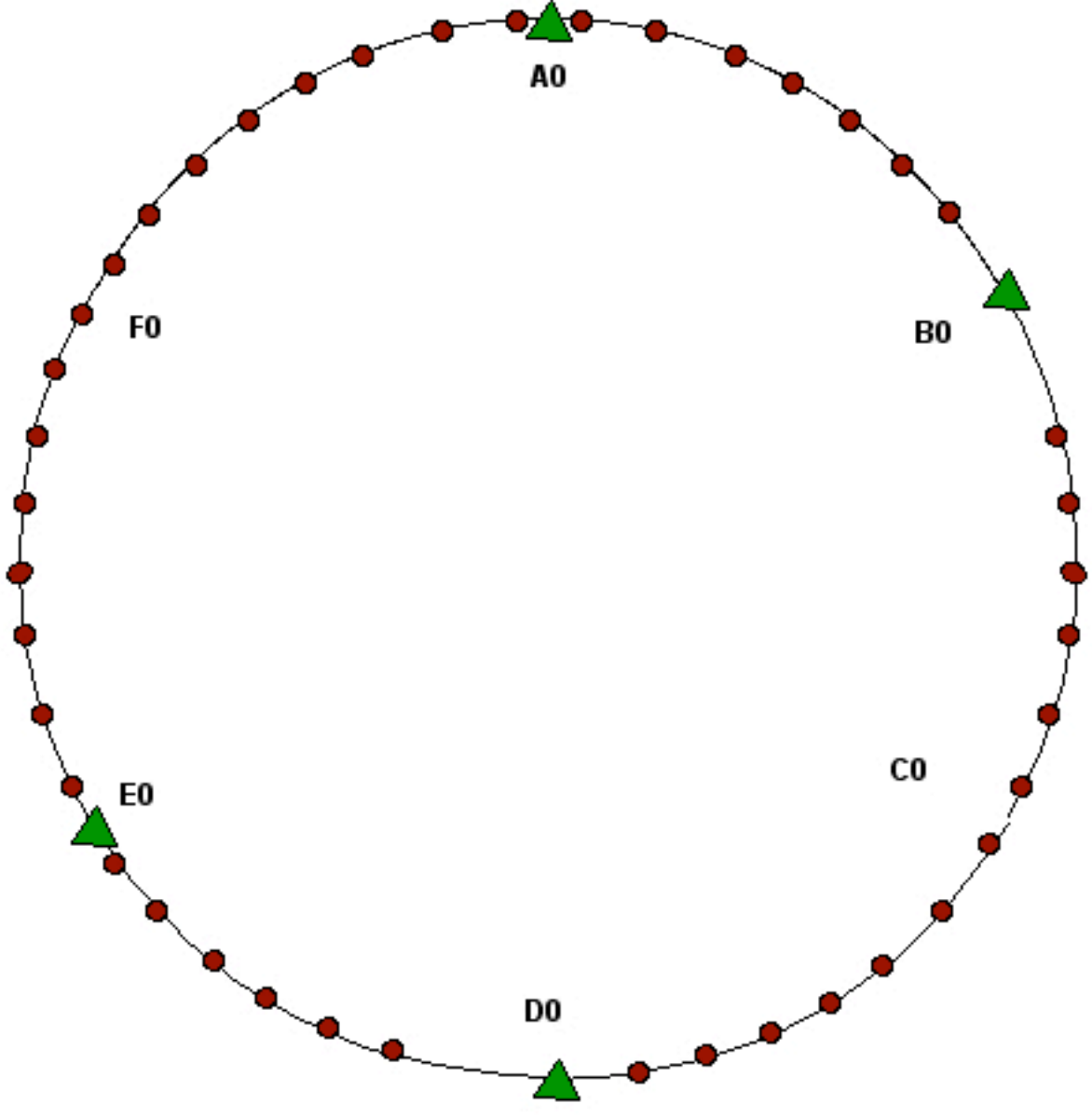
It should be pointed out that at 980 GeV the T:SQ circuit runs at -28 A, or 60% of its range. In addition, this is 50% stronger than the simple scaling of its 150 GeV current to 980 GeV.

Circuit	SQ	SQA0	SQA4	SQB1	SQD0	SQE0
Current (A)	-3.0	6.27	-5.17	0.56	0	0

Table 2: Table of currents in the skew quadrupole circuits in the Tevatron on February 18, 2003, corresponding to the data of Figure 1, taken at 150 GeV.

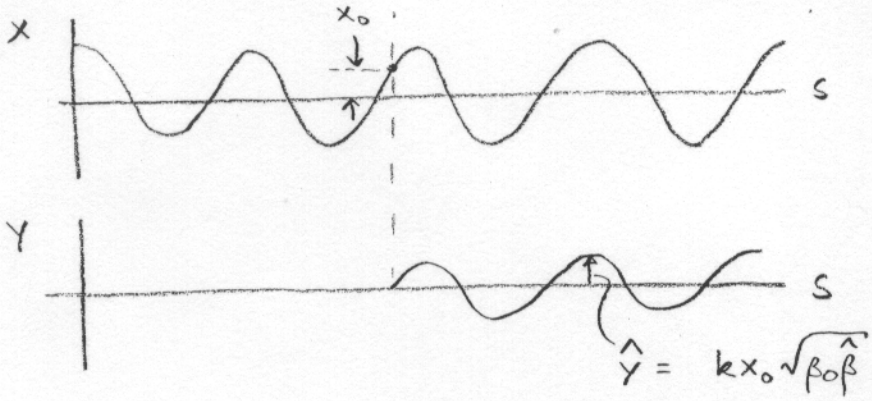

A0
B0
C0
D0
E0
F0


A0
B0
C0
D0
E0
F0


A0
B0
C0
D0
E0
F0

LOOK for Source(s) of Strong Coupling

Ex: kick beam horizontally, see where vertical oscillation begins...

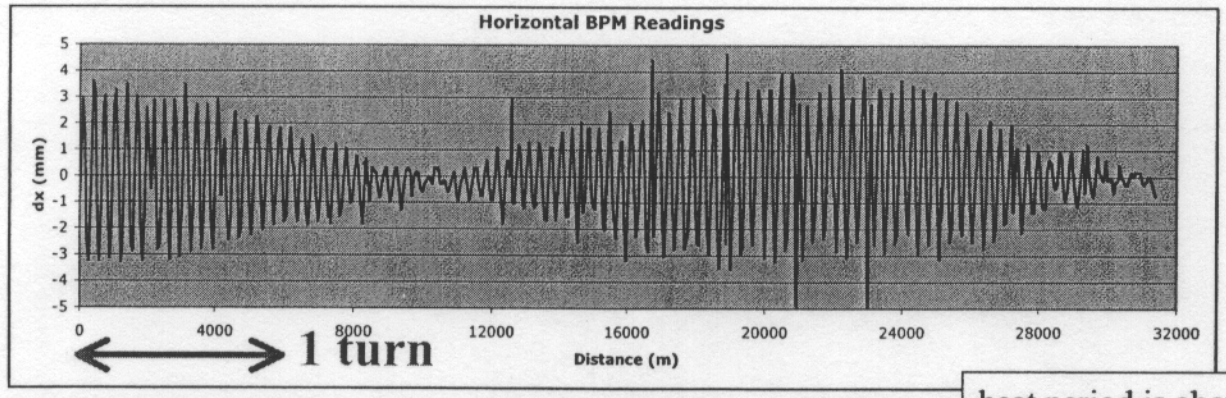


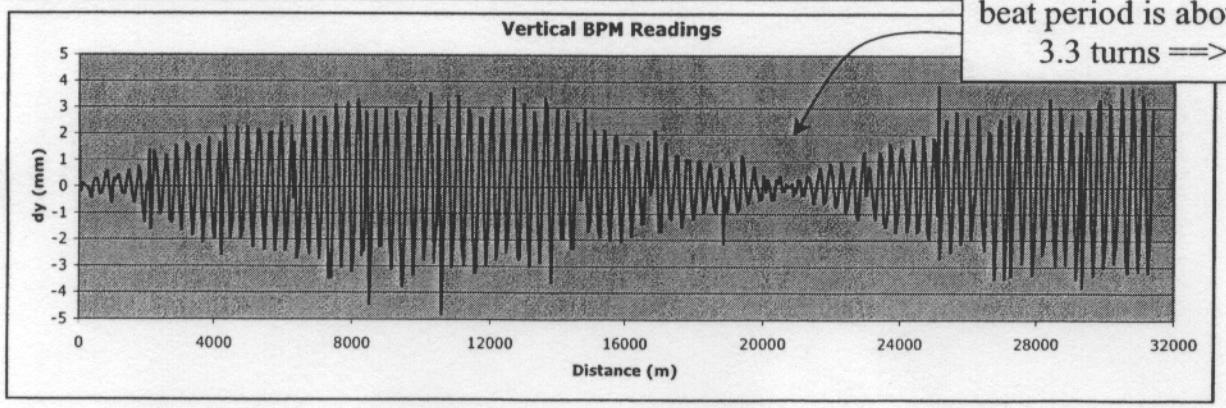
Performed Experiments Feb. 18, 27, 2003 (J. Annala)

- all skew quad correctors turned off
- inject with horizontal offset in trajectory...

Source is EVERYWHERE!

i.e., $k(s) = \text{constant}$





beat period is about 3.3 turns ==> $\Delta\nu \sim 0.3$

$k(s) = \text{constant} \Rightarrow$ (a) all 216^+ quads rolled by same amount in same direction

<u>or</u>, (b) dipole magnets have developed a skew quadrupole moment

$$B_y = B_0(1 + b_0 + b_1 x + b_2 x^2 + \dots) \quad \Leftarrow \text{ assumes } y=0$$

more general:
$$\Delta B_y + i\,\Delta B_x = B_0 \sum_n (b_n + i\,a_n)(x+iy)^n$$

"normal" "skew"

quad: $n=1 \Rightarrow$
$$\Delta B_y = B_0 b_1 x - B_0 a_1 y$$
$$\Delta B_x = B_0 b_1 y + B_0 a_1 x$$

$$a_1 = -\frac{\partial B_y}{\partial y}\frac{1}{B_0} = \frac{\partial B_x}{\partial x}\frac{1}{B_0}$$

Magnitude ? For $\Delta\nu_{min} \approx 0.3$ uncorrected,

then
$$\Delta\nu_{min} \approx \frac{1}{2\pi}\sum_i \sqrt{\beta_x\beta_y}\, k_i \qquad k = \frac{\partial B_x/\partial x \cdot \ell}{B\rho}$$
$$= \frac{1}{2\pi}\sum_i 2F \cdot \frac{B_0 a_1 \ell}{B\rho}$$
$$= \frac{1}{2\pi}\cdot 2F \cdot a_1 \cdot \underbrace{\underbrace{\frac{B_0\ell}{B\rho}}_{\theta_{dipole}} \cdot N_{dipoles}}_{2\pi}$$

$$\Rightarrow \Delta\nu_{min} \approx 2F a_1$$

$$\Rightarrow a_1 = \frac{\Delta\nu_{min}}{2F} = \frac{0.3}{2.25\,m} \cdot \frac{0.0254\,m}{in}$$

$$\Rightarrow \boxed{a_1 \approx 1.5 \times 10^{-4}\ /\text{inch}}$$

Meanwhile…

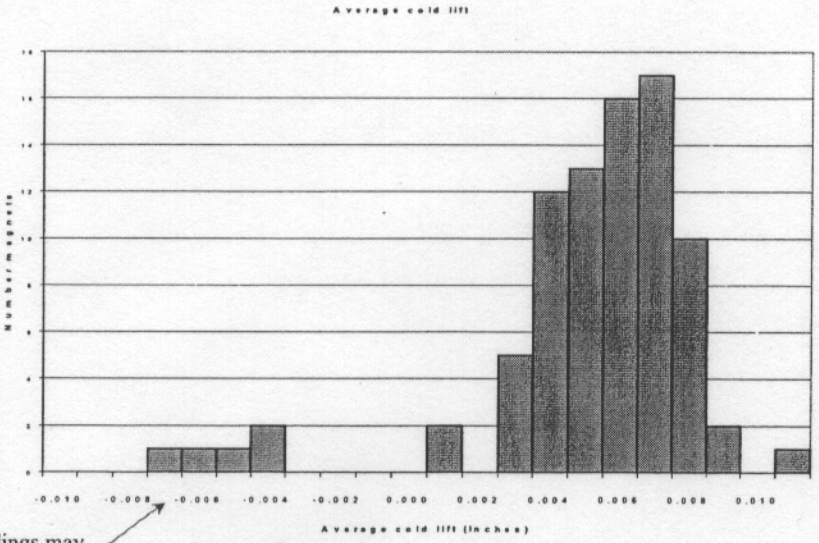
From: Michael Syphers syphers@fnal.gov
Date: Mon Feb 24, 2003 4:37:48 PM America/Chicago
To: harding@fnal.gov
Subject: Re: Tevatron coupling

Hi Dave,
A back-of-envelope calculation says that <a_1> = 1 unit might be enough to do the trick. I will be trying a more exact calculation tomorrow to see if this estimate holds up. I'll let you know...
Cheers,
-Mike

On Wednesday, February 19, 2003, at 03:09 PM, David Harding wrote:

Mike,If every Tevatron dipole had developed with age a skew quadrupole component,how big would it need to be to explain the effects that are currently seen?
-Dave Harding

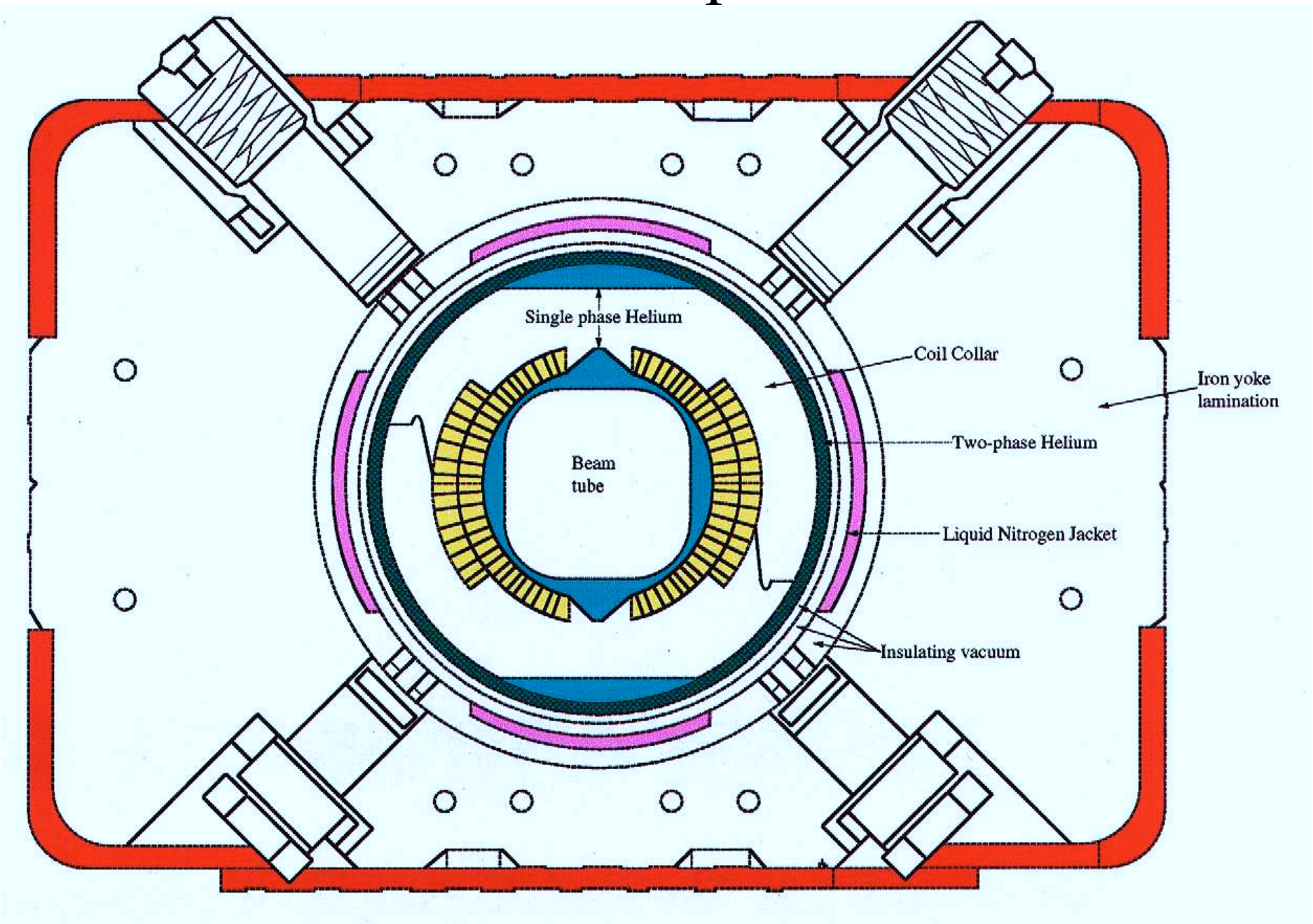
"Smart Bolts" in Tevatron magnets allow measurement of coil position relative to iron yoke.



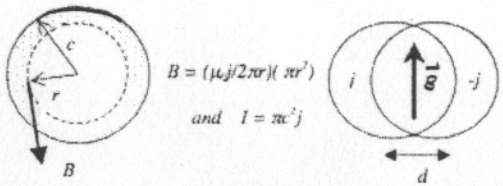
Believe these readings may have been off by 10 mils

Average increase in Smart Bolt expansions by 6 mil ==> $\Delta y = (\sqrt{2}/2)*6 = 4.2$ mil

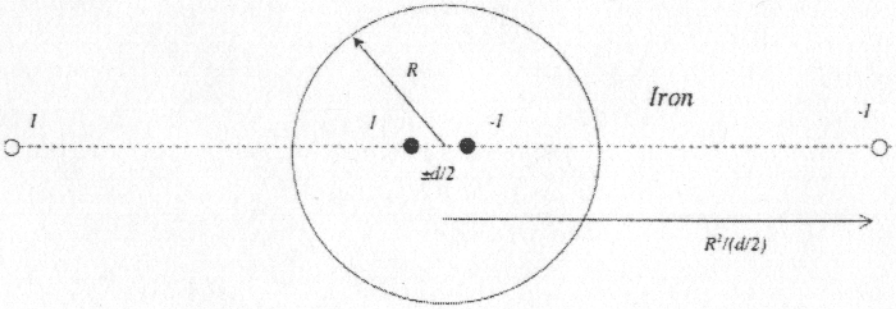
Tevatron Dipole



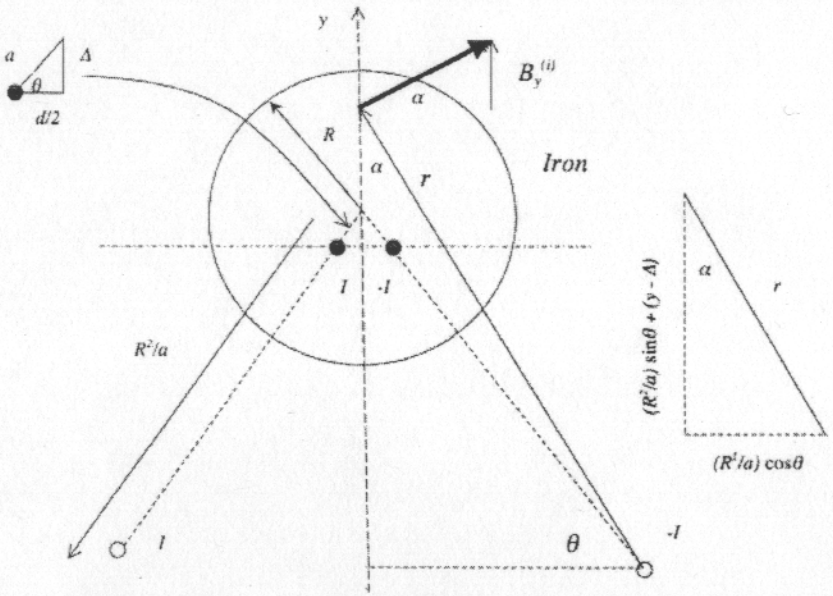
Start with uniform current density within a cylindrical cross section; look at field from 2 such cross sections, separated by distance d, and with opposite currents --> pure dipole field, B_c



Next, add an iron yoke of radius R and compute magnetic images, which will be located left and right, and which enhance the field in the center:
$B_0 \sim B_c[\ 1+(c/R)^2\ ]$

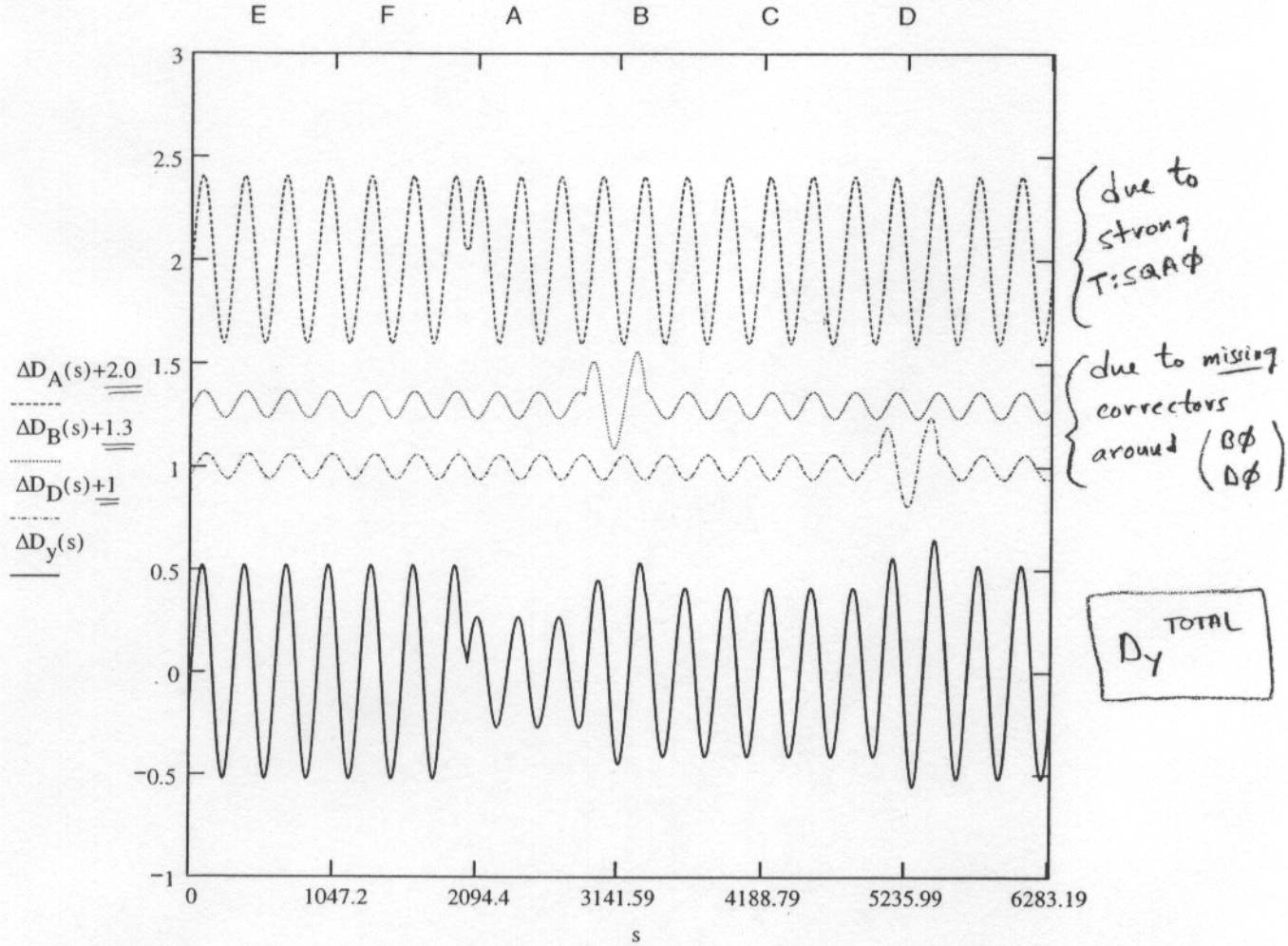


Finally, displace the center of the yoke with respect to the center of the coil by a distance Δ, and compute the resulting skew quadrupole component
$a_1 = (dB_x/dx) / B_0$

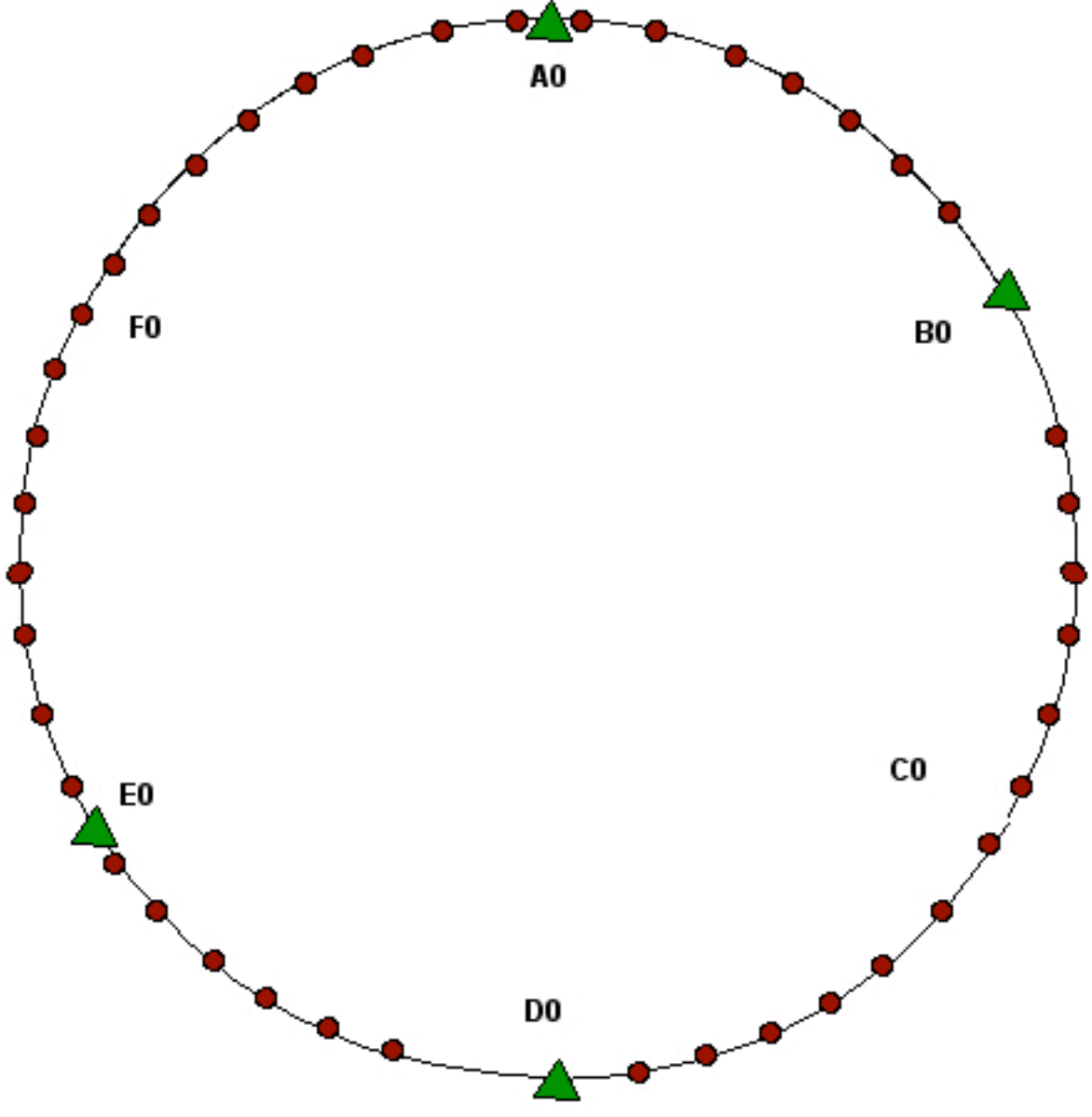


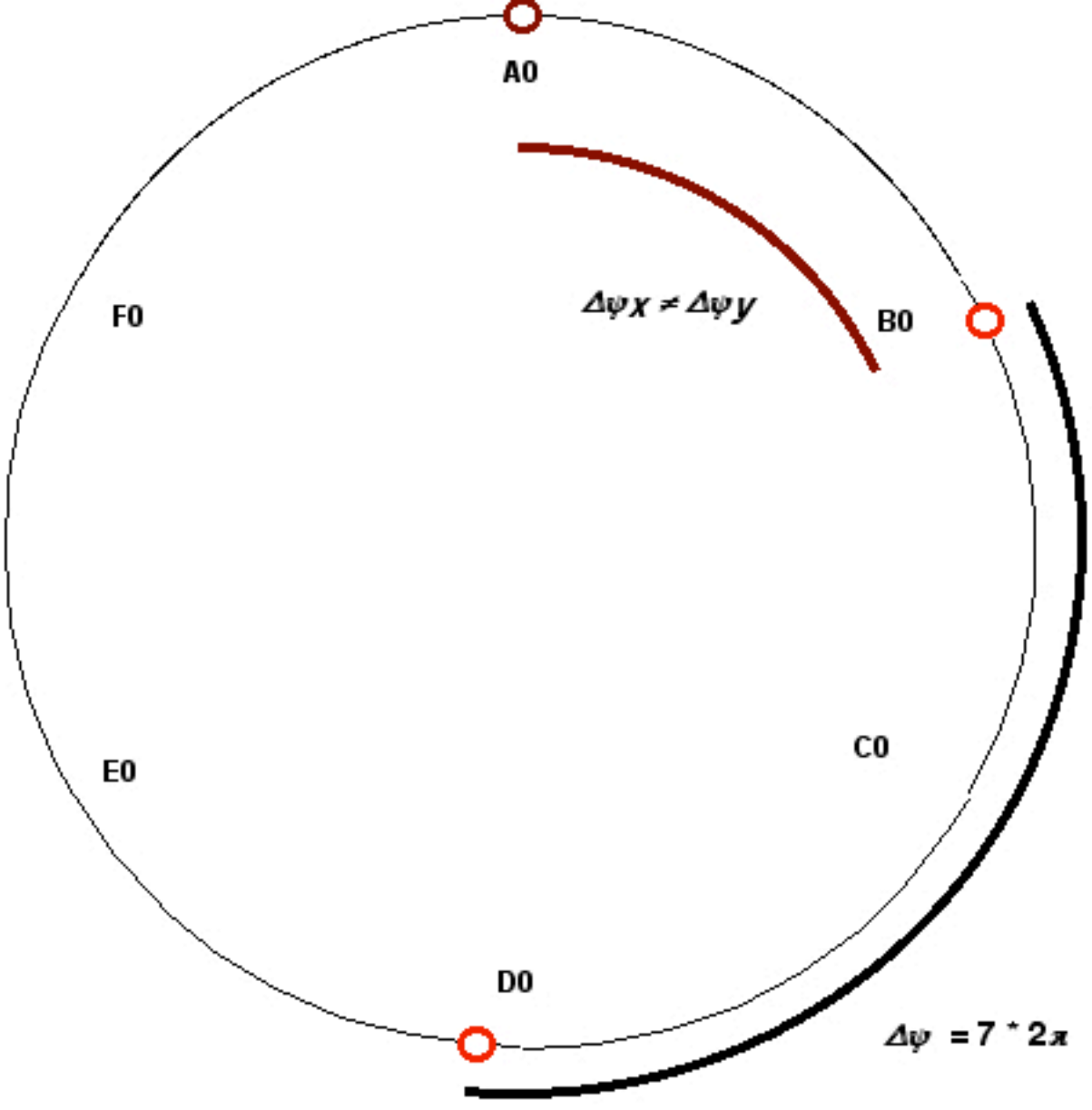
$$a_1 = 2\frac{(c/R)^2}{1+(c/R)^2}\frac{\Delta}{R^2} = 2\frac{0.25}{1.25}\frac{0.004}{(3.8)^2\text{in}} = 1.1\times10^{-4}\ /\text{in}$$

Also explains observed vertical dispersion...



SQ = −3 A0 = 6.27 A4 = −5.17 B1 = 0.56 D0 = 0 E0 = 0


A0
B0
C0
D0
E0
F0


A0
B0
C0
D0
E0
F0
Δψx ≠ Δψy
Δψ = 7 * 2π

TEVATRON SUMMARY

- STRONG COUPLING DUE TO SAGGING COILS WITHIN THE DIPOLE MAGNETS $\longrightarrow$ systematic a_1

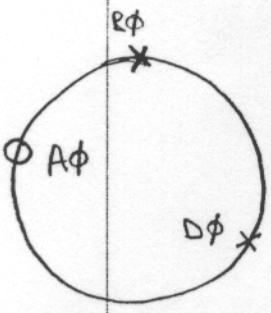
- CORRECTORS (SQ) REMOVED FROM BϕDϕ REGIONS FOR IR UPGRADES

- "CENTER-OF-MASS" OF THESE MISSING REGIONS ARE NOT IN PHASE WITH T:SQ CIRCUIT ;
  $\therefore$ T:SQAϕ (+/OR T:SQEϕ) REQUIRED IN ORDER TO MAKE $\Delta\nu_{min} \longrightarrow \phi$

- T:SQAϕ RUNS STRONG TO MAKE CORRECTION

  $\Rightarrow$ VERTICAL DISPERSION

  $\Rightarrow$ SINGLE LARGE CORRECTION @ Aϕ TO FIX TWO LARGE "ERRORS" @ Bϕ, Dϕ REGIONS DISTORTS LATTICE FUNCTIONS, GENERATES LARGE "LOCAL" COUPLING THROUGH LARGE REGIONS OF TEVATRON



$\therefore$ injection lattice mismatch
dispersion function mismatch } emittance growth @ injection
beam dampers – H + V coupled
⋮

- SHUTDOWN '03: FIXED coils in the unconnected Bϕ, Dϕ REGIONS (~ 100 dipoles)