FERMILAB-TM-2226

Efficiency of Transverse Instability Dampers

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Single and Couple-Bunch Description

- For the air-bag (=hollow beam) distribution $z = \hat{z}\cos\varphi$ single-bunch modes are described by a single head-tail wave number l. The transverse offset can be expanded over the modes as

$$x(\varphi) = \sum_l A_l \exp(il\varphi + i\chi\cos\varphi - i\Omega_l t)$$

  with complex amplitudes A_l and $\chi = \xi\hat{z}/(R\eta) \cong 1.5\xi\sigma_z/(R\eta)$ the head-tail phase.

- For coupled-bunch description, the modes are described by **two** numbers: intra-bunch head-tail number l and multi-bunch number μ. In this case,

$$x_r(\varphi) = \sum_l B_{l\mu} \exp(il\varphi + i\chi\cos\varphi + 2\pi ir\mu/M - i\Omega_{l\mu} t)$$

  When bunches do not talk to each other, the eigen-frequencies do not depend of the multi-bunch mode number: $\Omega_{l\mu} = \Omega_l$

Growth Rates

- Single-bunch instabilities are driven by the high-frequency impedance, $\omega \geq c/\sigma_z$ or $f \sim 50-200$ MHz for Tevatron. Coupled-bunch phenomena appears due to much lower frequency range of the impedance, $\omega_0 \leq \omega \leq M\omega_0$. In general, the growth rates are described like

$$\Lambda_{l\mu} = \Lambda_l^s + \Lambda_{l\mu}^c$$

where the single- and coupled-bunch contributions are calculated in the air-bag model as

$$\Lambda_l^s = -\frac{N_b r_0}{2\pi Z_0 \gamma \nu_\beta} \int_{-\infty}^{\infty} d\omega \, \mathrm{Re}\, Z(\omega) J_l^2(\omega \hat{z}/c - \chi)$$

$$\Lambda_{l\mu}^c = -\frac{M N_b r_0 \omega_0}{2\pi Z_0 \gamma \nu_\beta} J_l^2(\chi) \sum_{p=-\infty}^{\infty} \mathrm{Re}\, Z\left(\omega_0\left(\nu_\beta + pM + \mu\right)\right)$$

- The resistive wall impedance, slowly decreasing as $1/\sqrt{\omega}$, can provide both contributions being visible.

Resistive Wall

- For the resistive wall, the two contributions are

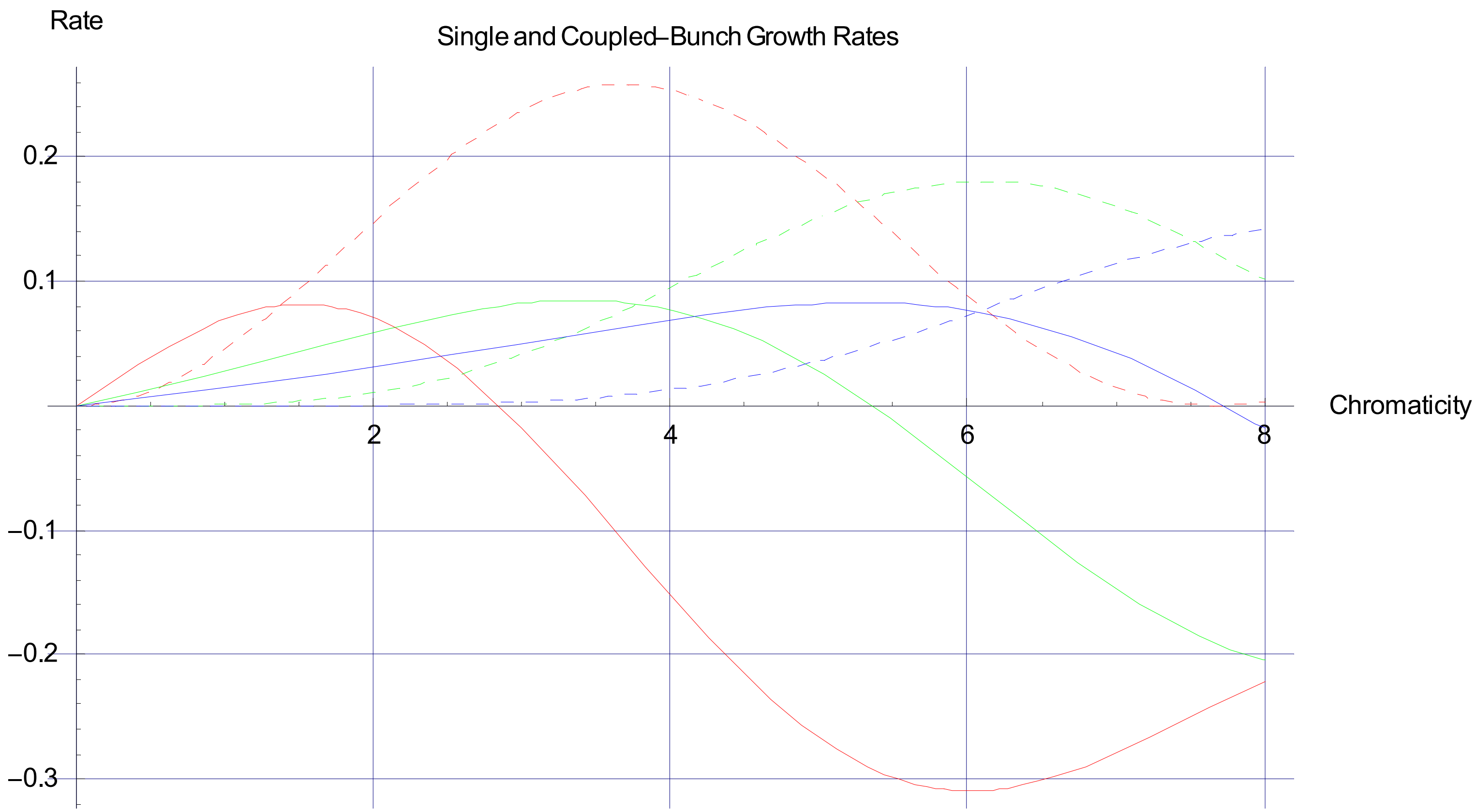
$$\Lambda_l^s = \hat{\Lambda} \int_0^\infty \frac{dw}{\sqrt{w}} \left[J_l^2(w-\chi) - J_l^2(w+\chi) \right]$$

$$\Lambda_{l\mu_0}^c = \hat{\Lambda} M J_l^2(\chi) \sqrt{\frac{\hat{z}}{\Delta_\beta R}}$$

where $\mu_0 = M - [\nu_\beta] - 1$ is the most coupled-bunch unstable mode.

RW: Single- and Coupled-Bunch Rate Contributions

- Single (solid lines)- and Coupled-Bunch (dashed) growth rate contributions are shown in the figure below for *l=1* (red), *l=2* (green) and *l=3* (blue), $\mu = \mu_0$.



Damper

- Damper principal scheme:
  - Pickup catches the beam signal
  - Sin/Cos – modulation
  - LPF
  - Amplification
  - Cos/Sin – modulation
  - Kicker acts back (with ~ 90 degree phase advance from pickup)
- The resulting rate is determined by a double scalar product modified with the couple-bunch mode number:

$$\Gamma_{l\mu} \propto \alpha_{\mu} \langle \boldsymbol{l} \,|\, \boldsymbol{p} \rangle \langle \boldsymbol{k} \,|\, \boldsymbol{l} \rangle$$

where the scalar products describe how well the mode $|\boldsymbol{l}>$ is seen by the pickup $|\boldsymbol{p}>$ and deflected by the kicker $|\boldsymbol{k}>$.

Damping Rates

- Feedback leads to damping rates. For air-bag distribution, the damping rates are calculated as

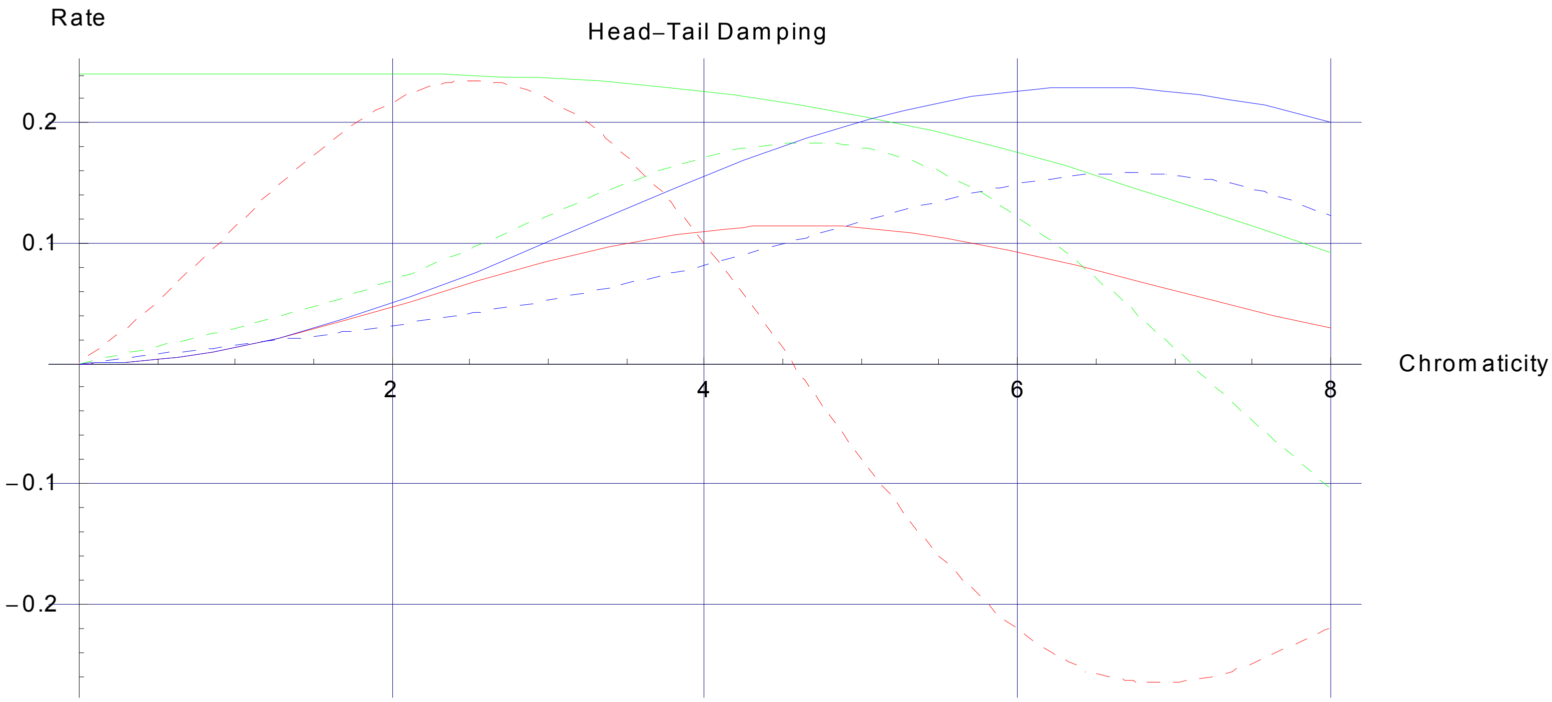
$$\Gamma_{l\mu} = g\left|J_l(\chi+q)e^{i\theta} + J_l(\chi-q)e^{-i\theta}\right|^2 \sum_{p=-\infty}^{\infty} \tilde{F}\left[\omega_0\left(\nu_\beta + pM + \mu\right)\right]$$

where g is a constant resulted from pickup and kicker impedances, amplification, etc, $q = \tilde{\omega}\hat{z}/c$ is a phase advance of modulation frequencies at pickup and kicker, and $\tilde{F}(\omega)$ is the Low Pass Filter transmission. The modulation is assumed as $\propto \sin(qz/\hat{z}+\theta)$ at the pickup and $\propto \cos(qz/\hat{z}+\theta)$ at the kicker, and the phase shift θ is a parameter for optimization. The pickup length assumed to be much smaller than the bunch length.

- Note that
  - this scheme makes all the head-tail modes damped/anti-damped simultaneously.
  - LPF width does not influence the sum of rates $\sum_\mu \Gamma_{l\mu}$; it starts to redistribute rates for $\Delta\omega_{LPF} \le M\omega_0 = 2.5$ MHz.

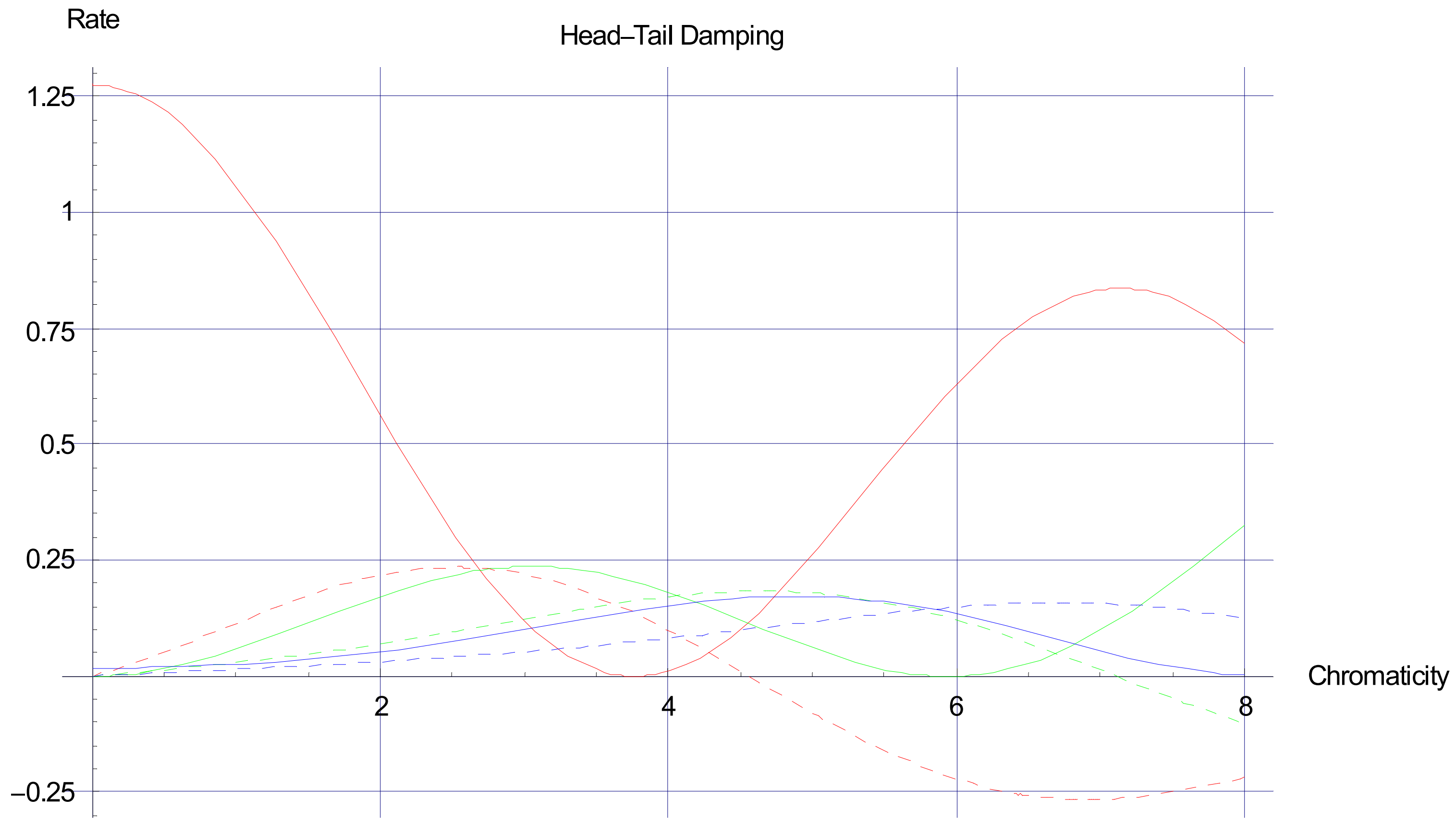
Damper against Instability: no phase shift

- For $\theta=0$, damping rates of odd modes (l=1,3…) vanish at low chromaticity as $\Gamma \propto \chi^2$, while the head-tail rates go down linearly, $\Lambda \propto \chi$. The main stopper is the lowest-order odd mode, $l=1$. The modulation phase advance $q=1.6$
- Relative behavior of damping (solid) and growth (dash) rates are illustrated in figure below for $l=1,2,3$ (red, green and blue lines).



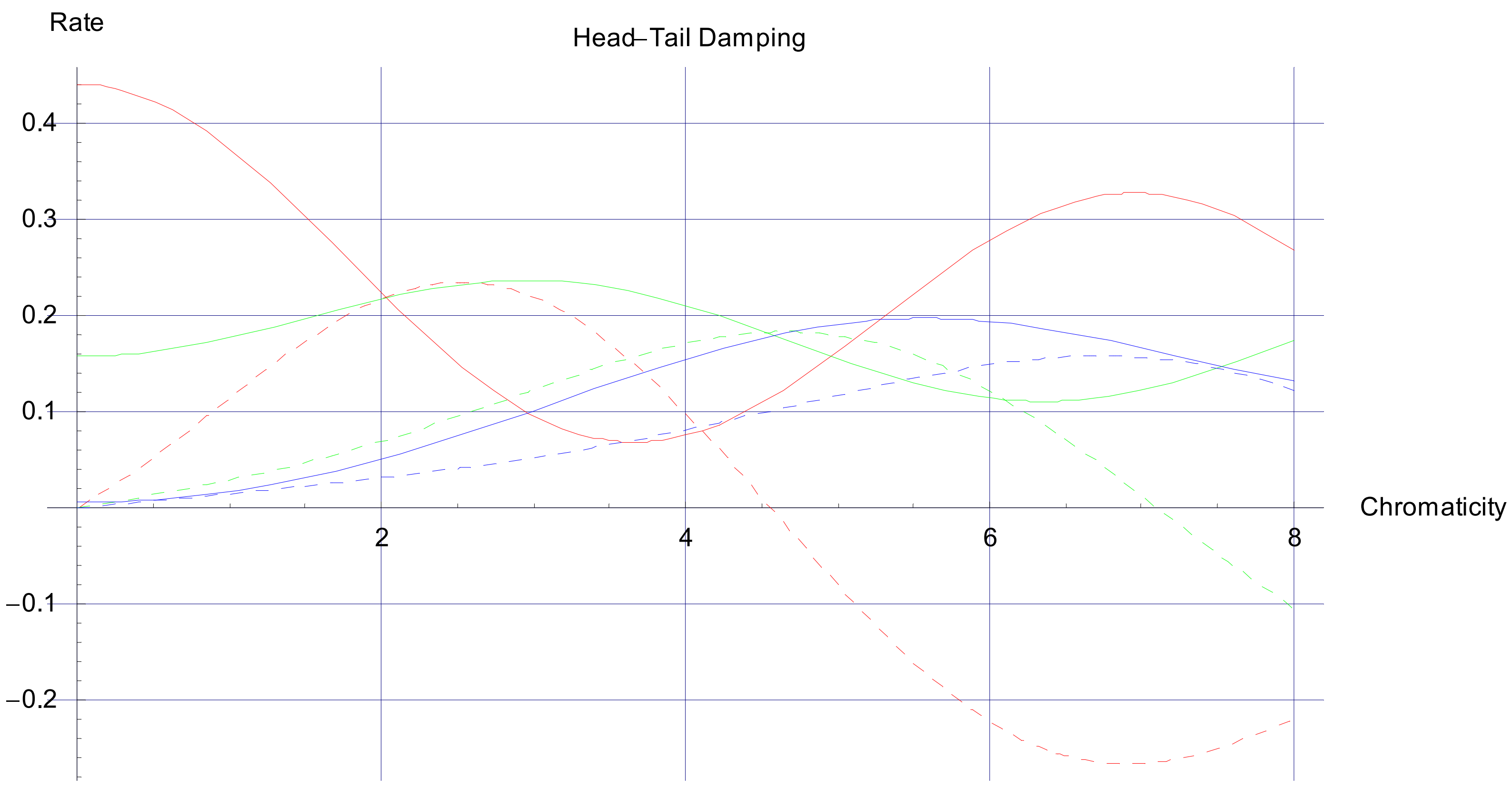
Damper against Instability: 90° phase shift

The same, for 90 degrees of the phase shift. Clearly, this phase shift lies from the other side from the optimum.



Damper against Instability: 36° phase shift

- The same figure for 36° of the phase shift θ shows this choice as close to optimal: all the modes can be effectively damped for all chromaticities.



Conclusions

- Effectiveness of the damper is a function of the modulation phase advance q and phase shift θ. Effective damping of the first 3 modes for all chromaticities requires the modulation phase advance $q \sim 1.5$ (-> mix frequency ~53 MHz) and the phase shift $\theta \approx 40°$.
- If the third mode is stable by itself (stabilized by Landau damping), the optimal modulation frequency is about 2 times lower, while the optimal shift is about same.