

# Measuring BPM Resolution

“Position resolution of  $7\ \mu$ ” –

- What does that really mean?
  - I will propose a concrete definition
  - Other definitions are very close to identical
  - It’s probably not worth arguing which is best to use.
  
- How can we verify that  $7\mu$  will be attained?
  - Measurements are each expensive.
  - How many measurements do we need to measure resolution to a given accuracy?
  - The accuracy might depend on the actual resolution.
  - What is an optimal scheme to do these measurements.

Not addressed by me

(and I believe never addressed by the requirements writers):

Based on beams physics consideration, how valuable would a resolution of  $x$  microns be?

What is the worst resolution that would do no appreciable harm to any measurements needed for physics purposes?

How big a sigma would start causing unacceptable problems?

Logical justification for choosing  $7\mu$ :

- Thought to be routine to reach this
- So much better than current resolution that clearly it will be good enough.

## Defining the Resolution

Given two measurements of position  $P_1$  and  $P_2$  in situations where the actual positions were at  $x$  and  $x + \delta x$  (with  $\delta x > 0$ ) there is some probability  $F(\delta x)$  that  $P_1 > P_2$ :

- A difference in position is discerned
- And the difference is in the correct direction.

$F(\delta x)$  can be fit to  $\text{erf}(\delta x / (\sigma\sqrt{2}))$ .

The  $\sigma$  that fits this best is defined to be the resolution of the device.

- Given that the device has a finite precision, the chance that the two measurements would fall into the same bin is folded into this definition, as being a “failure to discern the correct difference.”
- Taking this definition seriously, it is logically best to actually displace the beam when trying to evaluate  $\sigma$ .
- The  $s$  we are trying to measure is the resolution of some particular variety of measurement (say uncoalesced many-turn averaging). We will take measurements of that kind to evaluate  $\sigma$ .
- By “resolution” we refer to situations where measurements are made which are close in time.

An important difference between  
this definition and the  
Gaussian width definition

Gaussian width definition

- Take repeated measurements at  $P$ .
- Compute RMS deviation from mean.
- Glosses over discretization issue.

This definition is  $\sqrt{2}$  times larger.

The number  $7\mu$  in the requirements  
document needs to be changed to its  
equivalent in terms of the new definition:

**$10\mu$**

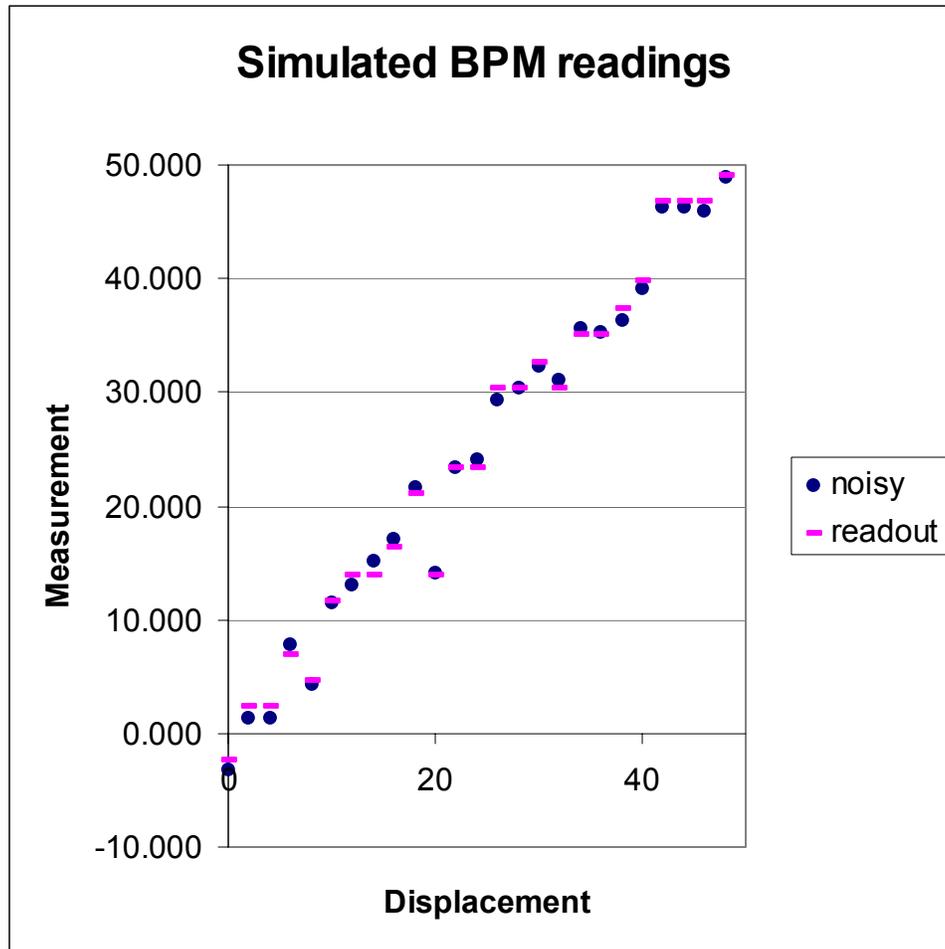
A much less important difference

The inaccuracy in this definition is dominated by  
the percent error in calibration of position offset  
against DFG current.

The inaccuracy in the Gaussian definition is  
dominated by calibration of new BPM reading per  
unit of actual displacement.

## Strategy to Measure $\sigma$

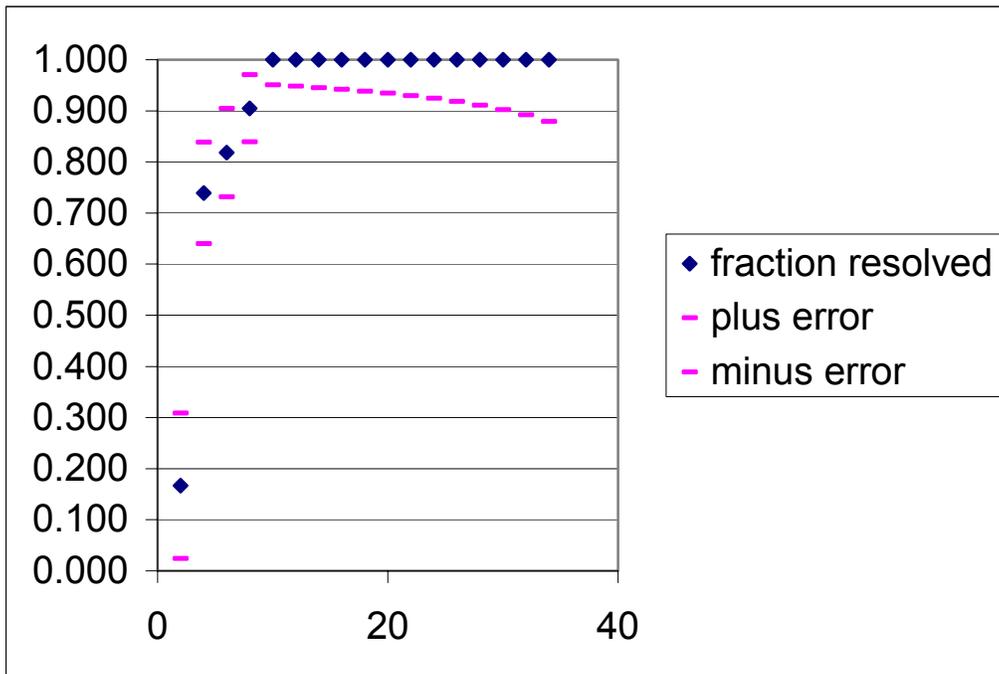
1. Will take  $N$  measurements (of the beam position)
2. Between each measurements, will adjust the currents of 3 DFGs to cause a 3-bump which has an expected displacement  $\rho$  at the prototype BPM.
  - It is assumed that very fine current adjustments can be made.
  - Linear response in that small range is assumed.
  - We know (or can easily calibrate) the response to 15% or so).
3. These  $N$  measurements at  $0, \rho, 2\rho, \dots, (N-1)\rho$  are combined in pairs to form  $N(N-1)/2$  “trials” at varying displacements  $\delta x$ .
4. Each displacement  $\delta x$  now has a “fraction of times the displacement is resolved correctly”  $F(\delta x)$ .
5. Fit  $F(\delta x)$  to the erf, to get  $\sigma$ .



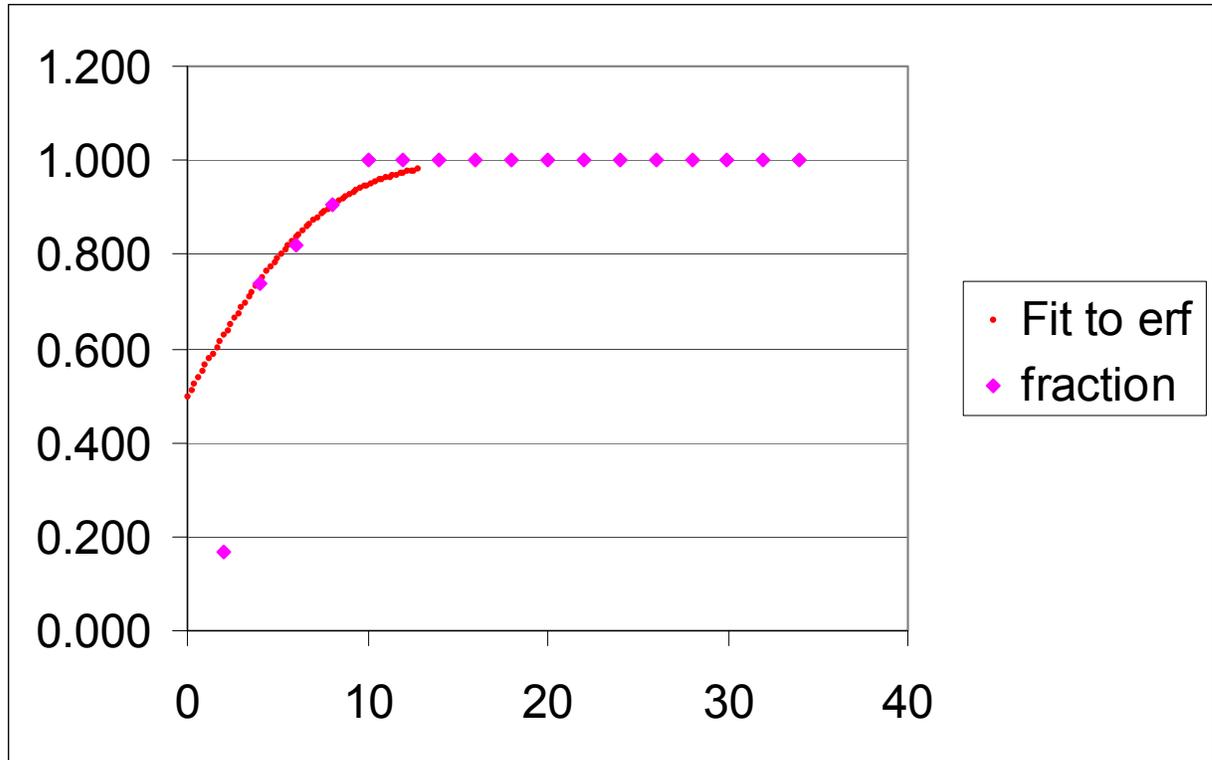
One simulated trial using 25 measurements.

The simulated  $s$  is  $10 \mu$ .

The lines represent the actual (discrete) readout values.



The algorithm groups pairs with like differences and calculates the fraction of correct resolution for each distance.



The best fit has  $\sigma = 8.667 \mu$ . This is the value that would be found for this trial data.

Note – in original talk the fit shown used this  $\sigma$  divided by root 2 (thus poor apparent fit).

After 1000-1000 simulated trials, we can understand accuracy and bias of this measurement strategy.

# Bottom Line

If  $\sigma$  is really about  $10\mu$ , to measure  $\sigma$  to accuracy of  $1.5\mu$  requires 35 measurements, base spaced at  $2.2\mu$  apart.

If  $\sigma$  is really about  $10\mu$ , to measure  $\sigma$  to accuracy of  $3\mu$  requires 14 measurements, base spaced at  $2.7\mu$  apart.

If  $\sigma$  is really some reasonably different value, from 7 – 30, these plans would still get reasonable accuracy.