Some Thoughts About June 9 Induced Instability at the Recycler Ring

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Below is my analysis of the growth in betatron oscillation that took place after the induced instability at the Recycler Ring on June 9, 2004 performed by J. Crisp and M. Hu. My analysis is for the growth during the second beam loss, before which the antiproton beam had the intensity of about $N_b = 28 \times 10^{10}$.

1. Length of beam was roughly $t_b = 3.5\ \mu$s. Thus the local current was roughly $I_{\rm local} = eN_b/t_b = 12.8$ mA.

2. Treating this as a coasting beam, the transverse microwave stability limit is given by

$$|Z_1^V| \lesssim \frac{4\sqrt{2\pi}\,\nu_V \sigma_E}{eI_{\rm local}\beta R} \left|\xi_V + \eta\big(|n| - [\nu_V]\big)\right| , \tag{1}$$

   where $R = 528.3$ m is the mean radius of the Recycler, βc is the velocity of the beam particle with c the velocity of light, ν_V is the vertical betatron tune, and $[\nu_V]$ is its decimal part. The last factor, $S_V = \left|\xi_V + \eta\big(|n| - [\nu_V]\big)\right|$ is referred to as the *effective chromaticity*, which includes Landau damping provided by the chromaticity ξ_V as well as the spread of the revolution frequency through the slip-factor $\eta = 0.008511$. With $\nu_V = 24.415$ and rms energy spread $\sigma_E = 3$ MeV, the stability limit becomes

$$|Z_1^V| \lesssim 109\, S_V \ \text{M}\Omega/\text{m} . \tag{2}$$

   The chromaticity was set at zero. Thus for the $1-Q$ and $2-Q$ betatron sidebands, the stability limits are

$$|Z_1^V| \lesssim \begin{cases} 0.54\ \text{M}\Omega/\text{m} & \text{for } (1{-}Q) , \\ 1.47\ \text{M}\Omega/\text{m} & \text{for } (2{-}Q) . \end{cases} \tag{3}$$

3. The beam pipe of the Recycler Ring is made of stainless steel of resistivity $\rho_{ss} = 7.4 \times 10^{-7}\ \Omega$m, and elliptical in cross section with horizontal and vertical diameters,

respectively, $2a = 3.75''$ and $2b = 1.75''$. For a cylindrical beam pipe of radius b, the monopole longitudinal impedance and dipole transverse impedance are

$$Z_0^\parallel\Big|_{\rm cyl} = [1 + j\,{\rm sgn}(\omega)]\frac{R\rho_{ss}}{b\delta_{ss}}\ , \qquad Z_1^\perp\Big|_{\rm cyl} = \frac{2c}{b^2}\frac{Z_0^\parallel}{\omega}\Big|_{\rm cyl}\ , \tag{4}$$

where δ_{ss} is the skip-depth into the stainless steel walls of the beam pipe.

For the elliptical beam pipe, the impedances are obtained from the cylindrical ones with multiplicative form factors $F_0 = 0.9591$, $F_{\scriptscriptstyle H} = 0.4501$, and $F_{\scriptscriptstyle V} = 0.8369$, for, respectively, the longitudinal, horizontal, and vertical. Thus we obtain the resistive wall impedances:

$$\begin{aligned} Z_0^\parallel &= (1+j)11.67\,n^{1/2}\ \Omega\ , \\ Z_1^{\scriptscriptstyle H} &= (1+j)11.79\,\big|n - [\nu_{\scriptscriptstyle H}]\big|^{-1/2}\ {\rm M}\Omega/{\rm m}\ , \\ Z_1^{\scriptscriptstyle V} &= (1+j)21.92\,\big|n - [\nu_{\scriptscriptstyle V}]\big|^{-1/2}\ {\rm M}\Omega/{\rm m}\ , \end{aligned} \tag{5}$$

where $n = 1,\ 2,\ 3,\ \cdots$ are revolution harmonics. Obviously, the limit of stability had been exceeded.

4. The growth rates of vertical instabilities without Landau damping is given by

$$\frac{1}{\tau} = \frac{ecI_{\rm local}\,\mathcal{R}e\,Z_1^{\scriptscriptstyle V}}{4\pi\nu_V E} = 1.40\,\mathcal{R}e\,Z_1^{\scriptscriptstyle V}\ \ {\rm M}\Omega/{\rm m}\ , \tag{6}$$

with $\mathcal{R}e\,Z_1^{\scriptscriptstyle V}$ in MΩ/m. We therefore have

$$\frac{1}{\tau} = \begin{cases} 40.2\ {\rm s}^{-1} & \text{for } (1-Q)\ , \\ 24.4\ {\rm s}^{-1} & \text{for } (2-Q)\ . \end{cases} \tag{7}$$

This implies that a sideband grows faster if it is associated with a lower revolution harmonic. Therefore, the relative power of the lower harmonic sidebands to the higher harmonic sidebands should increase as time goes on.

5. Since the beam has a length of $t_b = 3.5$ μs, the longitudinal wave excited in the beam should have a wavelength shorter than two-times its length. Thus the lowest frequency mode excited is $f \approx 1/(2t_b) = 143$ kHz. The $(1 - Q)$ sideband has the frequency 52.5 kHz and the $(2 - Q)$ sideband has the frequency 142 kHz. Thus the $(1 - Q)$ sideband can hardly be excited and the $(2 - Q)$ sideband should be excited mostly. This agrees with what was observed. Also its growth rate of 24.9 s^{-1} (growth time 41 ms) agrees with observation. Notice that the pre-amplifier on the VP522 beam-position monitor, where all the data were recorded, has a flat response in the bandwidth from 10 kHz to 10 MHz.

6. The rms momentum spread of the beam is $\sigma_p = 3$ MeV/c. Particles with this energy offset will drift by
$$\Delta t = |\eta| T_0 \frac{\sigma_p}{p_0} = 3.20 \times 10^{-5}\ \mu\text{s}\ , \tag{8}$$
or they take $2t_b/\Delta t = 219000$ turns to drift in both direction. At the rf voltage of 2 kHz, it takes 6000 turns to move inside the two barriers. Thus the synchrotron period of these particles is 2.5 s. Thus for time longer than 1.25 s, the head and tail of the beam exchange position and the beam can no longer be treated as a coasting beam.

7. The bunched-beam effect will be established gradually after approximately half a synchrotron oscillation. Now each revolution harmonic is no longer an independent eigenmode. Instead, the beam is described by bunch modes which depend on the length of the beam.

   (a) For the rigid dipole mode, the power spectrum extends to roughly $\pm 1/t_b = \pm 2.9$ MHz with a peak at zero frequency. The growth rate of this mode samples contribution from all the betatron sidebands inside the power spectrum. Since slow waves (negative frequencies) correspond to growth and non-slow waves (positive frequencies) correspond to damping, the growth rate is proportional to the difference between the $\mathcal{R}e\, Z_1^V$ at $(n-Q)$ and $(n-1+Q)$. This is the Robinson type of instability. Because $\nu_V = 24.415$, the $(0+Q)$ sideband is closer to zero frequency than the $(1-Q)$ sideband. This mode is therefore stable.

   (b) At zero chromaticity, the power spectra of all bunch modes are symmetric with respect to zero frequency. The higher modes are stable or unstable depending on the Robinson stability criterion. Even if they happen to be unstable, the growth rate should be extremely slow, especially $\mathcal{R}e\, Z_1^V$ decreases as $\omega^{-1/2}$, and does not vary as much from a $(n-1+Q)$ sideband to a $(n-Q)$ sideband.

   (c) Because of the above, it may be interesting to monitor the betatron sidebands for a few seconds and we should see the betatron sidebands get damped as the bunch-beam effect sets in.

8. The mechanism of the observed instability can be described as follows:

   (a) Before the induced instability using the quadrupole-correction loop, the beam at the intensity $\sim 28 \times 10^{10}$ has been completely stable. This is because the beam has been inside the Recycler for a long time (many synchrotron periods) and has been governed by bunched-beam effect rather than coasting-beam effect.

(b) The trigger by the quadrupole-correction loop suddenly changes the transverse position of the beam. This transverse offset produces a transverse wake which excites transverse oscillations of the beam particles following.

(c) The beam can be considered coasting at this moment. The sidebands of all revolution harmonics are good eigenmodes and will be excited independently, except for the $(1-Q)$ mode which has half-wavelength slightly longer than the bunch length. Beam loss occurs when the amplitude of oscillation exceeds the aperture of the Recycler.

(d) In a time comparable to the synchrotron period, the tail particles reach the head position of the beam, and several harmonics interfere with each other; for example, the damping power of the $(n+Q)$ upper sidebands start to cancel the growth the $(n-Q)$ lower sidebands. In other words, the beam has reached the steady state of a bunch, when bunch modes become eigenmodes instead of the individual revolution harmonics. Because of the residual betatron tune chosen, the rigid dipole mode is stable. The higher-order head-tail modes, if unstable, have very small growth rates, which may be damped by tune spread and/or stochastic cooling.

(e) For this reason, the instability can persist only for the duration of a synchrotron period.

9. The beam intensity and the beam transverse emittances are usually monitored every second. It should be monitored more frequently in future experiments so that we can tell exactly when beam loss occurs. At the same time, we will know when the transverse emittances start growing and when the growth subsides.

A more detailed report will be written later.