Procedure for Measuring β^* in the Tevatron

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Single Particle Dynamics

Hill's Equation: $$x'' + K(s)\,x = 0$$

… with solution:

$$x(s) = A\sqrt{\beta(s)}\cos\left[\psi(s) + \delta\right]$$

$$\rightarrow \quad \psi' = 1/\beta, \qquad K\beta = \gamma + \alpha',$$

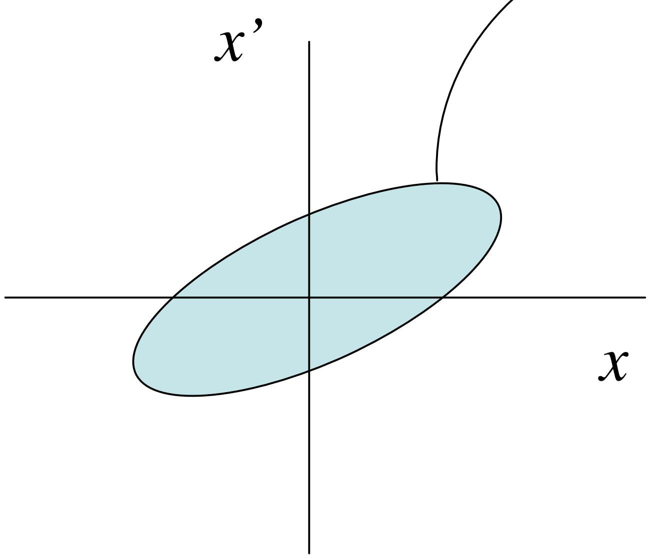
$$where \quad \alpha = -\frac{1}{2}\beta', \;\; \gamma = \frac{1+\alpha^2}{\beta}$$

α, β, γ are the Courant-Snyder parameters

Phase Space

Phase space ellipse:

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = A^2$$



Area of ellipse:

$$\epsilon = \pi A^2,$$

Amplitude of motion:

$$a = A\sqrt{\beta}$$

Moments of Single Particle Motion

from betatron oscillation

Position:

$$
\begin{aligned}
x &= A\sqrt{\beta}\cos\psi \\
x^2 &= A^2\beta\cos^2\psi
\end{aligned}
$$

$$
\begin{aligned}
\langle x^2\rangle &= A^2\beta\langle\cos^2\psi\rangle \\
&= \frac{1}{2}A^2\beta
\end{aligned}
$$

Averages are taken over time, or phase of oscillation

Slope:

$$
\begin{aligned}
x &= A\sqrt{\beta}\cos\psi \\
x' &= \frac{1}{2}A\beta^{-1/2}\cos\psi\ \beta' - A\sqrt{\beta}\sin\psi\ \psi' \\
&= -\frac{A}{\sqrt{\beta}}\left(\alpha\cos\psi + \sin\psi\right) \\
x'^2 &= \frac{A^2}{\beta}\left(\alpha^2\cos^2\psi + 2\alpha\cos\psi\sin\psi + \sin^2\psi\right)
\end{aligned}
$$

$$
\begin{aligned}
\langle x'^2\rangle &= \frac{A^2}{\beta}\left(\alpha^2\langle\cos^2\psi\rangle + \langle\sin^2\psi\rangle\right) \\
&= \frac{1}{2}A^2\left(\frac{\alpha^2+1}{\beta}\right) \\
&= \frac{1}{2}A^2\gamma
\end{aligned}
$$

Correlation Moment

$$\begin{aligned} x\;x' &= \left[A\sqrt{\beta}\cos\psi\right]\left[-\frac{A}{\sqrt{\beta}}\left(\alpha\cos\psi+\sin\psi\right)\right] \\ &= -A^2\left(\alpha\cos^2\psi+\cos\psi\sin\psi\right) \end{aligned}$$

$$\begin{aligned} \langle x\;x'\rangle &= -A^2\alpha\langle\cos^2\psi\rangle \\ &= -\frac{1}{2}A^2\alpha \end{aligned}$$

So, if could measure turn-by-turn position and slope of a single particle at a particular location, and compute the above moments from the data, could in principle determine Courant Snyder parameters at that location…

What about *A*? -- "Single Particle" Emittance

$$
\begin{aligned}
\langle x^2\rangle\langle x'^2\rangle - \langle x\, x'\rangle^2 &= (A^2\beta/2)(A^2\gamma/2) - (-A^2\alpha/2)^2 \\
&= A^4(\beta\gamma - \alpha^2)/4 \\
&= \left(\frac{A^2}{2}\right)^2 = \left(\frac{a^2}{2\beta}\right)^2 = \left(\frac{\epsilon}{2\pi}\right)^2 \\
\Rightarrow \epsilon/2\pi = A^2/2 &= \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle x\, x'\rangle^2}
\end{aligned}
$$

Thus, can substitute above expression for A^2 on previous pages to obtain…

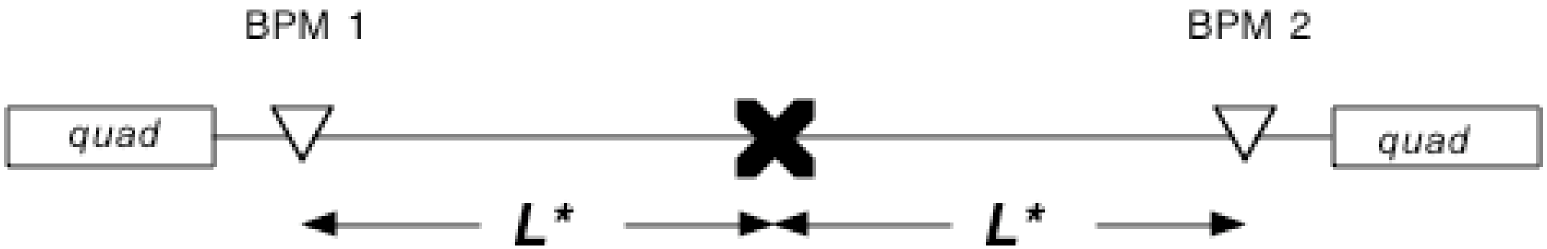
$$\beta = \frac{\langle x^2 \rangle}{\sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x\ x' \rangle^2}}$$

$$\gamma = \frac{\langle x'^2 \rangle}{\sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x\ x' \rangle^2}}$$

$$\alpha = -\frac{\langle x\ x' \rangle}{\sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x\ x' \rangle^2}}$$

Situation at CDF, D0 IP's

"Collision Point Monitors"



Measure x_1 and x_2, "Turn-By-Turn," then note,

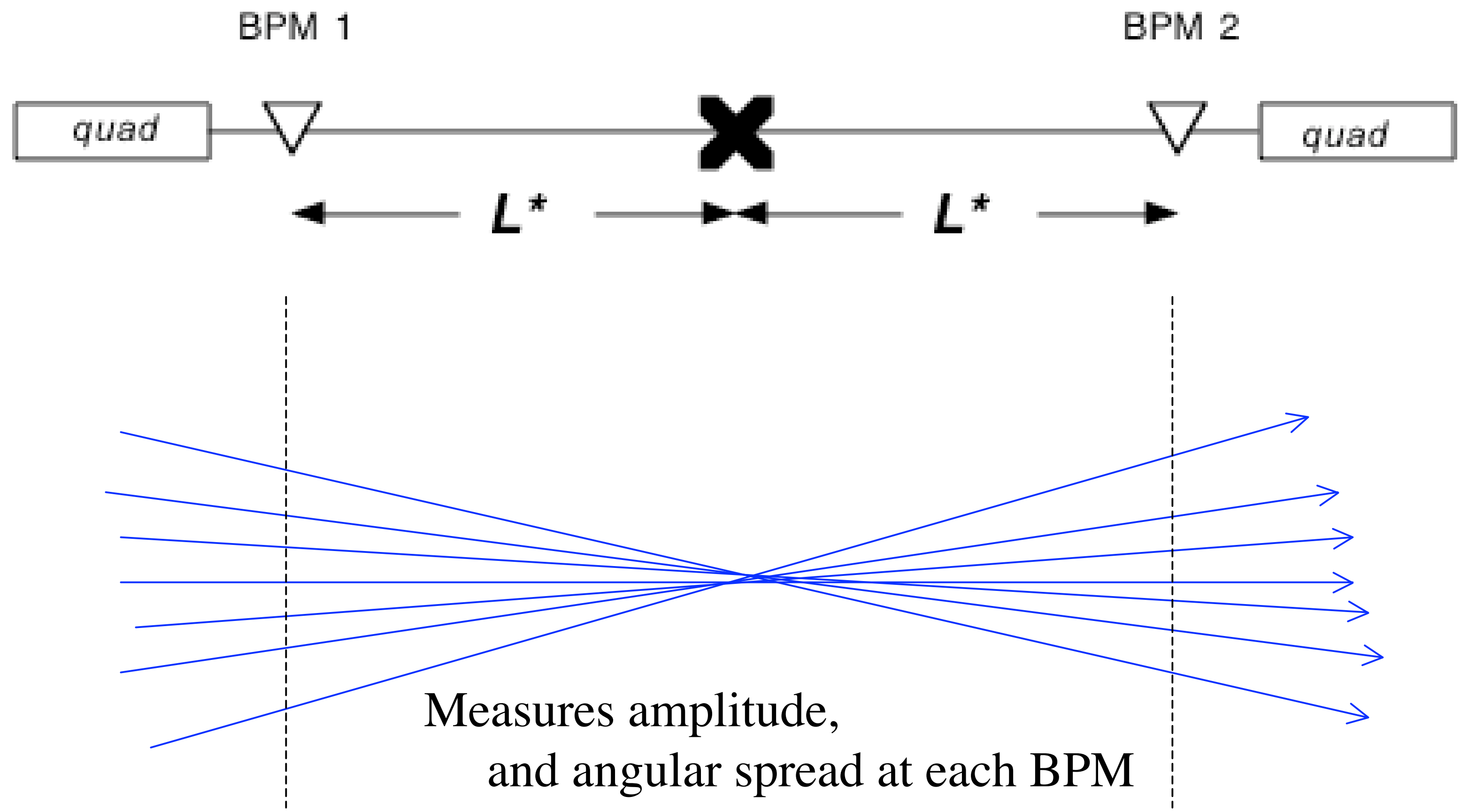
$$x' = \frac{x_2 - x_1}{L}, \qquad L = 2L^*$$

Then can compute α_1 and α_2 at the two BPM's

(and β_1, γ_1, β_2, γ_2, if desired)

Turn-by-turn BPM data

"Collision Point Monitors"



Computing β^*

Since we have a drift space between BPM1 and BPM2,

$$K\beta = \gamma + \alpha' = 0 \qquad \text{===>} \qquad \beta''' = 0$$

and so, the amplitude function is parabolic, the parameter γ is a constant through the region, and we can write

$$\begin{aligned} \beta(s) &= \beta_1 - 2\alpha_1 s + \gamma s^2 \\ &= \beta_2 - 2\alpha_2 (s - L) + \gamma (s - L)^2 \end{aligned}$$

$$and$$

$$\begin{aligned} \alpha(s) &= \alpha_1 - \gamma s \\ &= \alpha_2 - \gamma (s - L) \end{aligned}$$

The minimum of the amplitude function and its location relative to the upstream BPM are

$$\check{\beta} = \frac{L}{\alpha_1 - \alpha_2} \quad at \quad \check{s} = \frac{\alpha_1}{\alpha_1 - \alpha_2} L$$

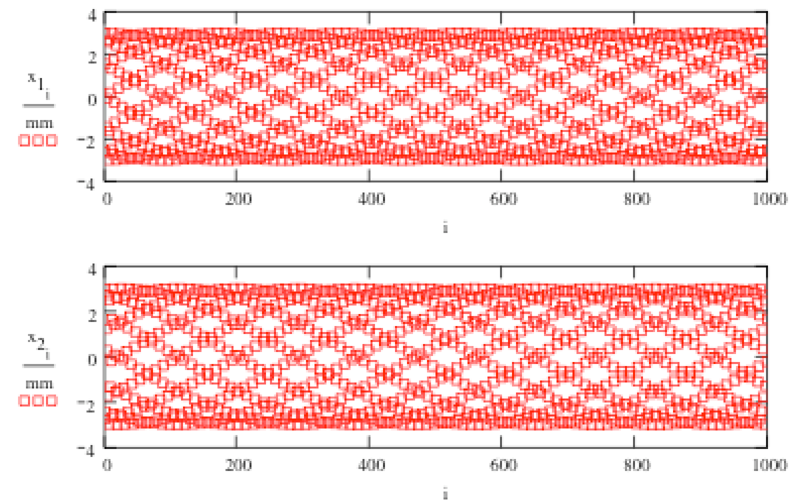
The value of the amplitude function and its slope at the mid-point of the straight section are given by

$$\beta^* = \frac{L}{\alpha_1 - \alpha_2} \left\{ 1 + \frac{1}{4}(\alpha_1 + \alpha_2)^2 \right\}$$

$$(\beta')^* = -2\alpha^* = -(\alpha_1 + \alpha_2)$$

Simulated Data

Simulated 1000-turn BPM data with infinite resolution



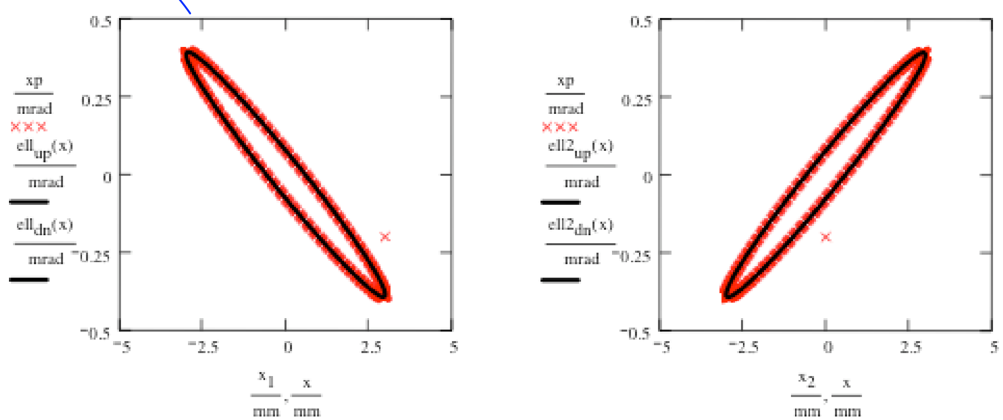
α determines the inclination of the ellipse

$$x' = x_2 - x_1 \quad \text{vs.} \quad x_1$$

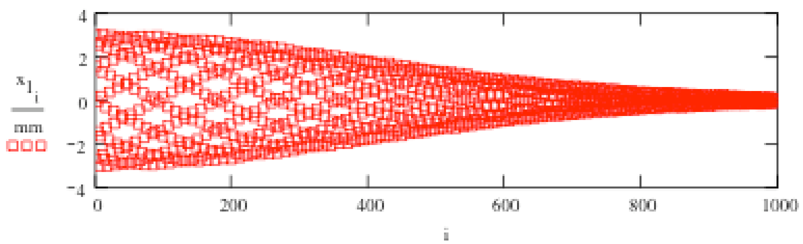
and

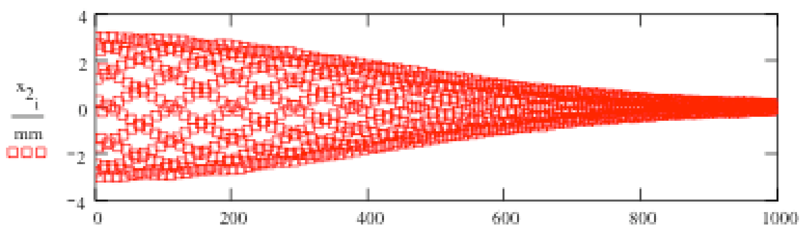
$$x' = x_2 - x_1 \quad \text{vs.} \quad x_2$$

ellipse parameters (giving the solid lines) are determined by the moments of the data

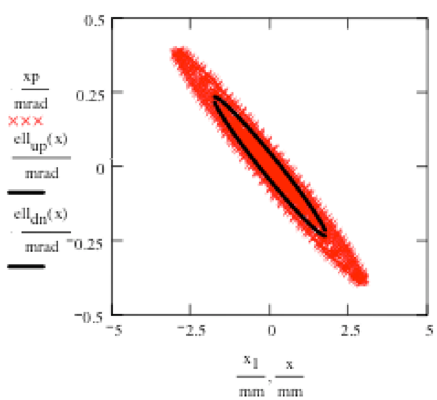


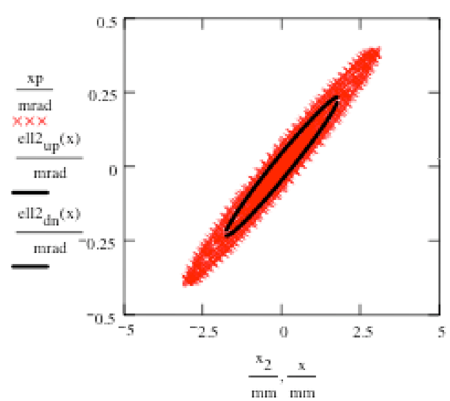
Simulation, with decoherence





Here, the (simulated) signal decoheres in about 400 turns due to nonlinear transverse fields; moments for the data, using all 1000 turns, are still used to determine the ellipse parameters





Can obtain about 3% accuracy of $\beta^* \sim 35$ cm with ~20 μm resolution on BPMs

Experimental Proposal

- Add T-B-T capability to CPM BPMs
- During studies, induce betatron oscillation and measure ~1000 turns
- First round:
  - Use existing hardware and digital scope can get ~100 μm resolution, measure β^* ~ 150 cm at injection optics; need a ~ 5mm in arcs to get ~5%
- Next round:
  - Upgrade resolution to ~20 μm (standard Tev Upgrade BPM system), repeat for β^* ~ 35 cm; here, would need a ~ 2.5 mm in arcs to get ~ 5%
- For now, can do X,Y separately; eventually, will want X,Y simultaneously --> can look at 4x4 matrix elements at BPMs, and therefore coupling effects

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