Gain Ramping for Transverse Stochastic Cooling Systems in the Debuncher

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Transverse Stochastic Cooling Equation

For transverse stochastic cooling, the cooling term is given as:

$$\frac{d(a^2)_{rms}}{dt} = -(a^2)_{rms} 2f_o K \sum_{m=-\infty}^{\infty} g_m \tag{1}$$

$$K = \frac{1}{pc/q} \sqrt{\frac{Z_{pu} Z_{kr}}{2}} \frac{\sqrt{\beta_{pu}\beta_{kr}}}{h_{pu}} e f_o \tag{2}$$

where f_o is the revolution frequency, pc is the momentum of the beam, Z_{pu} and Z_{kr} is the pickup and kicker impedance, β_{pu} and β_{kr} are the beta functions at the pickup and kicker, h_{pu} is the effective height of the pickup, and g_m is the electronic gain at the m^{th} betatron sideband. The heating term due to other particles is:

$$\frac{d(a^2)_{rms}}{dt} = (a^2)_{rms} N_p 2f_o K^2 \sum_{m=-\infty}^{\infty} \frac{|g_m|^2}{|m|} \frac{f_o}{\Delta f_o} \tag{3}$$

where N_p is the number of particles and Δf_o is the spread in revolution frequencies. The heating term due to amplifier noise is:

$$\frac{d(a^2)_{rms}}{dt} = \frac{(a^2)_{rms} N_p}{S((a^2)_{rms})} 2f_o K^2 \sum_{m=-\infty}^{\infty} |g_m|^2 \tag{4}$$

where $S((a^2)_{rms})$ is the average signal to noise and is given as:

$$S((a^2)_{rms}) = \frac{\frac{\beta_{pu}(a^2)_{rms}}{h_{pu}^2} N_p Z_{pu} (q f_o)^2}{k_B T_{pu} f_o} \tag{5}$$

where k_B is Boltzman's constant (1.36×10^{-23} Watts $K^{-1}Hz^{-1}$) and T_{pu} is the effective noise temperature of the pickup and pickup pre-amplifier. If the gain of the electronics is flat over a bandwidth W, then the r.m.s. betatron amplitude is given as:

$$\frac{d(a^2)_{rms}}{dt} = -4W(a^2)_{rms}\left(Kg(t) - (Kg(t))^2 N_p \left(M(t) + \frac{1}{S((a^2)_{rms})} \right)\right) \tag{6}$$

where M(t) is the mixing factor:

$$M(t) = \frac{f_o}{W} \frac{f_o}{\Delta f_o(t)} \ln\left(\frac{f_{max}}{f_{min}}\right) \tag{7}$$

where f_{max} and f_{min} are the maximum and minimum frequencies of the cooling system.

Gain Ramping

The output power of the cooling system is given as:

$$P(t) = \left(P_{bo}\left(\frac{P_b(t)}{P_{bo}}\right) + P_n \right)\left(\frac{Kg(t)}{Kg_o}\right)^2 \tag{8}$$

where P_{bo} is the initial beam power, P_n is the noise power, and g_o is the initial gain. The noise power can be written as the initial beam power divided by the initial signal to noise:

$$P_n = \frac{P_{bo}}{S_o} \tag{9}$$

The total initial power is:

$$P_o = P_{bo}\left(1 + \frac{1}{S_o}\right) \tag{10}$$

The ratio of beam power to initial beam power is equal to the ratio of the emittance to the initial emittance:

$$\frac{P_b(t)}{P_{bo}} = \frac{\left(a(t)^2\right)_{rms}}{\left(a_o^2\right)_{rms}} \tag{11}$$

Equation 8 becomes

$$P(t) = \frac{P_o}{1 + \frac{1}{S_o}}\left(\frac{\left(a(t)^2\right)_{rms}}{\left(a_o^2\right)_{rms}} + \frac{1}{S_o}\right)\left(\frac{Kg(t)}{Kg_o}\right)^2 \tag{12}$$

The gain profile as a function of power is:

$$\frac{Kg(t)}{Kg_o} = \sqrt{\frac{P(t)}{P_o}\,\frac{S_o + 1}{S_o\frac{\left(a(t)^2\right)_{rms}}{\left(a_o^2\right)_{rms}} + 1}} \tag{13}$$

Example

The gain ramping in the following example is set so that the total cooling system power is kept constant throughout the entire cycle. The parameters used in the example are shown in Table 1.

f_{min}	4.5	GHz
f_{max}	7.5	GHz
Production	15	$x10^{-6}$
MI Beam	5	$x10^{12}$
f_o	590020	Hz
η	0.006	
pc_o	8801	MeV
pc_{init}	80	MeV
pc_{asym}	5	MeV
τ_{pc}	1	Sec
ε_{95init}	20	π-mm-mrad
τ_{init}	1.3	Sec

Table 1.

The revolution frequency spread for the mixing factor is computed from the following:

$$\frac{\Delta f_o(t)}{f_o} = \eta \frac{(pc_{init} - pc_{asym})e^{-\frac{t}{\tau_{pc}}} + pc_{asym}}{pc_o} \tag{14}$$

The parameter τ_{init} is the initial cooling rate which is used to determine the initial electronic gain:

$$\frac{1}{\tau_{init}} = 4WKg_o \tag{15}$$

Figure 1 shows the emittance as a function of time for an initial average signal to noise of three with and without gain ramping. Using the same average initial signal to noise ratio, Figure 2 shows the ratio between the cooling to heating term with and without gain ramping. For this example, the heating term does not contribute substantially to the cooling rate. Figure 3 shows the emittance at the end of two seconds of cooling as a function of the initial signal to noise ratio. If the initial signal to noise ratio is three, then using gain ramping reduces the emittance by 35%.

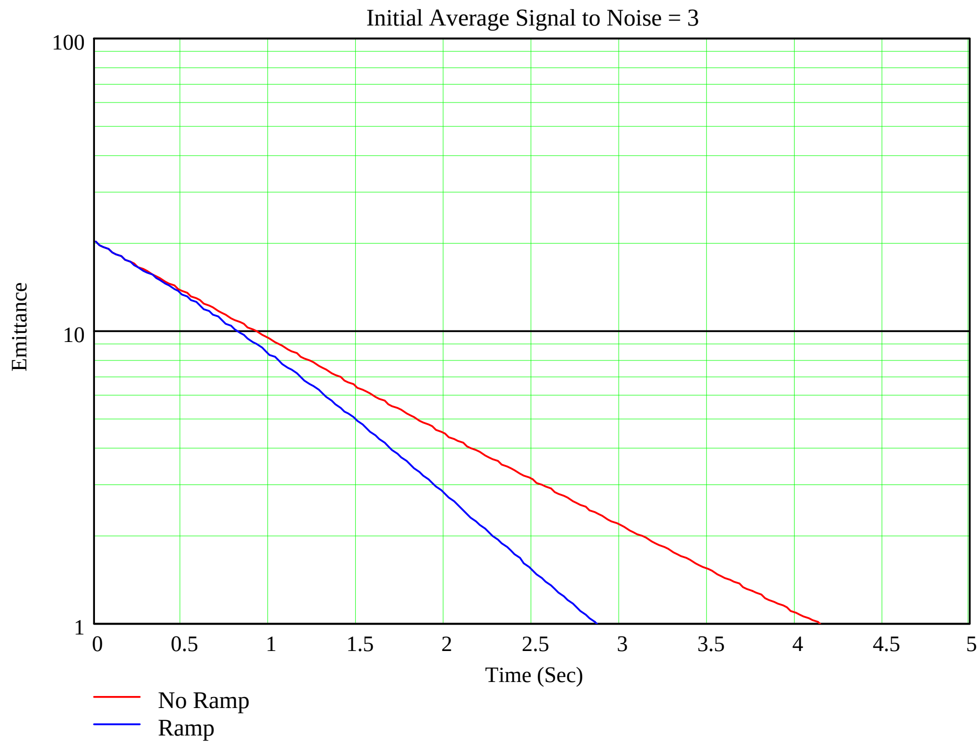

Figure 1. Emittance as a function of time for an initial average signal to noise of three with and without gain ramping.

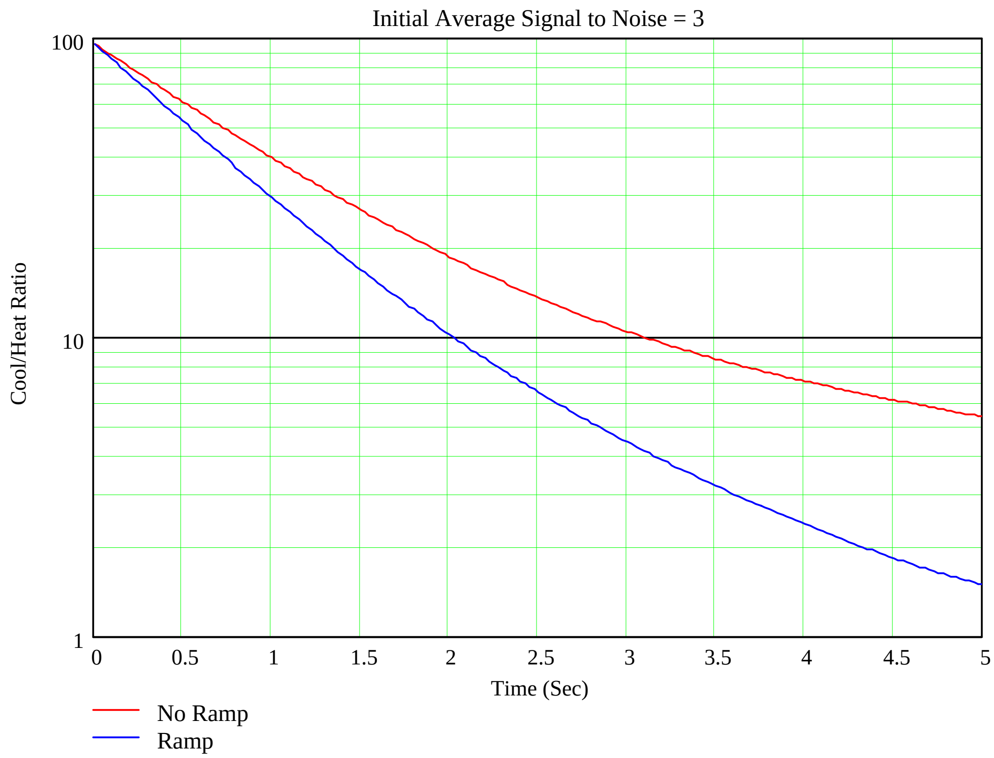

Figure 2. The ratio between the cooling to heating term with and without gain ramping.

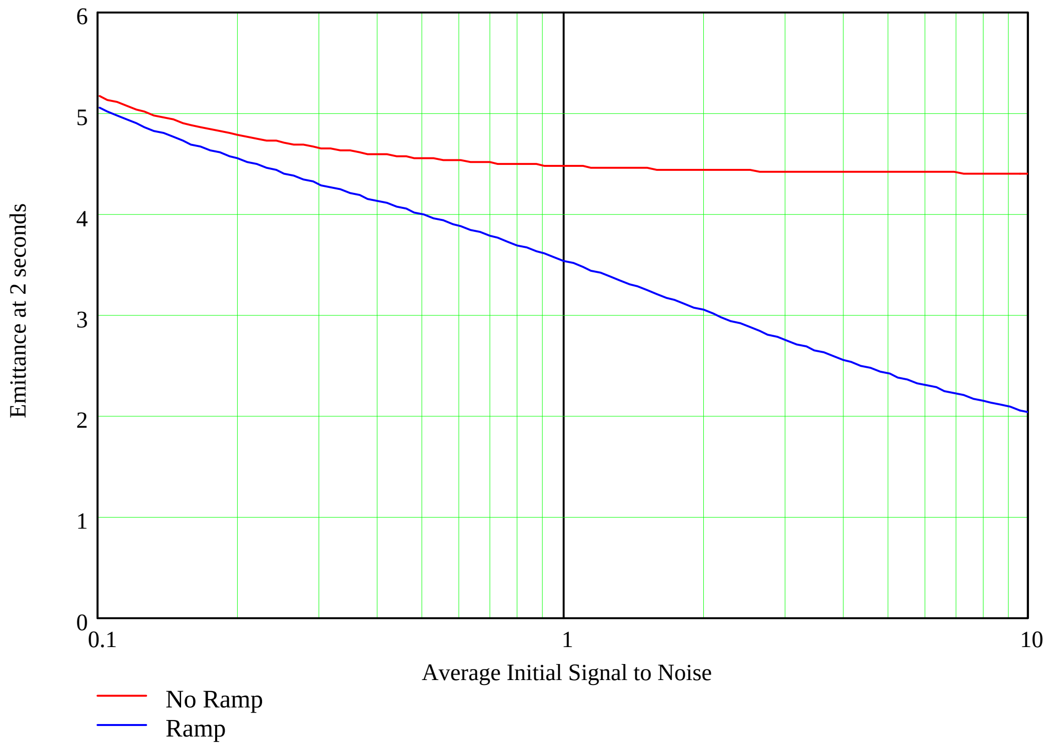

Figure 3. The emittance at the end of two seconds of cooling as a function of the initial signal to noise ratio