# Appendix 1.

# Detailed analysis of IBS study on Dec 14 2004 at 5am

The data analyzed here were accumulated on Dec. 14 2004 at about 5 am. The store consisted of 18 bunches. The first 15 were for studying the beam properties as a function of bunch intensity as shown in Fig. 1. However, there is much more data available from the SBD which was recording data every 10 minutes during the study. In addition there are some 15 sets of flying wire data. This more complete data set is used to carry out the calculations outlined in the first part of this paper.

## 1.0 Flying wire data.

We show here the 15 sets of flying wire data and a least squares linear fit to the data. The emittance is the vertical emittance calculated by T:PVEMIT. I have multiplied it by $\frac{80.8}{59.4} = 1.37$ which is the ratio of beta functions used in the calculation to that measured by Valerie.

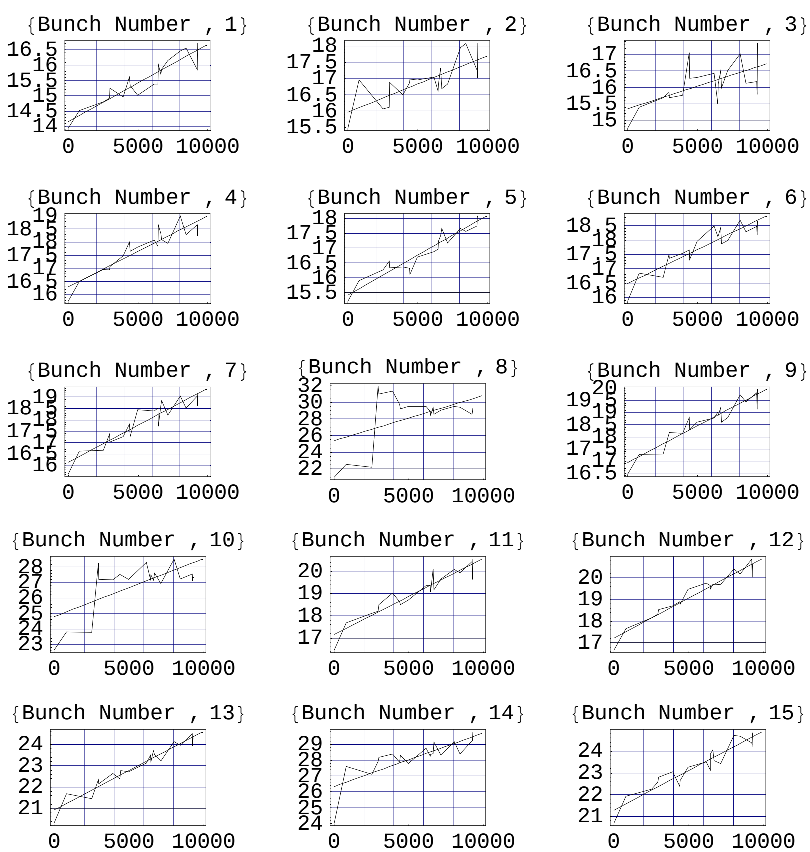


**Figure 1**

Bunches 8, 10 , and 14 blew up during the test and will not be used in the analysis. The time is in seconds from the end of scraping and the emittance is normalized. The fits are given in the Table 1 below. The first column is the fitted emittance at t=0, the second is the error, the 3 is the slope, and the fourth is the fractional error on the slope. Eliminating bunches 8, 10, 14, the slope error is of the order of 10%.

**VALUES AND ERRORS FOR $\epsilon_y$ LINEAR FITS**

| Value,t=0 | Error at,t=0 | Slope | SE on slope |
|---|---|---|---|
| 14.1597 | 0.146862 | 0.905851 | 0.0858856 |
| 15.9684 | 0.228475 | 0.623598 | 0.133614 |
| 15.3516 | 0.255472 | 0.494685 | 0.149401 |
| 16.2934 | 0.168298 | 0.975274 | 0.0984214 |
| 15.4179 | 0.124554 | 0.969007 | 0.0728396 |
| 16.4872 | 0.162543 | 0.856025 | 0.0950564 |
| 16.1319 | 0.192053 | 1.17407 | 0.112314 |
| 25.3861 | 1.34141 | 1.96243 | 0.784464 |
| 16.9379 | 0.161162 | 1.09792 | 0.0942482 |
| 24.7882 | 0.62297 | 1.35451 | 0.364316 |
| 17.1522 | 0.180259 | 1.23546 | 0.105416 |
| 17.2133 | 0.132013 | 1.33299 | 0.0772021 |
| 20.9263 | 0.151901 | 1.33081 | 0.0888326 |
| 26.3422 | 0.37928 | 1.21741 | 0.221805 |
| 21.2898 | 0.189555 | 1.30375 | 0.110853 |

**Table 1**

## 2. Longitudinal emittance evolution

The evolution of the longitudinal emittance is derived from the SBD which recorded data every 10 minutes during the store. A color coded plot of the mean square bunch width is shown below.

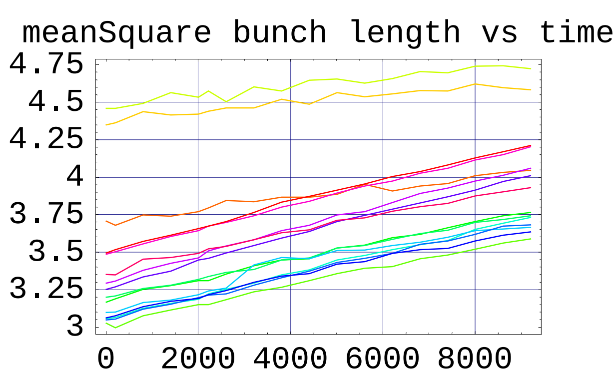


**Figure 2**

A Least Squares fit to the data above yields the following results. The first column is the bunch Mean Square of the bunch width at t=0, the second is the error, the third is the slope of the mean square width in $ns^2$ / hour and the last is the fractional error on the slope. This data set is the highest in accuracy.

**VALUES AND ERRORS FOR $\tau^2$ LINEAR FITS**

| Value,t=0 | Error at,t=0 | Slope $tau^2$ | SE on slope |
|---|---|---|---|
| 3.64887 | 0.000694615 | 0.118182 | 0.000465696 |
| 4.30649 | 0.000827692 | 0.0926474 | 0.000554915 |
| 4.40684 | 0.000904275 | 0.0897946 | 0.000606259 |
| 2.96167 | 0.000404666 | 0.224867 | 0.000271303 |
| 3.11942 | 0.0004011 | 0.226007 | 0.000268912 |
| 3.13563 | 0.000374671 | 0.221207 | 0.000251193 |
| 2.99919 | 0.000384765 | 0.269566 | 0.000257961 |
| 3.06204 | 0.000885342 | 0.239614 | 0.000593565 |
| 2.99187 | 0.000376882 | 0.259445 | 0.000252675 |
| 3.03623 | 0.000498673 | 0.234725 | 0.000334328 |
| 3.21928 | 0.000435422 | 0.307166 | 0.000291923 |
| 3.26257 | 0.000425353 | 0.310185 | 0.000285172 |
| 3.44726 | 0.000384996 | 0.293434 | 0.000258116 |
| 3.3285 | 0.000442646 | 0.241911 | 0.000296766 |
| 3.45747 | 0.000375536 | 0.294856 | 0.000251773 |

**Table 2**

## 4. Bunch intensity vs time

Bunch intensity vs time can be obtained from the SBD. The data here come from two sources. The first is from a careful analysis of the SBD raw data that was recorded every 10 minutes. There is a very small baseline correction that can be obtained by examining the base in front of the bunches. In addition, the bunches have not had time to spread appreciably and so I have integrated over just $+/-$ 7 ns around the center of the bucket. A plot of the data is below and one can see the blow up of bunches 8, 10, and 14. The lifetime in the machine due to gas interactions is of the order of 1000 hours and the test lasted about 3 hours, so one is looking for intensity changes of the order of a few tenths of a percent.

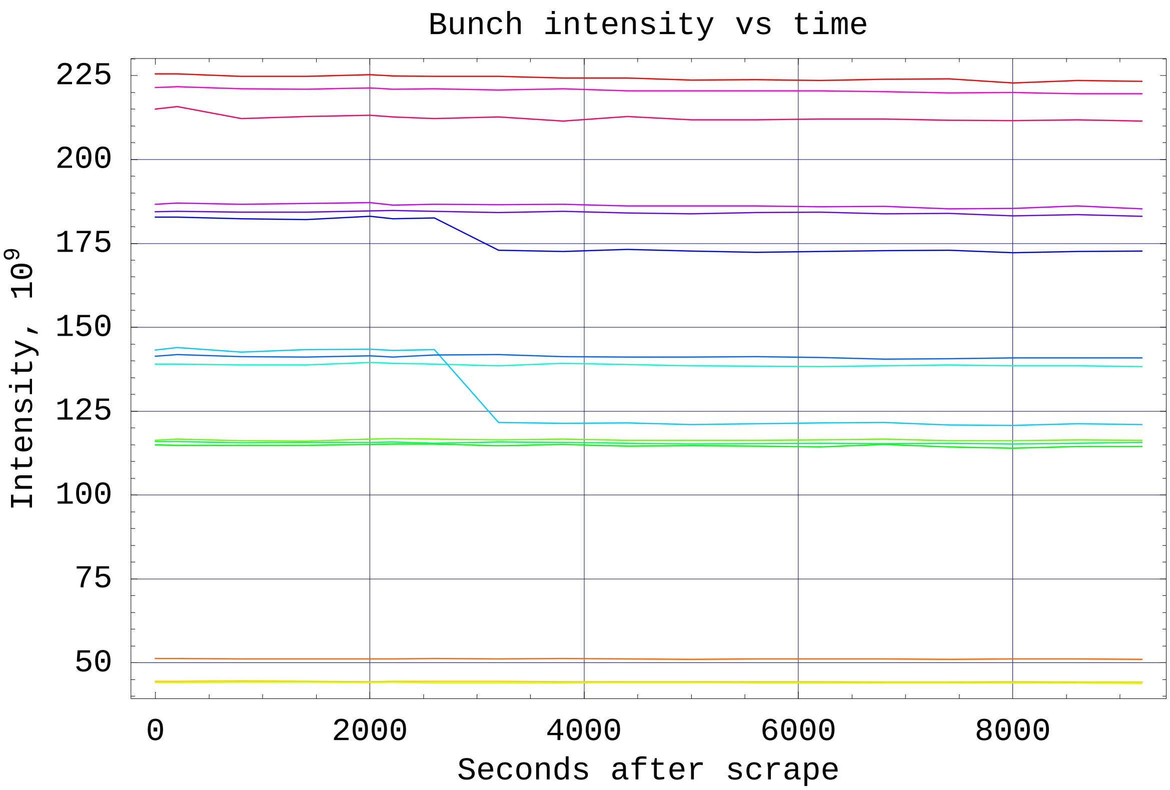


**Figure 3**

An additional source of data is provided by using the Array Plotter on D44 and obtaining the processed bunch intensities, T:SBDPIS. Since these are provided every second, the statistical accuracy is greater than the above. The raw scope traces are not available, so a comparison of this data and the above can be viewed as a check on the internal SBD analysis. A time plot of the 15 bunches obtained from T:SBDPIS is shown below.

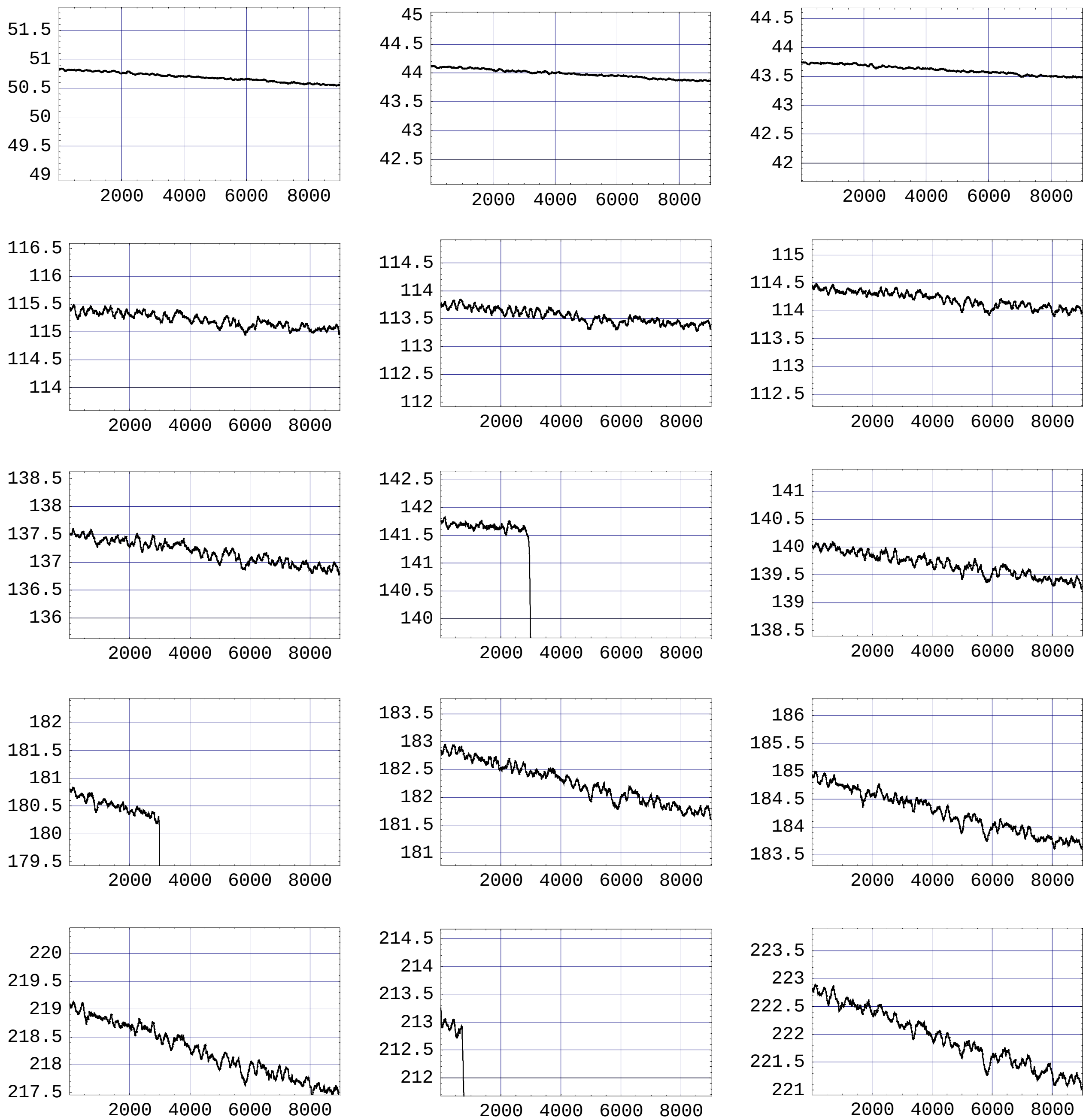
51.5 51 50.5 50 49.5 49
2000 4000 6000 8000
45 44.5 44 43.5 43 42.5
2000 4000 6000 8000
44.5 44 43.5 43 42.5 42
2000 4000 6000 8000
116.5 116 115.5 115 114.5 114
2000 4000 6000 8000
114.5 114 113.5 113 112.5 112
2000 4000 6000 8000
115 114.5 114 113.5 113 112.5
2000 4000 6000 8000
138.5 138 137.5 137 136.5 136
2000 4000 6000 8000
142.5 142 141.5 141 140.5 140
2000 4000 6000 8000
141 140.5 140 139.5 139 138.5
2000 4000 6000 8000
182 181.5 181 180.5 180 179.5
2000 4000 6000 8000
183.5 183 182.5 182 181.5 181
2000 4000 6000 8000
186 185.5 185 184.5 184 183.5
2000 4000 6000 8000
220 219.5 219 218.5 218 217.5
2000 4000 6000 8000
214.5 214 213.5 213 212.5 212
2000 4000 6000 8000
223.5 223 222.5 222 221.5 221
2000 4000 6000 8000

**Figure 4**

The data above have been fit to a linear decay. The results for the 15 bunches are given in the following table along with the standard errors. Again, disregard bunches 8, 10, and 14.

**VALUES AND ERRORS FOR N(t) LINEAR FITS**

| N(t=0) protons | SE on N | Slope $10^9$/hour | SE on slop |
|---|---|---|---|
| 50.8307 | 0.0012034 | -0.112938 | 0.00080683 |
| 44.1178 | 0.00124647 | -0.103873 | 0.00083570 |
| 43.7496 | 0.00136171 | -0.110243 | 0.00091296 |
| 115.404 | 0.00596495 | -0.152777 | 0.00399926 |
| 113.74 | 0.00594567 | -0.155746 | 0.00398633 |
| 114.421 | 0.00558924 | -0.176871 | 0.00374736 |
| 137.513 | 0.00655971 | -0.266569 | 0.00439802 |
| 141.416 | 0.120943 | -11.1707 | 0.0810875 |
| 140.017 | 0.00670075 | -0.269262 | 0.00449259 |
| 180.575 | 0.0508364 | -4.99913 | 0.0340837 |
| 182.86 | 0.00757865 | -0.480372 | 0.00508118 |
| 184.877 | 0.00763544 | -0.497137 | 0.00511926 |
| 219.029 | 0.0082763 | -0.631313 | 0.00554893 |
| 211.406 | 0.0138087 | -0.965352 | 0.00925819 |
| 222.787 | 0.00832298 | -0.68672 | 0.00558023 |

**Table 3**

The data from a direct analysis of the raw scope data shown above in Figure 3 is completely consistent with the data in Table 3. The lifetime in hours is for the bunches (grouped by 3) is:

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 450.074 | 424.727 | 396.846 |
| 4 | 5 | 6 |
| 755.378 | 730.289 | 646.92 |
| 7 | 8 | 9 |
| 515.861 | 12.6596 | 520.003 |
| 10 | 11 | 12 |
| 36.1213 | 380.663 | 371.885 |
| 13 | 14 | 15 |
| 346.942 | 218.993 | 324.422 |

**Table 4**

The behavior is a little curious and we will come back to this at the end.

## 5. Calculation of the "factor"

The time behavior of the transverse and longitudinal emittances is given by:

$$\frac{dE_T}{dt} = \gamma^{3/4}\, C_T\, \frac{N_p}{E_T^{1.5}\, E_L^{0.5}} \tag{1}$$

$$\frac{d\sigma_z^2}{dt} = \gamma^{-1/4}\, C_L\, \frac{N_P}{E_T^{1.5}\, E_L^{0.5}} \tag{2}$$

We will refer to the last factor in the above equations as "fac". The Touschek scattering depends on the same variable and so the bunch intensity decay should have a term proportional to this factor as well as a constant term from RF noise and from gas interactions.

Since the bunch is evolving with time, fac is a function of time and in principle,our fits should not be linear, but should have a $t^2$ term included. I have looked for this in the bunch width growth and it is too small to see. To take in to account the change of fac with time, we will use the average value of fac during the store. Since the values for $N_p$ , $E_T$ , and $E_L$ as a function of time are known from the fits above, we can easily calculate the function fac($t$) by substituting in equation 1 and 2.

## 6. $\tau^2$ vs fac

First we show the fit of the square of the bunch width vs fac. Bunches 8, 10, and 14 have been eliminated.

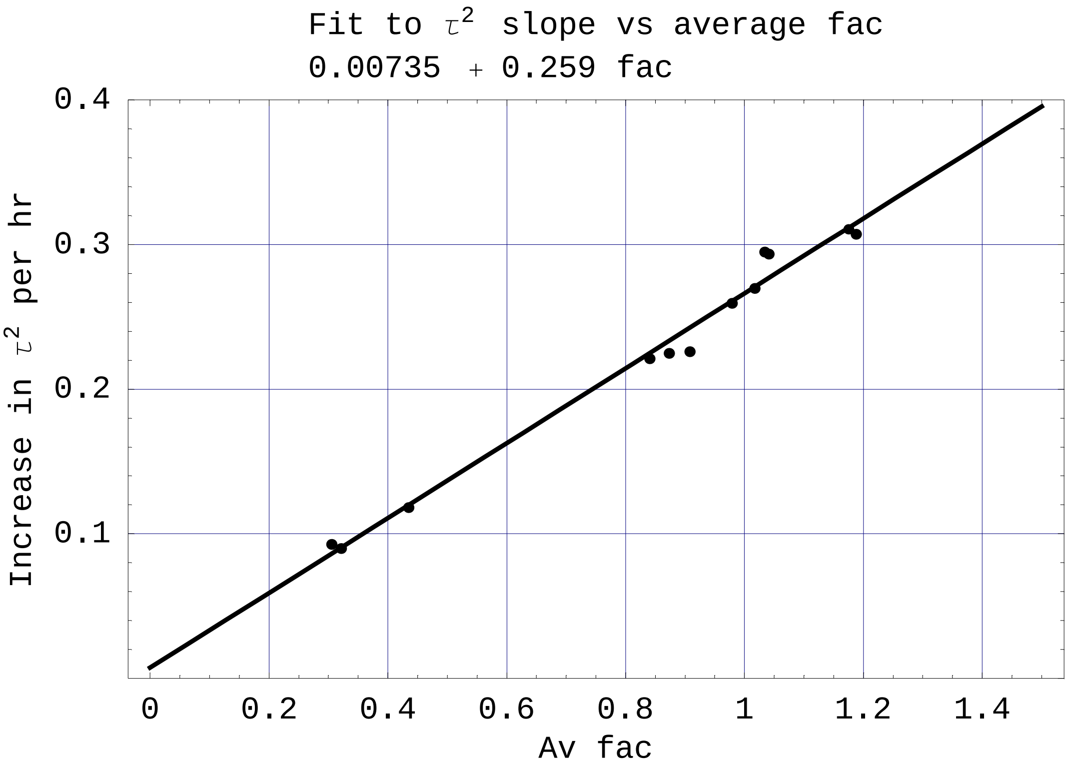


**Figure 5**

The fit is given by the equation:

$$\frac{d\tau^2}{dt} = 0.00735^{+.009}_{-.009} + 0.259^{+.01}_{-.01} \text{ fac ns}^2 / \text{hour} \tag{3}$$

The errors shown are for the fit only, that is the errors on the slope of $\tau^2$ are not included as they are so small. See Table 2.

## 7. $\epsilon_y$ vs fac

The fit of the transverse y emittance is shown below:

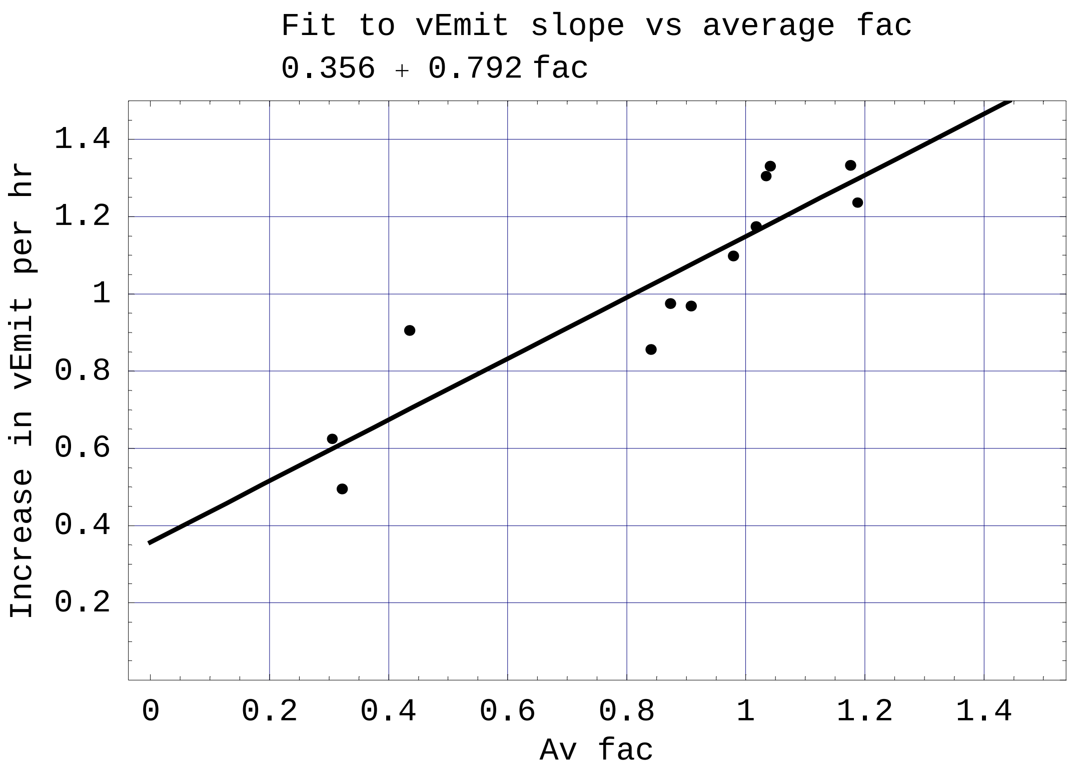


**Figure 6**

The fit is given by the following equation

$$\frac{d\epsilon_y{}^2}{dt} = 0.356^{+0.1}_{-0.1} + 0.792^{+0.12}_{-0.12}\ \text{fac}\ \ \pi\,\text{mm mr} / \text{hour} \quad (4$$

The errors would be a little larger if the errors on the slope of $\epsilon_y$ were included.

# 8. $\frac{dN_P}{dt}$ vs fac

The rate of loss of protons per hour is shown below.

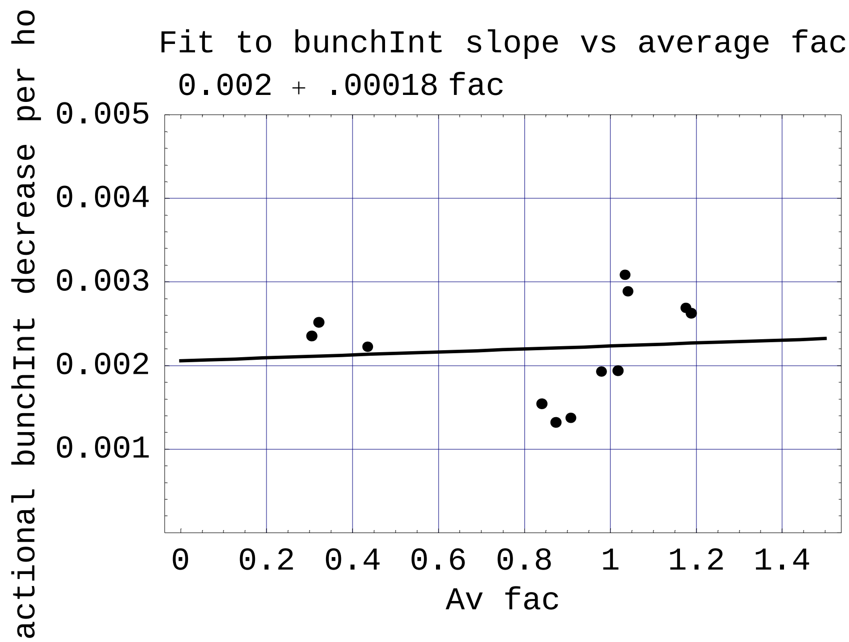


**Figure 7**

This data set is the worst of the three. The run was about 2.5 hours long. We know from uncoalased beam studies that the vacuum lifetime is 1000 hours or greater. Thus the intercept in the plot above of .002 + / − .0005, which corresponds to 500 hours seems to be at variance with other measurements. I have looked at the base line variations and other features of the SBD and could not find any thing that seems to indicated trouble with the SBD. The accuracy of the measured slopes are much smaller than variations in the above plot suggest. As will be shown below, the Touschek lifetime is very long which correctly accounts for the flatness of the fitted curve.

So there are two mysteries:

1. Why is the indicated vacuum life time so short. Was the vacuum really that bad at the time of the measurements? The long lifetimes are measured with uncoalesed beam using the DCCT. The SBD drifts away from T:BEAM during a store about 1%. The store lifetime is short, so the effect is not large, but if the same effect is going on here where the lifetime is 500 hours, it could be important. For instance, a gas lifetime of 1000 hours combined with an instrumental error lifetime of 1000 hours would combine to give the observed 500 hour lifetime.
2. What is causing the large fluctuations in the measurements?

## 9. Relation between longitudinal and transverse emittance growth.

Lebedev[1]] gives the relation connecting transverse and longitudinal emittance growth when IBS is the cause. For an uncoupled machine, the vertical emittance growth is essentially zero. For a coupled machine we will use the following equations:

$$\frac{d\epsilon_x}{dt} = (1-\kappa)\, A_x\, \frac{d\delta p_z^2}{p^2\, dt} \tag{5}$$

$$\frac{d\epsilon_y}{dt} = \kappa\, A_x\, \frac{d\delta p_z^2}{p^2\, dt} \tag{6}$$

The only emittance data we have is for the y direction and so we will assume the above relations and calculate $\kappa$. $A_x$ is 0.190 meters. Using this number and the measured growth rates given in Eq(3) and Eq(4), we find that $\kappa$ is 0.41.

## 10. Longitudinal emittance growth from RF noise.

The intercept in Eq(3), 0.00735 $ns^2$/ hr, represents the bunch growth due to other causes than IBS. We can attribute this to phase noise on the RF and changing units in Eq(3), we have 0.82 e-3 $rad^2$ /hour as the measured growth of the beam. This is to be compared with the analysis in Ref. [1] of 1.87 e-3 $rad^2$/hour.

## 11. Transverse emittance growth from gas scattering.

The intercept in Eq[4] gives the transverse emittance growth due to things other than IBS. The most obvious sourse is coulomb scattering in the residual gas in the beam tube. An empirical fit has been made to the interaction cross section of the various elements listed in the PDG from Hydrogen up thru Argon as a function of A, the atomic weight. Using this information and the gas laws, a plot can be made of average beam tube pressure vs lifetime for the various elements. This is shown in Fig.5

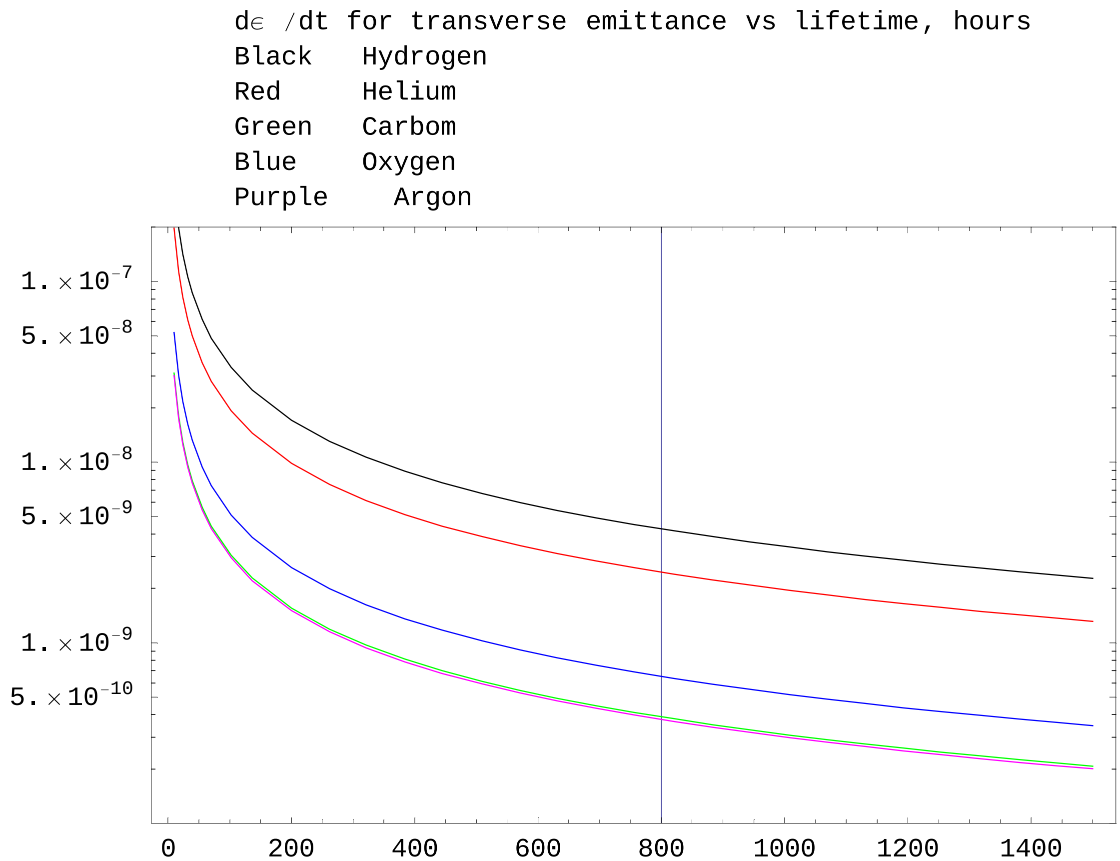


**Figure 8**

The vertical line is drawn for a lifetime of 800 hours. Note that this is not the number measured in Section 8. However, the recently measured lifetimes using T:BEAM for both bunched and unbunched beams is over 1000 hours. Next, we can use radiation length $X_o$ as given in the PDG to calculate the emittance growth for the same elements given above. A convenient plot is one of transverse emittance growth vs lifetime for the various elements. This is shown in Fig. 9.

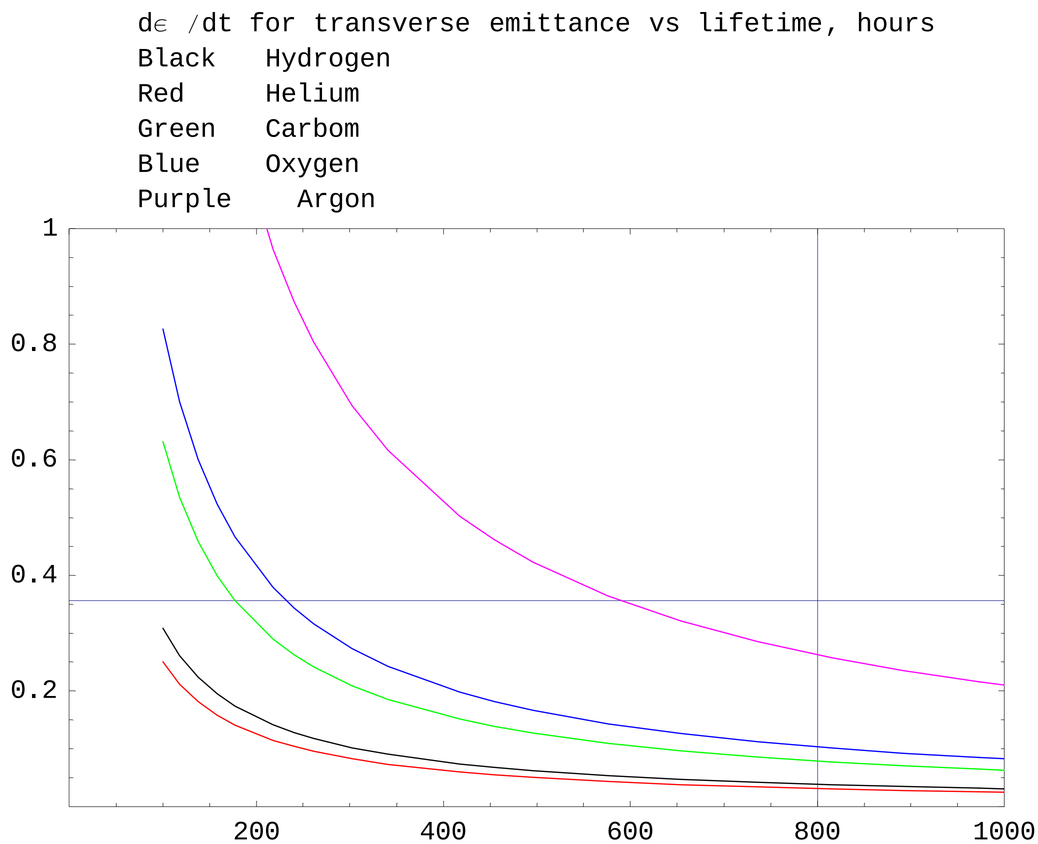


**Figure 9**

The horizontal line is at 0.356 mm mr/hr given by the intercept of Eq.[4]. The point marked by the intersection of the horizontal and vertical lines characterizes the A and Z of the residual gas. Argon is the only gas that comes close the satisfying the measurements. I believe that the lifetime is actually longer and so the disagreement ay be larger. This difference would indicate that there is an additional source of transverse noise. If the measurements could be increased in accuracy, this could be a valuable tool for diagnosing magnet noise induced transverse growth.

## 12. Touschek scattering.

I have calculated the Touschek scattering lifetimes for the various bunches in this study. I have used the formulas in Wiedemann[2]. The values are rather large. I have divided the calculated values by the factor, fac, given in Section 5. This should result in a constant if the Touschek scattering depended strictly on fac. As can be seen below, this is approximately true. Although all three processes we consider depend on the phase space density in the same way, there are some additional factors that depend on other features of the machine that I believe lead to the differences observed here.

| Bunch | lifetime,hours |
|---|---|
| 1 | $18.\times 10^3$ |
| 2 | $23.\times 10^3$ |
| 3 | $21.\times 10^3$ |
| 4 | $11.\times 10^3$ |
| 5 | $9.8\times 10^3$ |
| 6 | $11.\times 10^3$ |
| 7 | $8.8\times 10^3$ |
| 8 | $15.\times 10^3$ |
| 9 | $9.2\times 10^3$ |
| 10 | $11.\times 10^3$ |
| 11 | $6.9\times 10^3$ |
| 12 | $6.8\times 10^3$ |
| 13 | $7.1\times 10^3$ |
| 14 | $10.\times 10^3$ |
| 15 | $7.2\times 10^3$ |

**Table 5**

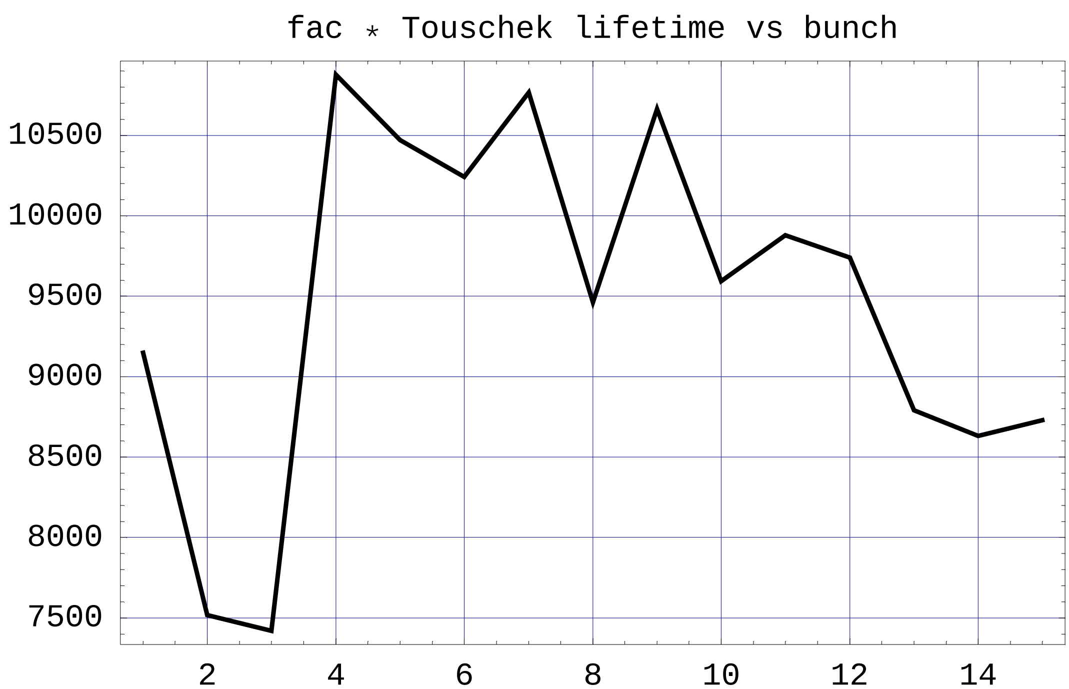


**Figure 10**

## References

1. Valerie Lebedev RunII meeting Dec 16, 2004. BeamDoc 1509-v1
2. Particle Accelerator Physics II, H. Wiedmann pp328-330*e*