LPI Design Note
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Calculated Doubler Position Detector Response to Centered Beam

The purpose of this note is to calculate the amplitude of the induced signal on the pickup electrode for a centered beam of 1×10^{10} protons per bucket in batch mode.

Average and Peak Beam Current.

Suppose the beam bunch may be represented by a Gaussian shape

$$I_b(t) = I_o \exp\left(\frac{-t^2}{2\sigma^2}\right) \qquad (1)$$

where σ = RMS bunch width. Then the charge Q per bunch is

$$Q_b = \int_{-\infty}^{\infty} I_b(t)\,dt = I_o \int_{-\infty}^{\infty} \exp\left(\frac{-t^2}{2\sigma^2}\right) dt = \sqrt{2\pi}\,\sigma\, I_o \qquad (2)$$

If the repitition rate (i.e inverse of bunch spacing) is $\frac{\omega_o}{2\pi}$, then the average current for a full ring is

$$< I_b > = \sqrt{2\pi}\,\sigma\, I_o \frac{\omega_o}{2\pi} = \frac{\sigma\,\omega_o I_o}{\sqrt{2\pi}} \qquad (3)$$

Current and Voltage Pulse from Pickup Electrode

When the beam bunch approaches the upstream end of the pickup electrode, a charge $-FQ_b$ is induced on the inside surface of the electrode. The fraction F represents approximately the azimuthal fraction of the beam pipe covered by the electrode.

As the electrode is not grounded, no net charge can be induced on it. Hence on the outside of the plate, a charge $+FQ_b$ is generated. Assuming both ends of the electrode are coupled to transmission lines which have the same characteristic impedance as the electrode, this induced charge divides equally between two electromagnetic waves, one travelling down along the outside of the electrode at the same velocity as the beam, and the other out the upstream port along the transmission line. A similar effect occurs at the downstream end. However there is complete cancellation of the signal travelling out the downstream port (ideally). The induced travelling upstream along the electrode and eventually out the upstream port is the same amplitude but opposite polarity to the original signal, and is delayed with respect to the first one by $2t_0$ where t_0 is the electrical length of the pickup electrode. Hence the current signal observed at the upstream port is:

$$I_{pu}(t) = \frac{F}{2}\left[I_b(t+t_0) - I_b(t-t_0)\right]$$

$$= \frac{FI_0}{2}\left\{\exp\left[\frac{-(t+t_0)^2}{2\sigma^2}\right] - \exp\left[\frac{-(t-t_0)^2}{2\sigma^2}\right]\right\} \qquad (4)$$

where t_0 is the electrical length of the pickup.

If the electrode and the transmission lines have a characteristic impedance Z_0, then the observed output voltage is:

$$V_{pu}(t) = \frac{Z_0 F I_0}{2}\left\{\exp\left[\frac{-(t+t_0)^2}{2\sigma^2}\right] - \exp\left[\frac{-(t-t_0)^2}{2\sigma^2}\right]\right\} \qquad (5)$$

Fourier Sine Transform of Pulse.

If the bipolar doublet signal occurs at regular intervals, then it can be expressed as a sum of harmonics of the fundamental frequency using a Fourier sine series. Suppose.

$$V_{pu}(t) = Z_o\, I_{pc} + \sum_{n=1}^{\infty}{}' V_n \sin n\omega_o t \qquad (6)$$

then

$$\int_{-\pi}^{+\pi} V_{pu}(t) \sin m\omega_o t \; d(\omega_o t) = \int_{-\pi}^{+\pi} \sin m\omega_o t \sum_{n=1}^{\infty} V_n \sin n\omega_o t \; d(\omega_o t)$$

$$= \sum_{n=1}^{\infty} V_n \int_{-\pi}^{\pi} \sin m\omega_o t \sin n\omega_o t \; d(\omega_o t)$$

$$= \pi V_m \qquad (7)$$

Hence from (7) and (5)

$$V_m = \frac{Z_o F I_o}{2\pi} \int_{-\pi}^{\pi} \left\{ \exp\left[-\frac{(t+t_o)^2}{2\sigma^2}\right] - \exp\left[-\frac{(t-t_o)^2}{2\sigma^2}\right] \right\} \sin m\omega_o t \; d(\omega_o t) \qquad (8)$$

As long as the Gaussian pulse is short compared to the integration range, this integral may be rewritten

$$V_m = \frac{Z_o F I_o}{2\pi} \int_{-\pi}^{\pi} \exp\left\{\frac{-t^2}{2\sigma^2}\right\} \left\{ \sin m\omega_o (t+t_o) - \sin m\omega_o (t-t_o) \right\} d(\omega_o t) \qquad (9)$$

but

$$\sin m\omega_o (t+t_o) - \sin m\omega_o (t-t_o) = \sin m\omega_o t \cos m\omega_o t_o + \cos m\omega_o t \sin m\omega_o t_o$$
$$- \sin m\omega_o t \cos m\omega_o t_o + \cos m\omega_o t \sin m\omega_o t_o$$

$$= 2 \sin(m\omega_o t_o) \cos(m\omega_o t) \qquad (10)$$

Equation (9) may now be rewritten:

$$V_m = \frac{Z_o F I_o}{\pi} \sin(m\omega_o t_o) \int_{-\pi}^{\pi} \exp\left(\frac{-t^2}{2\sigma^2}\right) \cos(m\omega_o t)\, d(\omega_o t)$$

Extending the integration range from $\pm\pi$ to $\pm\infty$ allows performing the integration:

$$V_m = \sqrt{\frac{2}{\pi}}\, Z_o F I_o \omega_o \sigma \sin(m\omega_o t_o) \exp\left[\frac{-m^2\omega_o^2\sigma^2}{2}\right] \qquad (12)$$

this may be rewritten in terms of the average current eqn (3):

$$\boxed{V_m = 2 Z_o F \langle I_b \rangle \sin(m\omega_o t_o) \exp\left[-\frac{m^2\omega_o^2\sigma^2}{2}\right]} \qquad (13)$$

Application to Doubler

(see IEEE NS # 28, page 2290 (1981) for description of Doubler pickup)

We now use eqn (13) to calculate the signal amplitude $V_m(\sigma)$ (peak voltage) for the $m=1$ harmonic of 53 MHz for 1×10^{10} ppb. The parameters are

$m = 1$ $\quad (m\omega_o = 2\pi \times 53$ MHz$)$

$Z_o = 50$ ohms

$F = 110^\circ/360^\circ = 0.300$ (geometrical)

$t_o = 19\text{cm}/3\times10^{10}\text{cm/sec} = 6.33\times10^{-10}$ sec

$\sigma = 1$ nsec $\quad$ (FWHM = 2.36 nsec)

$\langle I_b \rangle = (1\times10^{10}) \times (1.6\times10^{-19}) \times 53.1\text{ MHz} = 85.0$ milliamps

$V_m(\sigma)$ is plotted on next page using above values of Z_o, F, $\langle I_b \rangle$, t_o, and ω_o, for various values of m and σ.

$$\therefore \quad V_1 = 2 Z_o F \langle I_b \rangle \sin(m \omega_o t_o) \exp\left[\frac{-m^2 \omega_o^2 \sigma^2}{2}\right]$$

$$= 2 \times 50 \times .3 \times 85 \times 10^{-3} \sin(2\pi \times 53.1 \times 10^6 \times 6.33 \times 10^{-10}) \times \exp\left[-\frac{(2\pi \times 53.1 \times 10^6 \times 10^{-9})^2}{2}\right]$$

$$= 0.535\, e^{-.0556} = 0.506 \text{ volts} \qquad (14$$

The total power output is $P(\sigma) = \sum_m P_m(\sigma) = \sum_m \frac{V_m^2(\sigma)}{Z_o}$ (15)

For the signal in eqn (14), the power in the 53 MHz component is

$$P_1 = \frac{1}{2} \frac{V_1^2}{Z_o} = 2.56 \text{ mW} = +4.1 \text{ dbm} \qquad (16)$$

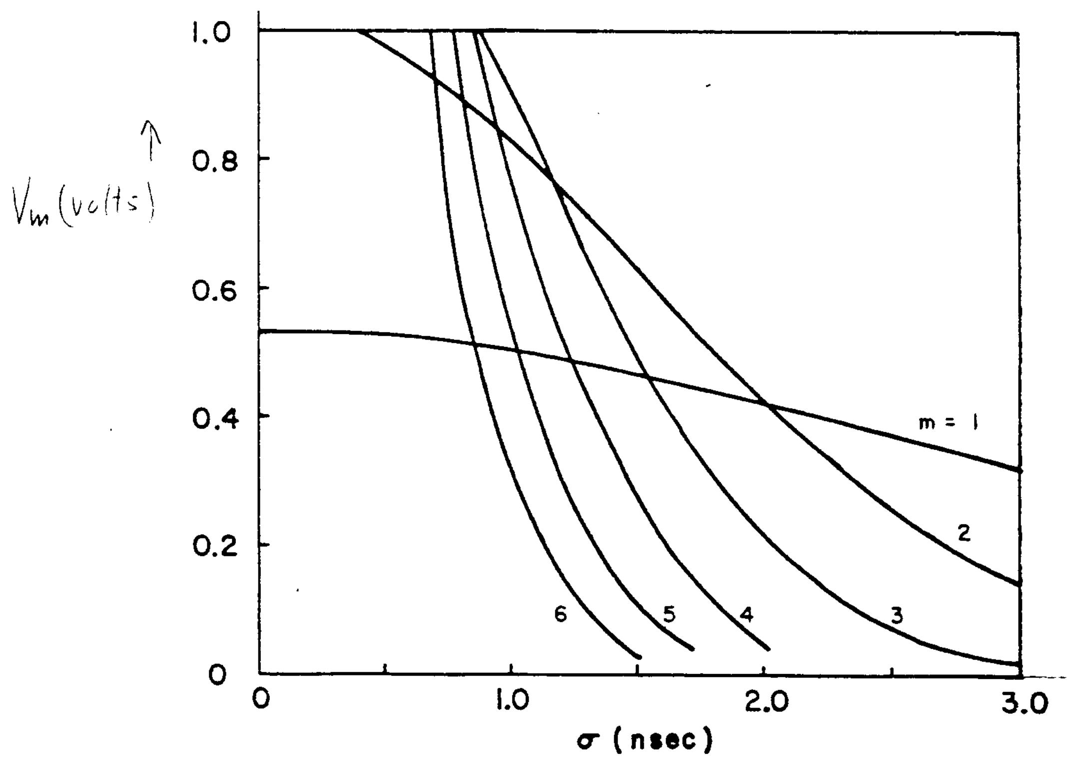

Figure 1

$V_m(\sigma)$ vs σ for Dwner detector for 1×10^{10} pro.