Evaluation and correction of nonlinear effects in FNPL beam position monitors

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Abstract

It was recently recognized that FNPL beam position monitors (BPMs) have a strong nonlinear response when the beam is at large offset with respect to the electric axis of the BPM [3]. This nonlinearities spoil, for instance, difference orbit type measurements and need to be corrected. We present the correction implemented for the on-line orbit display software and compare with off line correction based on Reference [4].

1 Nonlinear response of FNPL BPMs

FNPL is equipped with 8 (and soon 12) beam position monitors (BPMs) [1]. The beam position monitors are of capacitive type: four button-type electrodes located 90° apart from each other are used to measured the electric field induced by the electron bunch (see Fig. 1). The potential angular distribution for an off-centered charge [2]:

$$\Phi(\rho,\theta) = \Phi_0 \frac{a^2-\rho^2}{a^2+\rho^2-2a\rho\cos(\theta-\theta_0)}, \tag{1}$$

where Φ_0 is a constant, $\rho \equiv (x^2+y^2)^{1/2}$ is the radial position of the charge, a the BPM vacuum chamber radius and the angles θ and θ_0 are defined in Fig.1. At FNPL, the beam position is calculated from the induced potential $\Phi_{L,R,B,T}$ on the left, right, bottom and top electrodes via:

$$\begin{aligned} x_{bpm} &= k_x \frac{\Phi_R-\Phi_L}{\Phi_R+\Phi_L}, \text{ and,} \\ y_{bpm} &= k_y \frac{\Phi_T-\Phi_B}{\Phi_T+\Phi_B}, \end{aligned} \tag{2}$$

where $k_{x,y}$ are calibration constants. Let's consider the horizontal position and assume the buttons are point-like. We have

$$\Phi_R \pm \Phi_L = \Phi_0(a^2-\rho^2)\frac{(a^2+\rho^2+2a\rho\cos\theta_0) \pm (a^2+\rho^2-2a\rho\cos\theta_0)}{(a^2+\rho^2-2a\rho\cos\theta_0)(a^2+\rho^2+2a\rho\cos\theta_0)}, \tag{3}$$

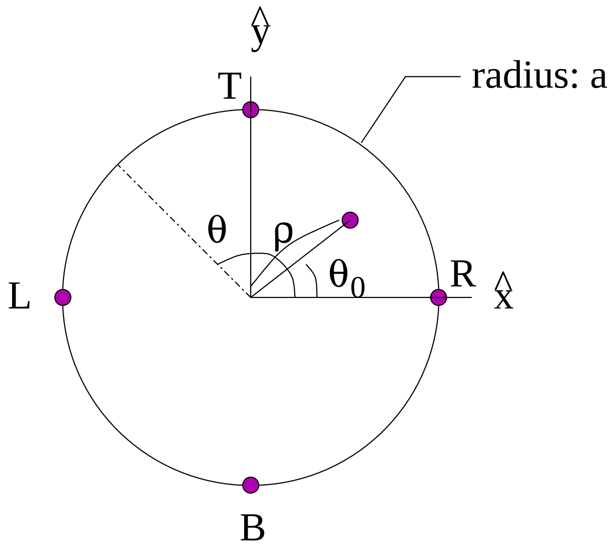

Figure 1: overview of BPM coordinate system. The letters T, B, L, and R represents the locations of the top, bottom, left, and right electrodes.

so that

$$\frac{x}{1+\frac{x^2+y^2}{a^2}} = \frac{a}{2}\frac{\Phi_R-\Phi_L}{\Phi_R+\Phi_L} \equiv x_{bpm}. \tag{4}$$

Thus if $x \ll a$ we have $x_{bpm} \simeq x$. In case of large offset ($x \ll\ a$ not satisfied) we must write

$$\frac{\Phi_R-\Phi_L}{\Phi_R+\Phi_L} \simeq \frac{2}{a}x\left(1-\frac{x^2+y^2}{a^2}+...\right). \tag{5}$$

To evaluate the importance of the nonlinear contribution in the BPM response (Eq. 5), we follow the technique used at CERN and DESY [5]. The BPM is treated as a two-dimensional electrostatic problem with POISSON [6] using the exact geometry. One antenna, e.g. the top antenna, is excited to a potential Ψ_0 and the corresponding potential within the beam pipe $\Psi_T(x,y)$ is computed (see Fig.2). By rotation we can infer the potentials $\Psi_{L,R,B}$ induced when the left, right and bottom electrodes are excited to the potential Ψ_0. Conversely, given a charge with transverse coordinate (x,y) we can compute the induced potentials on each electrode. From the calculated potentials we can infer the beam position (either from the linear approximation of Eq. 5 or from more elaborated algorithm). The simulations are done for a series of charge located on a grid $\{x,y\} \in \{[-2,2],[-2,2]\}$ cm (see Fig. 3) and the results (Fig. 3 right) indicate the linear approximation, nominally used in the on-line orbit display at FNPL, is valid only when the beam offset is less than 5 mm. It is however common, e.g. during difference orbit measurements, to have orbit excursions at offsets of $\sim$1 cm.

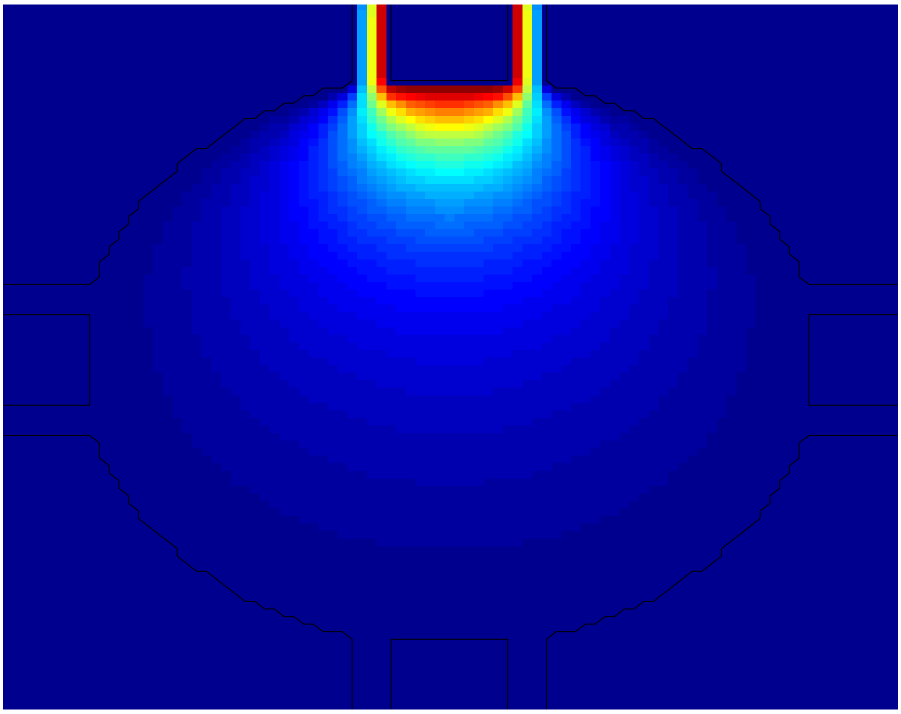

Figure 2: Potential induced in the beam pipe when the top electrode is excited to a potential; two dimensional electrostatic calculations performed with POISSON.

2 Including nonlinear BPM response

In order to include the nonlinear response in the algorithm that calculates the beam position given the potentials on the four electrodes, two methods are considered:

- use of a minimization algorithm as described in Ref. [4],
- approximate the BPM response by a high order polynomial [5].

The first technique is very precise but requires an interpolation of the potential obtained from POISSON and a minimization algorithm. Given the induced potential on the four electrode $\Phi^R_{L,R,T,B}$ we must search for the triplet (x, y, λ) so that the quantity

$$\begin{aligned}\chi^2 \quad &\equiv (\lambda\Psi_L(x,y) - \Phi^R_L)^2 + (\lambda\Psi_R(x,y) - \Phi^R_R)^2 + \\ &(\lambda\Psi_T(x,y) - \Phi^R_T)^2 + (\lambda\Psi_B(x,y) - \Phi^R_B)^2 \end{aligned} \tag{6}$$

is minimized. This technique was implemented using MATLAB and, as expected, was found to be slow (because of the required interpolation and minimization) but precise over the full area of interest as illustrated in Fig. 4. It is well suited for precise off-line analysis of the BPM data.

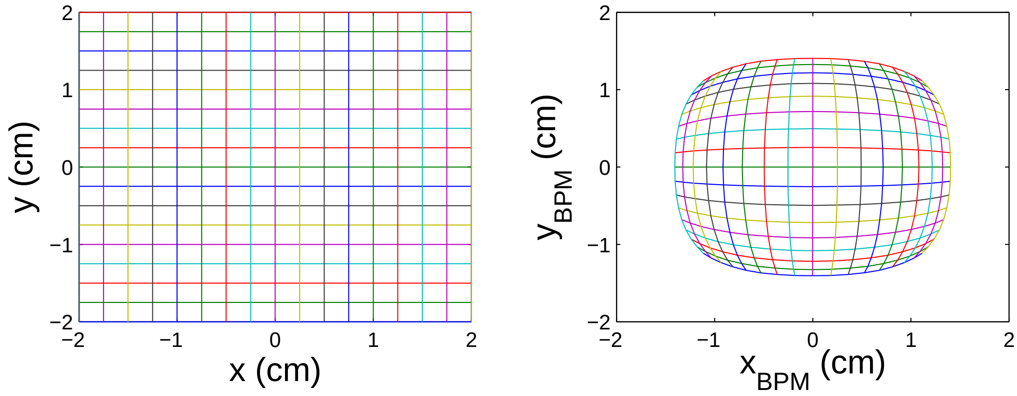

Figure 3: Initial particle position distribution (left) and corresponding BPM readback position using a linear response for the BPM (right).

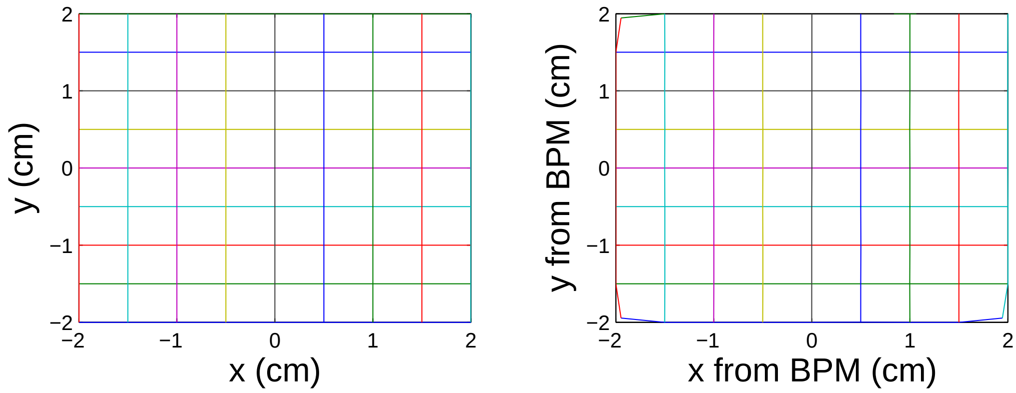

Figure 4: Initial particle position distribution (left) and corresponding BPM readback position computed using the minimization technique of Ref. [4](right).

The second technique was also tested. When the linear response of the BPM is used a beam with offset (x, y) in real space appear to have an offset (x_{bpm}, y_{bpm}) from the BPM readback. We seek a set of polynomials P and Q to correct the BPM readback, i.e. so that $P(x_{bpm}, y_{bpm}) = x$ and $Q(x_{bpm}, y_{bpm}) = y$. It is straightforward to show that $Q(x, y) = P(y, x)$ because of the symmetries. The polynominal P is written as $P(x, y) = \sum_{i=1}^{n} \alpha_i x^i \sum_{j=1}^{n} \beta_j y^j$ and a set of initial position and corresponding linear BPM readback (see Fig. 5 top row) are used to obtain the coefficients of P from a two-dimensional fit. In order to obtain a difference of less than 100 μm between the computer and original beam position over the area of interest we need $n = 11$. The corresponding results are summarized in Fig. 5 (bottom row). This method is very fast and can be implemented in the on-line orbit display.

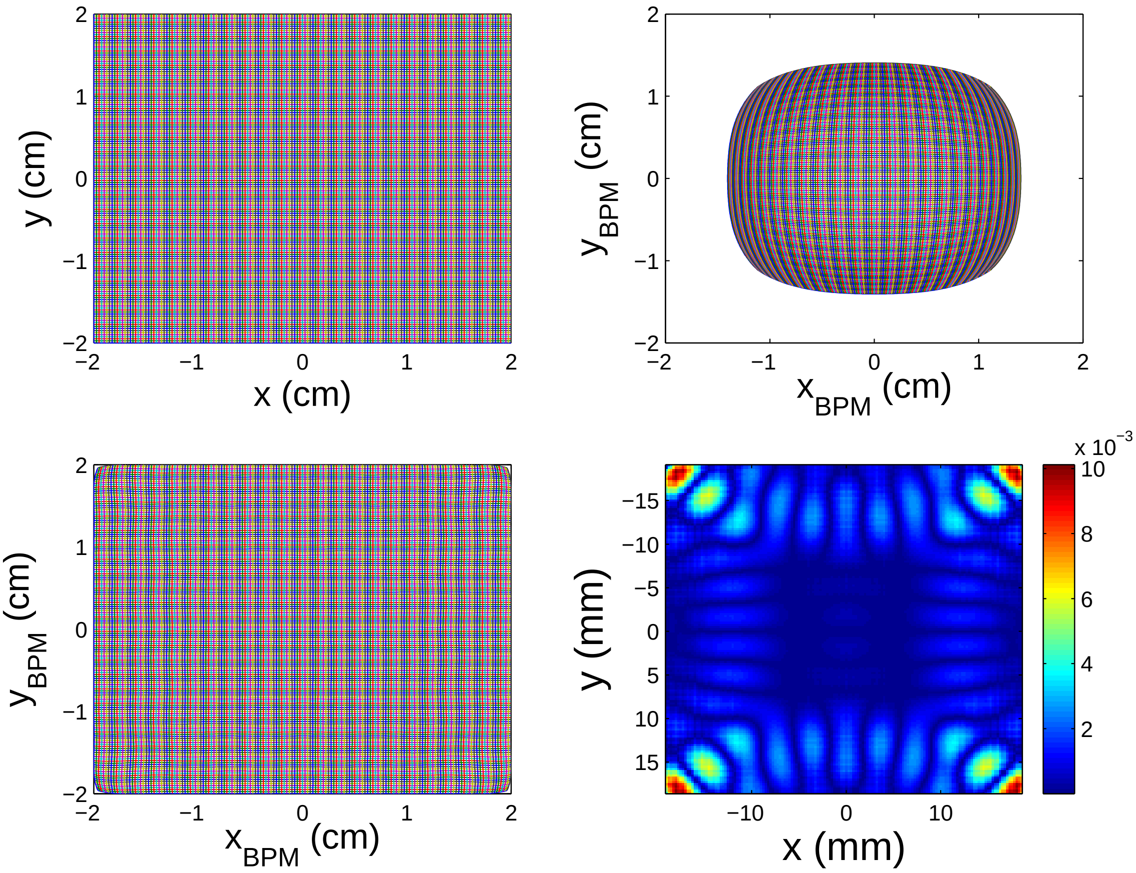

Figure 5: Initial particle position distribution (upper left), corresponding BPM readback position using a linear response for the BPM (upper right), correction of the BPM linear response using a 11th degree polynominal (bottom left) and (bottom right) difference between original and corrected positions [defined as $\delta_x(x, y) = |x - P(x, y)|$ the same error function is observed for the y-position (units for the false color plot are cm)]

References

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