

Fourier Analysis of MI TBT data

E.Gianfelice

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Beam position at the j -th BPM after a *single* kick:

$$z_n^j = \sqrt{\beta_z^j} e^{i\Phi_z^j} A_z e^{iQ_z(\theta_j + 2\pi n)} + c.c. \quad (z = x, y)$$

$$n \equiv \text{turn number} \quad A_z = |A_z| e^{i\delta_z} \equiv \text{constant of motion}$$

$$\Phi_z \equiv \int_0^\theta d\theta' \frac{R}{\beta_z} - Q_z \theta \quad (\text{periodic phase function})$$

Twiss functions may be computed through a Fourier analysis:

$$Z_j(Q_z) \equiv \text{Fourier component of } z_j$$

$$\beta_z^j = |Z_j(Q_z)|^2 / A_z^2 \quad \mu_z^j = \arg(Z_j) - \delta_z$$

Amplitude fit:

$$|A_z|^2 = \frac{\sum_j 1/\beta_{0z}}{\sum_j 1/|Z_j(Q_z)|^2}$$

TBT measurement goal for MI (besides optics measurement):

- measure dependence of tunes and phase advance upon bunch current
- measure dependence of closed orbit upon bunch current

D.Brandt et al.,
PAC95

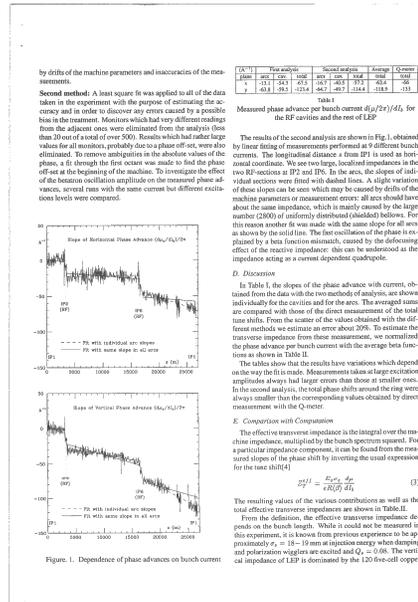


Figure 1. Dependence of phase advances on bunch current

| Section | Horizontal phase advance slope (deg/A) | | Vertical phase advance slope (deg/A) | |
|---------|--|-------------|--------------------------------------|-------------|
| | 0.3 e12 | 1.2 e12 | 0.3 e12 | 1.2 e12 |
| IP1 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP2 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP3 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP4 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP5 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP6 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP7 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP8 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP9 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP10 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP11 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP12 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP13 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP14 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP15 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP16 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP17 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP18 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP19 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP20 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP21 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP22 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP23 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP24 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP25 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP26 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP27 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP28 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP29 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP30 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP31 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP32 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP33 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP34 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP35 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP36 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP37 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP38 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP39 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP40 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP41 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP42 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP43 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP44 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP45 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP46 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP47 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP48 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP49 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |
| IP50 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 | 0.11 ± 0.01 |

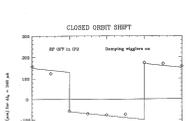


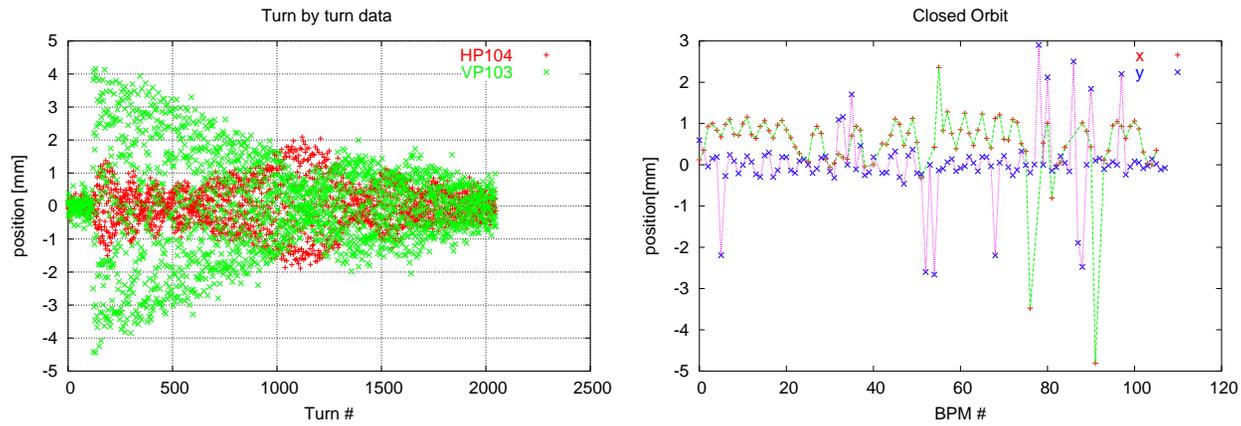
Figure 2. Difference orbits for 2 values of current

Two data sets are here analysed (30 bunches, machine status 15)

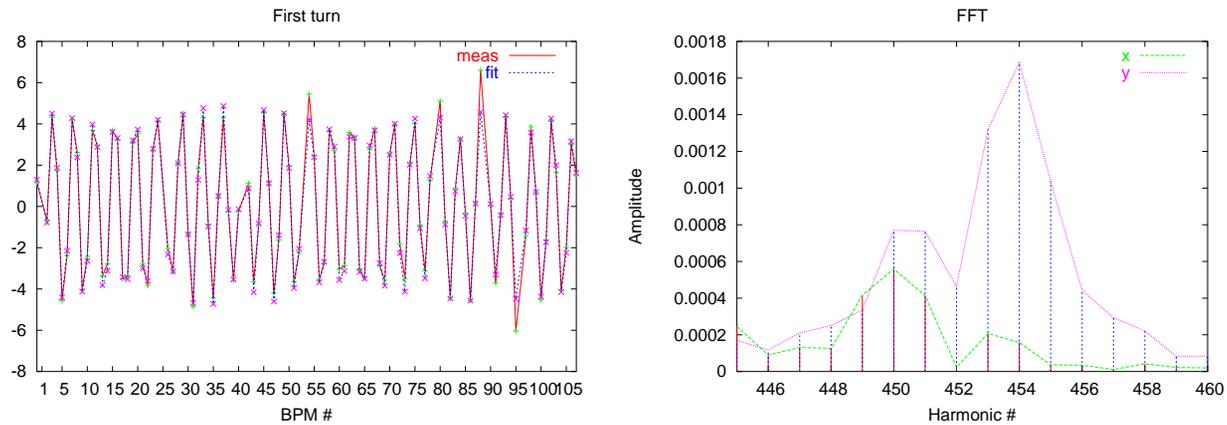
- 4 TBT data with 0.3 e12 particles
- 4 TBT data with 1.2 e12 particles

The large aperture quadrupole BPMs have been excluded from the analysis.

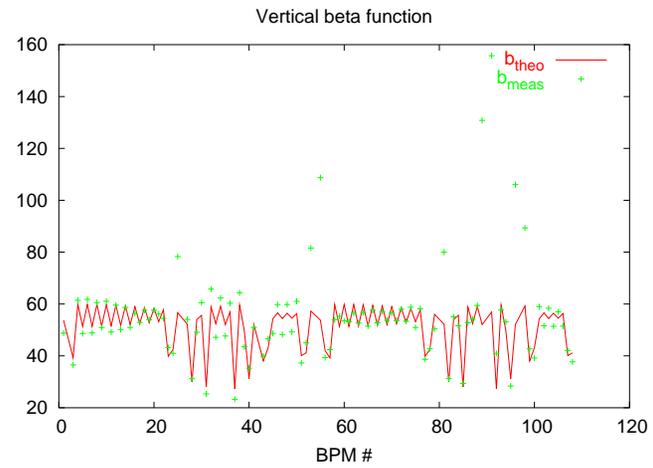
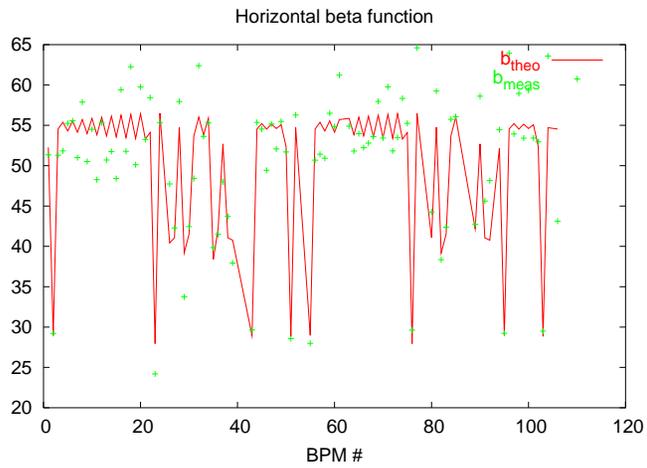
MI data (October 26)



The BPMs measuring a closed orbit larger than 5 mm have been excluded.



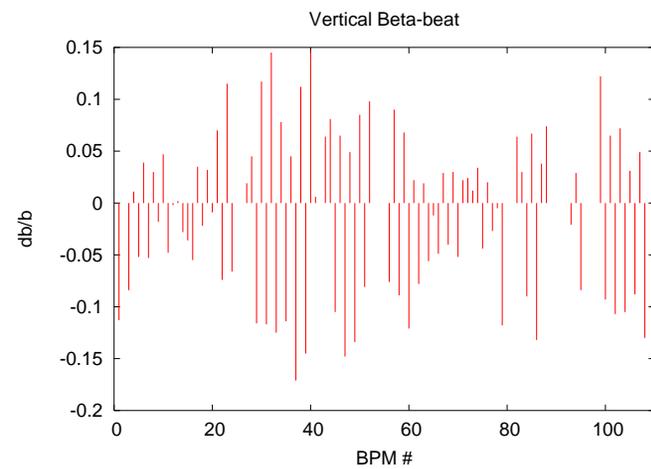
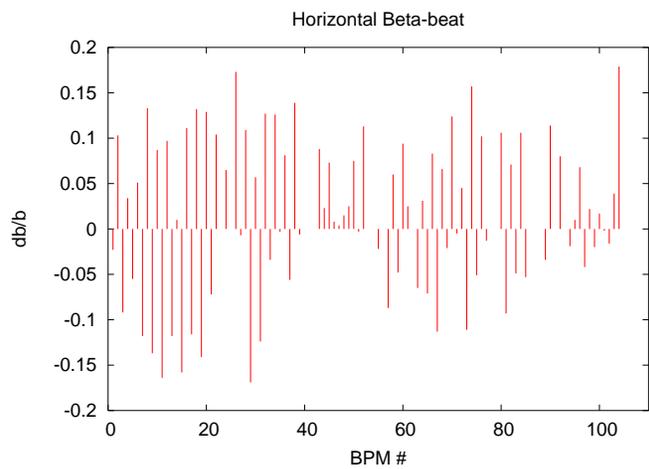
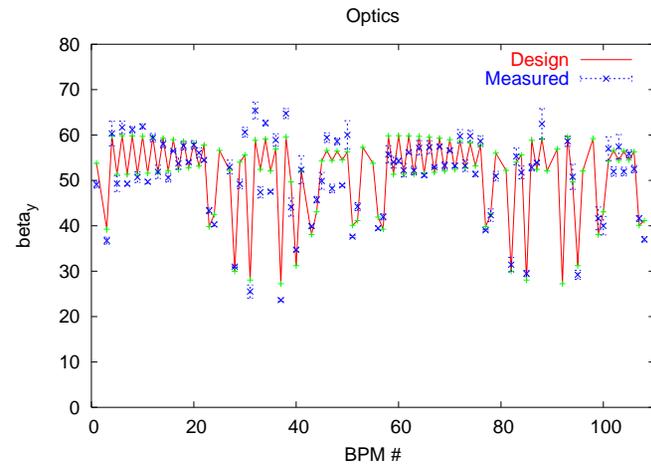
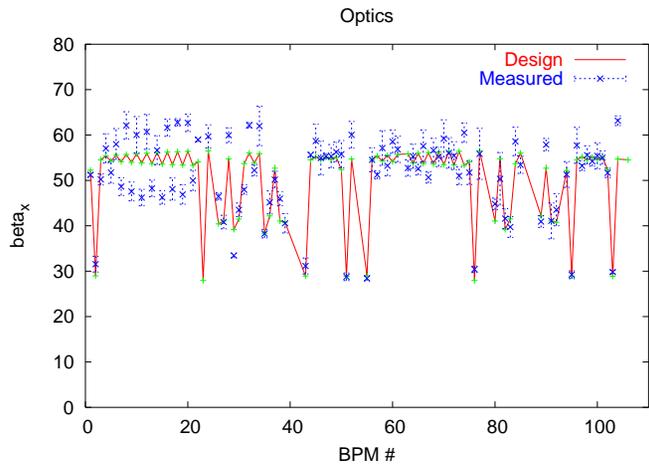
The first turn fit indicates that the “turn order uncertainty” has been correctly removed.

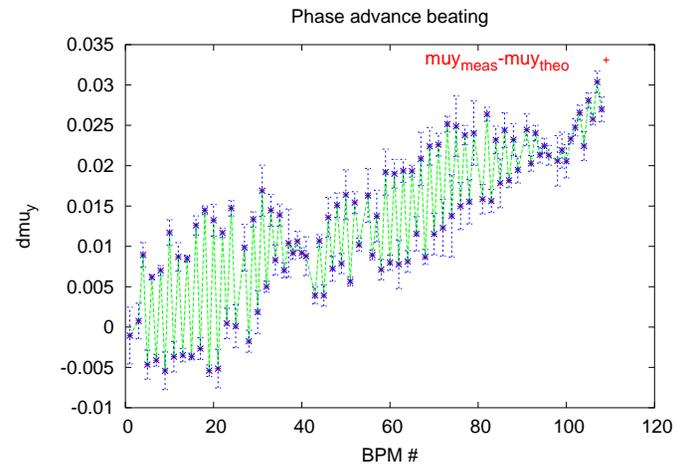
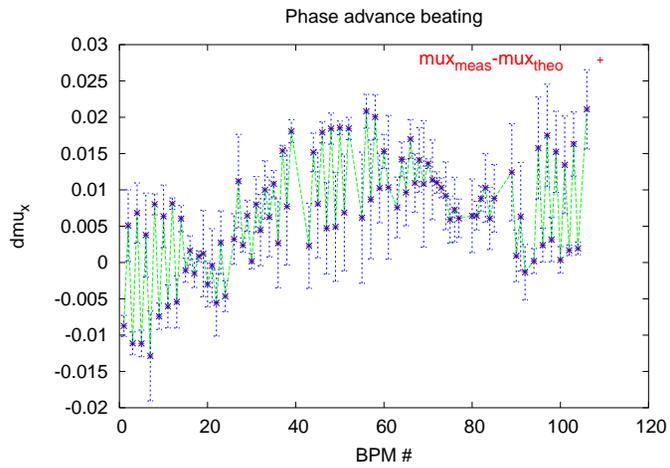


Some vertical BPMs show a “systematic” (understood!) calibration error in the TBT mode:

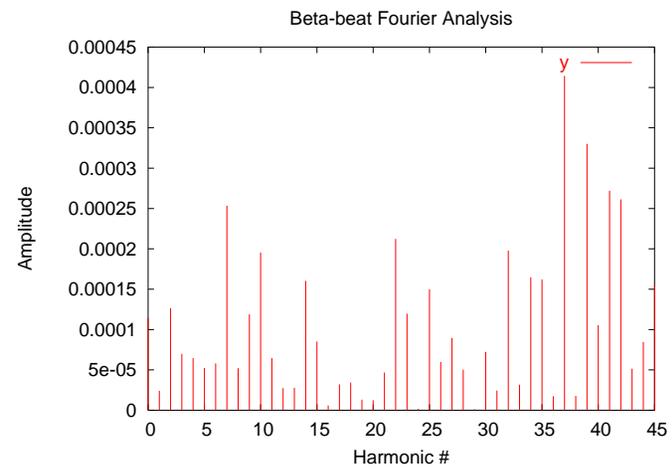
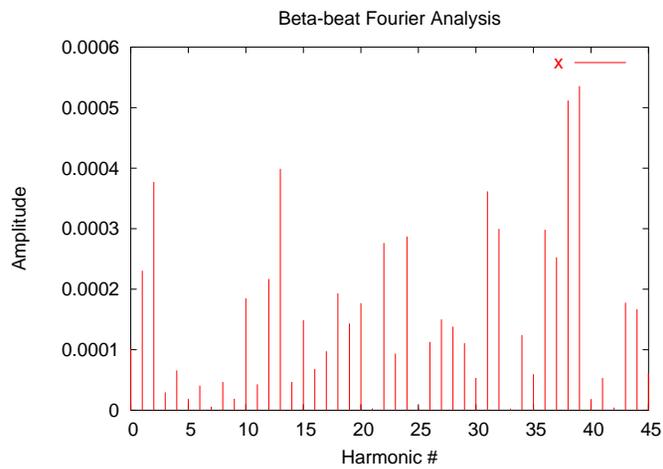
| name | db/b | closed orbit(mm) |
|---------|------|------------------|
| I:VP401 | 0.5 | -2.6 |
| I:VP403 | 1.0 | -2.7 |
| I:VP523 | 0.5 | 2.1 |
| I:VP607 | 1.5 | -2.5 |
| I:VP609 | 1.9 | 1.9 |
| I:VP611 | 0.5 | 0.1 |
| I:VP619 | 1.0 | 0.0 |
| I:VP621 | 0.6 | 2.2 |

Averaging over all low current measurements and excluding “bad” BPMs:



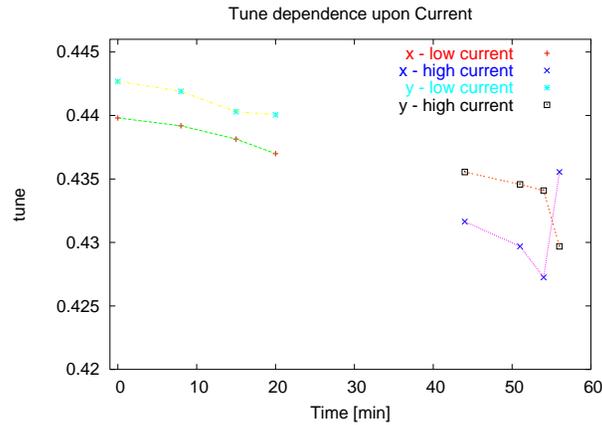


Fourier Analysis of beta-beat (BPMs with $\text{db}/\text{b} < 0.3$ retained)

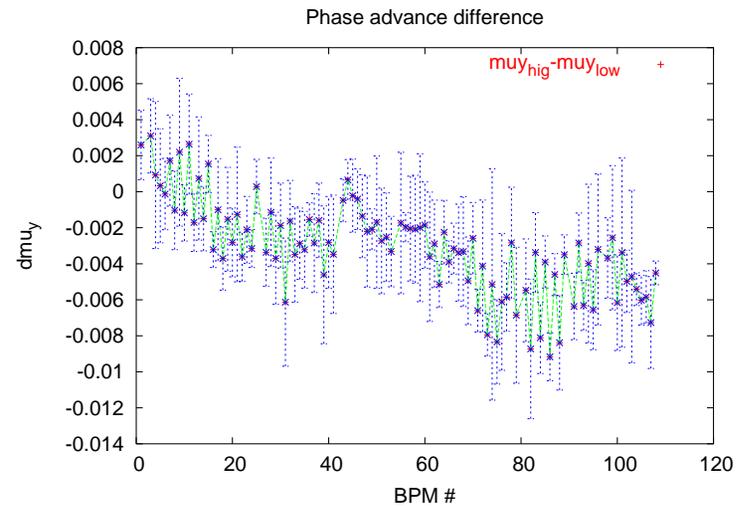
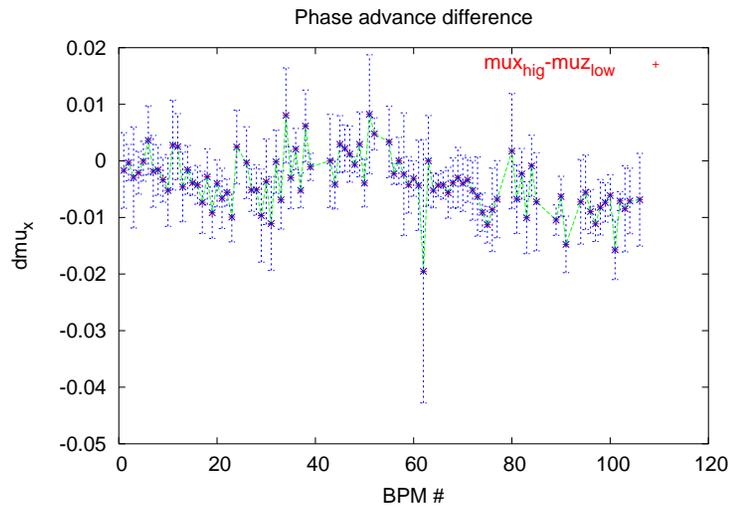


Peaks expected at 39 and 41.

Dependence of phase advance upon beam current

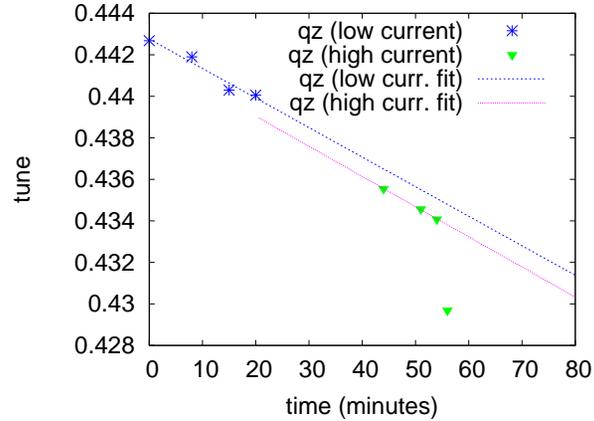


← Tunes drifting with time?



Last TBT data set has been excluded. Phases have been *rescaled* with total averaged tune corresponding to the same beam current. No large evidence of a localised reactive impedance.

Making for instance a *linear* fit of the low current vertical tune values vs. time and extrapolating, it looks as *there is* a dependence upon bunch current:



Effective transverse impedance:

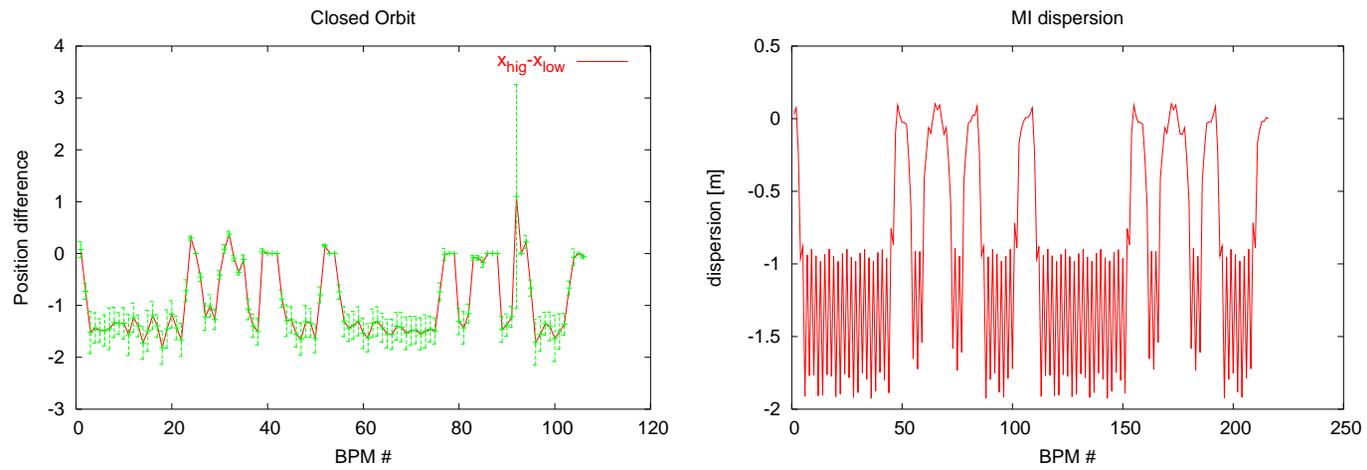
$$Z^T = \frac{E\sigma_\ell}{eR \langle \beta \rangle} \frac{d\mu}{dI_b}$$

For MI at injection:

$$Z^T = \frac{8.9 \text{ GeV} \times 2.5 \text{ ns} \times 3 \times 10^8 \text{ m s}^{-1}}{528 \text{ m} \times 28 \text{ m}} \frac{d\mu}{dI_b} = 0.45 \times 10^6 \text{ V m}^{-1} \frac{d\mu}{dI_b}$$

Due to the longer bunches, the factor relating phase advance slope to the impedance is much larger when compared to lepton machines: the same impedance produces a much smaller slope. With $dQ = 2\pi \times 0.00081$ and $dI = 0.434 \text{ mA}$ is $Z^T = 5.3 \text{ M}\Omega \text{ m}^{-1}$ (!).

Dependence of closed orbit upon beam current

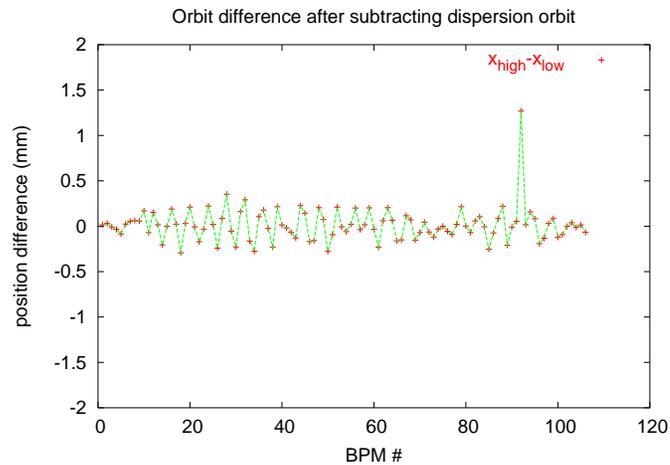


This is the horizontal dispersion: the energy of the high current beam was smaller ($\simeq 0.1\%$) than the energy of the low current beam?!

Fit to the theoretical dispersion:

| Δt (min) | dp/p (%) | $[x_{meas} - x_{fit}]_{rms}$ |
|------------------|------------|------------------------------|
| 8 | -0.002 | 0.02 |
| 15 | 0.001 | 0.02 |
| 20 | 0.001 | 0.01 |
| 44 | 0.065 | 0.14 |
| 51 | 0.065 | 0.14 |
| 54 | 0.091 | 0.14 |
| 56 | 0.095 | 0.22 |

A *step* rather than a drift. Subtracting the contribution of the dispersion orbit:



$$\text{Loss factor: } K_L = \frac{E f_{rev}}{e D_x} \frac{\Delta x_{co}}{\Delta I_b}$$

$$\text{For MI: } K_L < 383 \text{ V pC}^{-1}$$

Not a very tight upper limit...

Can the step observed in the vertical tune be explained by this energy step rather than by the beam current change? It can well be:

$$\Delta Q = \xi \frac{\Delta p}{p} \quad \rightarrow \quad \xi = \frac{2\pi \times 0.00081}{0.1\%} \simeq +5$$

A sensible number.

Conclusions

- A possible current dependence of the total tunes has been washed out by a tune drift with time. The tune drift is not related to the energy change observed in the closed orbit.
- The step observed in the tunes can be explained by the machine energy change, but as the chromaticity has been not measured, it is not known up to which amount.
- A possible closed orbit dependence upon current has been overwhelmed by a relatively large beam energy change.

What can we do better?

- For this measurement, the bunchlength should be kept constant by increasing the RF voltage when injecting larger currents.
- Increase current range decreasing the low current.
- Lower chromaticity to get longer lasting oscillations and minimise tune dependence upon energy.
- Can we have better control of tunes and machine energy? If not, we must improve the measurement strategy: take more data, alternating low and high current.
- Suggestions?