Parametrization of the Driven Betatron Oscillation

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An AC dipole is a magnet which produces a sinusoidally oscillating dipole field and excites coherent transverse beam motion in a synchrotron. By observing this driven coherent motion, the linear optical parameters can be directly measured at the beam position monitor locations. The driven oscillation induced by an AC dipole will generate a phase space ellipse which differs from that of free oscillations. If not properly accounted for, this difference can lead to a misinterpretation of the actual optical parameters, typically, 6% or more in the cases of the Tevatron, RHIC, or LHC. This paper shows that the effect of an AC dipole on the observed linear optics is identical to that of a thin lens quadrupole. By introducing a new amplitude function to describe this new phase space ellipse, the motion produced by an AC dipole becomes easier to interpret. The introduction of this new amplitude function also helps measurements of the normal Courant-Snyder parameters based on beam position data taken under the influence of an AC dipole. This new parameterization of the driven motion is presented and is used to interpret data taken in the FNAL Tevatron using an AC dipole.

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I. INTRODUCTION

A sinusoidally oscillating dipole magnetic field produced by an AC dipole excites coherent transverse beam motion in a synchrotron for machine diagnosis (Fig 1) [1]. Unlike a conventional single turn kicker/pinger magnet, it drives the beam close to the betatron frequency, typically, for several thousands of revolutions. If the amplitude of its oscillating magnetic field is adiabatically ramped up and down, it can create a large coherent oscillation without decoherence or emittance growth [1]. This property makes it a useful diagnostic tool for a synchrotron. AC dipoles have been employed in the BNL AGS and RHIC [1–3], CERN SPS [4, 5], and FNAL Tevatron [6–8]. There is an ongoing project to develop AC dipoles for LHC as well [9].

When the beam is driven by an AC dipole, the beam motion is governed by two driving terms and the influence of the lesser driving term makes driven oscillation response different from that of free oscillations. Although this difference has typically been ignored in previous analyses [3, 10], it could affect the interpretation of the linear optics more than 12% in the Tevatron and 6% in the RHIC and LHC.

This paper proceeds as follows. Section II discusses the two driving terms produced by an AC dipole and presents a new formulation of driven motion which is suited to simultaneously treat the influences of the two driving terms. Section III discusses the difference between free and driven betatron oscillations and the influence of this difference on measurement of the linear optical parameters, based on an analogy between driven motion and motion under the influence of a thin gradient error [13]. Section IV presents a few examples of the difference between free and driven oscillations observed in the Tevatron and explains how to measure the β-function corresponding to free oscillation by appropriate reduction of driven oscillation data.

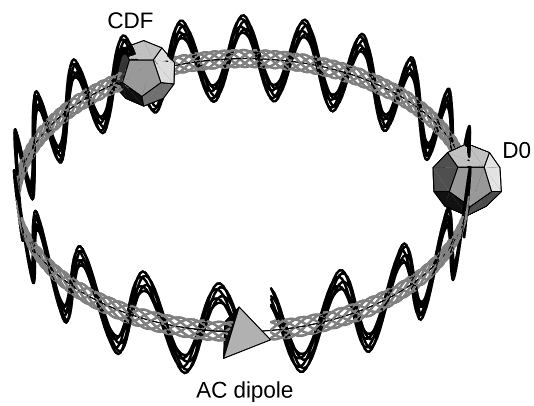

FIG. 1: A diagram of incoherent free oscillations (gray) and excited coherent oscillations (black) in the Tevatron. Since individual particles within the beam oscillate incoherently, coherent oscillations must be excited to observe betatron motion and measure optical parameters. An AC dipole [1] is a tool to excite sustained coherent oscillations.

II. A MODEL OF DRIVEN OSCILLATIONS

A. Two Driving Terms of an Oscillating Dipole Field

The tune of an AC dipole ν_{acd} is defined as the ratio between the frequencies of the AC dipole f_{acd} and the beam revolution f_{rev}: $\nu_{\mathrm{acd}} \equiv f_{\mathrm{acd}}/f_{\mathrm{rev}}$. In the following, for any tunes, only their fractional parts are considered. For instance,

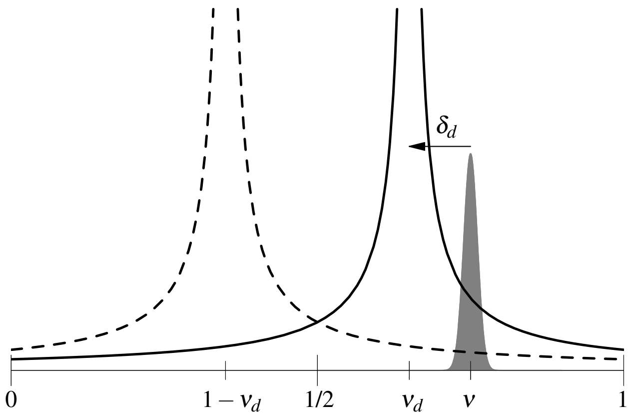

FIG. 2: The amplitude of the driven motion versus the machine tune. A circulating beam is influenced by both (solid and dashed) of the resonant amplitudes. In typical operations of an AC dipole, realizable δ_d is limited by the tune spread of the beam (shaded area).

if $f_{\text{acd}}/f_{\text{rev}}$ is larger than one, ν_{acd} means the fractional part of $f_{\text{acd}}/f_{\text{rev}}$. Since the beam sees an AC dipole only once in a revolution, the beam is driven by a pair of driving terms at ν_{acd} and $1-\nu_{\text{acd}}$ (cf. Nyquist sampling theorem). Obviously, the driving term closer to the machine tune ν has a bigger effect on a particle. In the following, the driving term closer to ν is called the primary and the other is called the secondary. A symbol ν_d is used for the primary driving tune:

$$\nu_d \equiv \begin{cases} \nu_{\text{acd}} & \text{when} \quad |\nu_{\text{acd}}-\nu| < |(1-\nu_{\text{acd}})-\nu| \\ 1-\nu_{\text{acd}} & \text{when} \quad |(1-\nu_{\text{acd}})-\nu| < |\nu_{\text{acd}}-\nu| \ . \end{cases} \quad (1)$$

For example, the frequencies of the AC dipole and beam revolution in the Tevatron are $f_{\text{acd}} \simeq 20.5$ kHz and $f_{\text{rev}} \simeq 47.7$ kHz and hence the tune of the AC dipole is $\nu_{\text{acd}} = 20.5/47.7 \simeq 0.43$. Since the machine tune of the Tevatron is $\nu \simeq 0.58$, $1-\nu_{\text{acd}} \simeq 0.57$ is the primary driving tune and $\nu_{\text{acd}} \simeq 0.43$ is secondary.

The difference between the primary driving term and the machine tune $\delta_d \equiv \nu_d - \nu$ is an important parameter of the driven betatron oscillation. As $\delta_d \to 0$, the influence of the primary driving term becomes dominant and the secondary driving term can be ignored. However, a finite tune spread of the beam can cause beam losses if $|\delta_d|$ is too small (Fig 2). In the Tevatron, without special tune-up, the limit of $|\delta_d|$ is about 0.01 to prevent beam losses.

When the amplitude of the AC dipole field is constant, the position of the driven beam x_d is given by [10, 11]

$$\begin{aligned} x_d(nC+\Delta s) \simeq\ & \frac{\theta_{\text{acd}}\sqrt{\beta_{\text{acd}}}}{4\sin[\pi(\nu_{\text{acd}}-\nu)]}\sqrt{\beta(\Delta s)}\cos[2\pi\nu_{\text{acd}}n+\psi(\Delta s)+\pi(\nu_{\text{acd}}-\nu)+\chi_{\text{acd}}] \\ & + \frac{\theta_{\text{acd}}\sqrt{\beta_{\text{acd}}}}{4\sin[\pi((1-\nu_{\text{acd}})-\nu)]}\sqrt{\beta(\Delta s)}\cos[2\pi(1-\nu_{\text{acd}})n+\psi(\Delta s)+\pi((1-\nu_{\text{acd}})-\nu)-\chi_{\text{acd}}]\ , \end{aligned} \quad (2)$$

where n is an integer for the revolution number, C is the circumference of the ring, Δs $(0 \le \Delta s < C)$ is the longitudinal distance from the AC dipole, θ_{acd} is the maximum kick angle of the AC dipole, β_{acd} is the β-function at the location of the AC dipole, ψ is the phase advance of the free oscillation from the location of the AC dipole to the observation point, and χ_{acd} is the initial phase of the AC dipole field. The two terms in Eq 2 are symmetric and represent the influences of the two driving terms [14]. To quantify the effect of the secondary driving term, it is useful to define a parameter to describe the ratio between the larger and smaller modes in Eq 2:

$$\lambda_d(\delta_d) \equiv \frac{\sin[\pi(\nu_d-\nu)]}{\sin[\pi((1-\nu_d)-\nu)]} = \frac{\sin(\pi\delta_d)}{\sin(2\pi\nu+\pi\delta_d)}\ . \quad (3)$$

This parameter λ_d depends on not only δ_d but also the machine tune ν. When $|\delta_d| = 0.01$, $|\lambda_d| \simeq 0.06$ for the Tevatron ($\nu \simeq 0.58$) and about half as much for the RHIC and LHC ($\nu \simeq 0.7$ and 0.3).

B. A New Parametrization of the Driven Betatron Oscillation

We note that Eq 2 can be written in the following compact form which includes the influences of the both driving terms:

$$x_d(s;\delta_d) = A_d(\delta_d)\sqrt{\beta_d(s;\delta_d)}\cos(\psi_d(s;\delta_d) \pm \chi_{\text{acd}})\ . \quad (4)$$

Here, $s \equiv nC + \Delta s$ is the longitudinal position, A_d is a constant of motion with dimensions of $(\text{length})^{1/2}$:

$$A_d(\delta_d) \equiv \frac{\theta_{\text{acd}}}{4\sin(\pi\delta_d)}\sqrt{(1-\lambda_d^2)\beta_{\text{acd}}}\ , \quad (5)$$

and the sign in front of χ_{acd} is positive when $\nu_d = \nu_{\text{acd}}$ and negative when $\nu_d = 1-\nu_{\text{acd}}$. The quantity β_d is a newly defined amplitude function of the driven oscillation:

$$\beta_d(s;\delta_d) \equiv \frac{1+\lambda_d^2-2\lambda_d\cos(2\psi-2\pi\nu)}{1-\lambda_d^2}\beta \quad (6)$$

and ψ_d is a newly defined phase advance of the driven oscillation from the location of the AC dipole to the observation point:

$$\psi_d(s;\delta_d) \equiv \int_0^s \frac{d\bar{s}}{\beta_d(\bar{s};\delta_d)} \ . \tag{7}$$

The increment of ψ_d is $2\pi\nu_d$ (mod 2π) in a single revolution. In this way, the driven oscillation can be parametrized in the same form as the free oscillation even when the influences of the both driving terms are included. Differences between the free and driven oscillations are characterized by the amplitude function β_d and phase advance ψ_d. In the limit of $\nu_d \to \nu$, λ_d becomes zero and β_d and ψ_d converge to β and ψ. Fig 3 shows the numerical calculations of β_d/β based on Eq 6.

If the lesser mode in Eq 2 is ignored, the oscillation phase has an apparent jump by $2\pi\delta_d$ at the location of the AC dipole. However, if the influences of both driving terms are properly included, the phase advance ψ_d is continuous at the location of the AC dipole. A relation between ψ and ψ_d is given by

$$\begin{aligned}\tan(\psi_d - \pi\nu_d) &= \frac{1+\lambda_d}{1-\lambda_d}\tan(\psi - \pi\nu) \\ &= \frac{\tan(\pi\nu_d)}{\tan(\pi\nu)}\tan(\psi - \pi\nu) \ . \end{aligned} \tag{8}$$

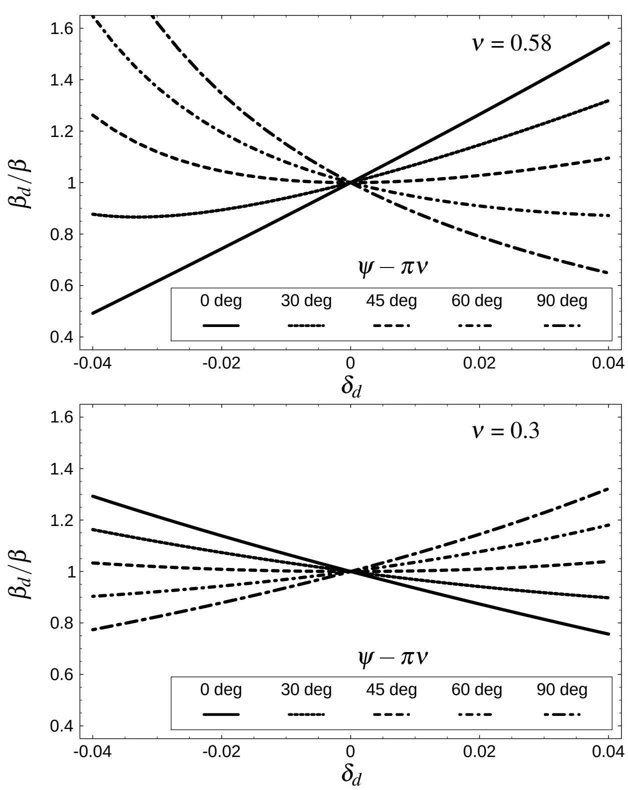

FIG. 3: Ratio between the amplitude functions of the driven and free oscillations, β_d and β. Based on Eq 6, β_d/β is calculated for different tune separations $\delta_d = \nu_d - \nu$ and the phase advances ψ. When $\nu = 0.58$ like the Tevatron, compared to when $\nu = 0.30$ like RHIC and LHC, β_d/β deviates larger from the unity and has stronger nonlinearity because of the larger influence of the secondary driving term.

III. DIFFERENCE BETWEEN FREE AND DRIVEN BETATRON OSCILLATIONS

Measurement of the β-function using an AC dipole requires careful understanding of the difference between the amplitude functions of the free and driven oscillations, β and β_d. For the free betatron oscillation, the machine tune ν and amplitude function β are correlated and a change of the tune induces a change of the amplitude function and vice versa. The correlation between the oscillation tune and amplitude function applies to the driven oscillation, too. When the beam is driven, the oscillation tune ν_d is different from the machine tune ν and the amplitude function β_d also differs from β depending on the oscillation tune ν_d. The relation between $\delta_d = \nu_d - \nu$ and $\beta_d - \beta$ is formally the same as the tune shift and the change of the β-function produced by a thin gradient error. This analogy helps to interpret data of the driven motion.

A. Review of a Thin Gradient Error

If a synchrotron has a gradient error, its machine tune ν and amplitude function β change to ν_q and β_q [12]. A thin gradient error with gradient B_1 and length ℓ at $\Delta s = 0$ results in an equation of motion

$$x'' + K(s)x = -q_{\rm err}x \sum_{n=-\infty}^{\infty} \delta(s - Cn) \ , \tag{9}$$

where the prime denotes the derivative with the longitudinal position s, $K(s)$ is the effective focusing function, $q_{\rm err} = B_1\ell/(B\rho)$ is the effective strength of the gradient error, and δ is the Dirac delta function.

By comparing the single turn transfer matrices with and without the gradient error, ν_q and β_q satisfy the following two equations:

$$q_{\rm err} = 2\frac{\cos(2\pi\nu) - \cos(2\pi\nu_q)}{\beta_{\rm err}\sin(2\pi\nu)} \tag{10}$$

$$\beta_q(s;\delta_q) = \frac{\sin(2\pi\nu) - q_{\rm err}\beta_{\rm err}\sin\psi\sin(2\pi\nu - \psi)}{\sin(2\pi\nu_q)}\beta \ , \tag{11}$$

where $\beta_{\rm err}$ is the β-function at the location of the gradient error, $\delta_q \equiv \nu_q - \nu$ is the tune shift caused by the gradient error, and ψ is the phase advance of the free oscillation from the location of the gradient error. By substituting the first equation into the second, β_q is given by

$$\beta_q = \frac{1 + \lambda_q^2 - 2\lambda_q\cos(2\psi - 2\pi\nu)}{1 - \lambda_q^2}\beta \ . \tag{12}$$

Here, λ_q is defined as a parameter with a similar form to λ_d:

$$\lambda_q(\delta_q) \equiv \frac{\sin(\pi\delta_q)}{\sin(2\pi\nu + \pi\delta_q)} \ . \tag{13}$$

When λ_q is small, the new and original amplitude functions satisfy

$$\frac{\beta_q - \beta}{\beta} \simeq -2\lambda_q\cos(2\psi - 2\pi\nu) \ . \tag{14}$$

This quantity behaves like a standing wave in a synchrotron and is called the β-beat (or sometimes β-wave). The amplitude of the β-beat is $2|\lambda_q|$ at lowest order.

It may be seen from from Eqs 6 and 12 that the relation between β_d and δ_d for driven motion is the same as the relation between β_q and δ_q when there is a thin gradient error. Hence, relative to β, we expect β_d will beat with amplitude of $2|\lambda_d|$.

B. Analogy between an AC Dipole and Gradient Error

This section explains why an oscillating dipole field changes the observed phase space motion, much like a gradient error. When driven by the AC dipole, the equation of motion is given by

$$x'' + K(s)x = -\theta_{\rm acd} \sum_n \cos(2\pi\nu_d n \pm \chi_{\rm acd})\delta(s - Cn) \ . \quad (15)$$

The right-hand-side describes the kicks by the AC dipole located at $\Delta s = 0$. The summation runs over the time period when the amplitude of the AC dipole field is constant and the sign in front of the initial phase $\chi_{\rm acd}$ follows the same convention as Eq 4. Eq 4 is the particular solution of this inhomogeneous Hill's equation when the amplitude of the AC dipole field is adiabatically ramped up to a constant amplitude. Since the phase advance of the driven oscillation ψ_d increases by $2\pi\nu_d$ in one revolution, the position of the driven oscillation at the location of the AC dipole $s = Cn$ is given by

$$x_d(Cn;\delta_d) = A_d(\delta_d)\sqrt{\beta_d(0;\delta_d)}\cos(2\pi\nu_d n \pm \chi_{\rm acd}) \ . \quad (16)$$

Here, the phases in Eqs 15 and 16 are the same. Hence, the AC dipole field is in sync with the position of the driven oscillation when the beam passes the AC dipole. The situation is analogous to a quadrupole magnet whose field is proportional to position x. The phases of the driven oscillation and the AC dipole field are synchronized like this only when the amplitude of the AC dipole field is constant after the adiabatic ramp up. Since x_d is the solution of Eq 15, it formally satisfies the following equation

$$x_d'' + K(s)x_d = -q_{\rm acd} x_d \sum_n \delta(s - Cn) \ . \quad (17)$$

On the right-hand-side, Eq 16 is used to rewrite $\cos(2\pi\nu_d n \pm \chi_{\rm acd})$ with x_d. The parameter $q_{\rm acd}$ is a constant given by

$$q_{\rm acd} = \frac{\theta_{\rm acd}}{A_d\sqrt{\beta_d(0;\delta_d)}} = 2\frac{\cos(2\pi\nu) - \cos(2\pi\nu_d)}{\beta_{\rm acd}\sin(2\pi\nu)} \ . \quad (18)$$

Eq 17 has exactly the same form as the equation of motion when there is a thin gradient error, Eq 9. By comparing Eqs 9, 10, 17, and 18, it is trivial that the relation between β_d and δ_d is the same as the relation between β_q and δ_q.

C. Ring-wide Behavior of the Amplitude Function β_d

When turn-by-turn beam positions at all BPMs are given for the free oscillation, the relative β-function can be determined by simply comparing the square of the oscillation amplitude at each BPM. If the same analysis is applied to the turn-by-turn data of the driven oscillation, what is calculated is β_d which is different from the real β at most by $2|\lambda_d|$ depending on the BPM location. When $|\delta_d|$ is 0.01, $2|\lambda_d|$ is about 12% for the Tevatron and 6% for the RHIC and LHC. Furthermore, since the beating of β_d relative to β cannot be distinguished from the real β-beat caused by gradient errors, the real β-beat cannot be measured in this way without relying on a machine model. Fig 4 shows β and β_d when $\delta_d = -0.01$ measured in the Tevatron.

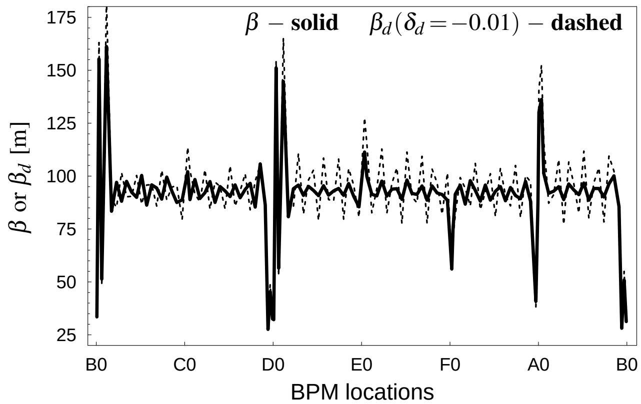

FIG. 4: The amplitude functions of the free and driven oscillations, β (solid) and β_d when $\delta_d = -0.01$ (dashed), measured in the Tevatron. As expected, β_d shows 10-15% beating relative to β. From multiple data sets of driven motion, the true β-function can be extrapolated.

As explained in the following section, by using multiple data sets of the driven motion, the influences of the primary and secondary driving terms can be separated and the linear optical parameters can be measured without depending on a machine model.

IV. EFFECT ON β-FUNCTION MEASUREMENT

A. Rotation of the Phase Space Ellipse

The previous section discussed the amplitude function of the driven motion β_d. Parameters corresponding to the other Courant-Snyder parameters α and γ can be also defined as for the free oscillation:

$$\alpha_d(s;\delta_d) \equiv -\frac{1}{2}\frac{d\beta_d(s;\delta_d)}{ds} \quad (19)$$

$$\gamma_d(s;\delta_d) \equiv \frac{1 + \alpha_d(s;\delta_d)^2}{\beta_d(s;\delta_d)} \ . \quad (20)$$

The explicit forms of these parameters are given by

$$\alpha_d = \frac{1 + \lambda_d^2 - 2\lambda_d\cos(2\psi - 2\pi\nu)}{1 - \lambda_d^2}\alpha - \frac{2\lambda_d\sin(2\psi - 2\pi\nu)}{1 - \lambda_d^2} \quad (21)$$

and

$$\gamma_d = \frac{1+\lambda_d^2+2\lambda_d\cos(2\psi-2\pi\nu+2\arctan\alpha)}{1-\lambda_d^2}\gamma\,. \tag{22}$$

When β_d, α_d, γ_d, and A_d are defined this way, they satisfy the Courant-Snyder invariance:

$$A_d^2 = \gamma_d x_d^2 + 2\alpha_d x_d x_d' + \beta_d x_d'^2\,. \tag{23}$$

Hence, the turn-by-turn position and angle of the driven oscillation also form an ellipse in phase space, like the free oscillation. Since not only A_d but also the Courant-Snyder-like parameters β_d, α_d, and γ_d depend on δ_d, both the area and shape of the phase space ellipse changes with δ_d for the driven oscillation. Since β_d, α_d, and γ_d converge into β, α, and γ in the limit of $\lambda_d \to 0$, this change of the shape is due to the secondary driving term.

In two collision straight sections of the Tevatron, B0 and D0, there are pairs of BPMs with no magnetic element in-between. The beam travels along straight lines between these pairs and, hence, both position and angle can be directly measured at these locations. Fig 5 shows the phase space ellipses of the driven oscillations measured by using a pair of such BPMs. The location is the B0 interaction point. In these measurements, δ_d was set to ±0.04 and ±0.02, while the kick angle of the AC dipole $\theta_{\rm acd}$ was kept the same. As expected, the shape of the phase space ellipse changes with δ_d.

By fitting Eq (23) to an ellipse in Fig 5, its area πA_d^2 and the parameters β_d, α_d, and γ_d can be determined. Fig 6 shows β_d determined from the fits to ellipses in Fig 5 (and three more). The curve in the figure is the fit of Eq 6 to the data with parameters β and ψ. In the figure, the true β-function at this location in the synchrotron is obtained by extrapolation of the value of β_d to the case of $\delta_d = 0$. For the Tevatron, the β-function at low-β locations, β^*, can be directly measured this way. The model of Eq 6 fits well to the data.

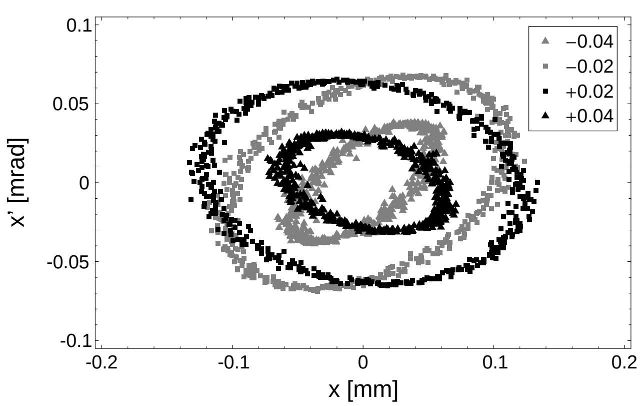

FIG. 5: Phase space ellipses of the driven oscillations when $\delta_d = \pm0.02$ and ±0.04. The location is one of the low-β point (B0) in the Tevatron where α is zero by design. Since the Courant-Snyder-like parameters of the driven motion β_d, α_d, and γ_d depend on δ_d, not only the areas but also the shapes of the ellipses are different.

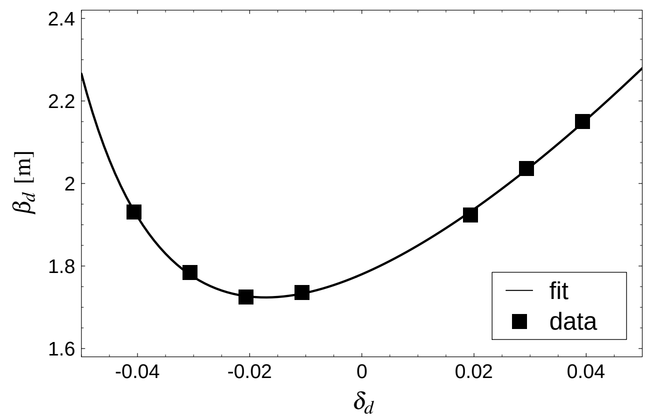

FIG. 6: Relation between β_d and δ_d determined from the fits to the phase space ellipses in Fig 5. The curve is fit of Eq 6 to the data points. The true β-function of this location is the value of β_d when $\delta_d = 0$.

B. Asymmetric Amplitude Response

When the influence of the secondary driving term is negligible, by ignoring the smaller term of Eq 2 or taking the limit of $\lambda_d \to 0$ in Eqs 4, 5, and 6, the amplitude of the driven oscillation can be approximated by

$$a_d^{(0)}(s;\delta_d) = \frac{|A_d|\sqrt{\beta}}{\sqrt{1-\lambda_d^2}} = \frac{\theta_{\rm acd}\sqrt{\beta_{\rm acd}\beta}}{4|\sin(\pi\delta_d)|}\,. \tag{24}$$

In this case, the amplitude of the driven oscillation depends on the primary driving tune ν_d only through $|\sin(\pi\delta_d)|$ and is symmetric around the machine tune ν. From Eqs 4, 5, and 6, the amplitude including the influence of the secondary driving term $a_d(s;\delta_d)$ is given by

$$a_d(s;\delta_d) = a_d^{(0)}\sqrt{1+\lambda_d^2-2\lambda_d\cos(2\psi-2\pi\nu)}\,. \tag{25}$$

Here, the amplitude a_d depends on ν_d through the factor $[1+\lambda_d^2-2\lambda_d\cos(2\psi-2\pi\nu)]^{1/2}$ as well. To the first order of δ_d,

$$a_d \simeq a_d^{(0)}\left[1-\frac{\pi\cos(2\psi-2\pi\nu)}{\sin(2\pi\nu)}\delta_d\right]\,. \tag{26}$$

Hence, the secondary driving term makes the ν_d dependence of the amplitude asymmetric around the machine tune ν. The magnitude of this asymmetry at each location is determined by the factor $\cos(2\psi-2\pi\nu)$.

Fig 7 shows the relation between the amplitude of the driven oscillation and ν_d at three BPM locations in the Tevatron. The dashed and solid lines represent the fits of Eq 24 and Eq 25 to the data. The fit parameters are $\theta_{\rm acd}(\beta_{\rm acd}\beta)^{1/2}$ and ν for Eq 24 and $\theta_{\rm acd}(\beta_{\rm acd}\beta)^{1/2}$, ν, and ψ for Eq 25. At two locations where $|\cos(2\psi-2\pi\nu)|$ is close to one, the asymmetry around the machine tune ($\nu \simeq 0.5785$) is large and the fits ignoring the secondary driving term effect based on Eq 24 is not well matched. From the fit of Eq 25, the β-function at each BPM location is determined up to a constant $\theta_{\rm acd}(\beta_{\rm acd})^{1/2}$.

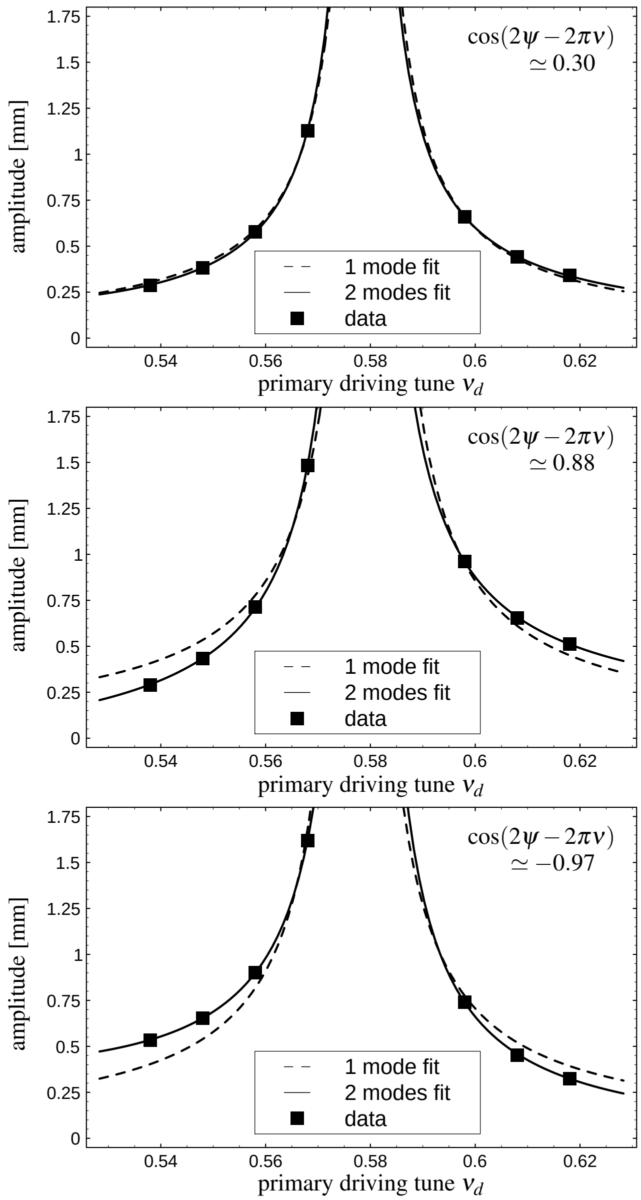

FIG. 7: Amplitude response of the driven motion to the primary driving tune ν_d measured at three BPM locations in the Tevatron. The solid and dashed curves represent fits with and without the effect of the secondary driving term. The asymmetry around the machine tune $\nu \simeq 0.5785$ depends on $\cos(2\psi - 2\pi\nu)$.

This constant can be determined from the analysis in the previous section which uses a pair of BPMs in a collision straight section. By combining these two types of analyses, the ring-wide β-function can be directly measured from multiple data sets of the driven motion with different ν_d.

Although the influence of the secondary driven term effect is clear in Fig 7, there is even better evidence that Eq 25 fits the data better than Eq 24. From the fits in Fig 7, the machine tune ν can be determined at each BPM location. Fig 8 shows machine tunes at all BPM locations determined this way. The solid curve includes the influence of the secondary driving term and the dashed curve does not. Since the machine tune ν is a global parameter of a synchrotron, the variation of the determined machine tune over BPMs shows the inaccuracy of the measurements and data analyses.

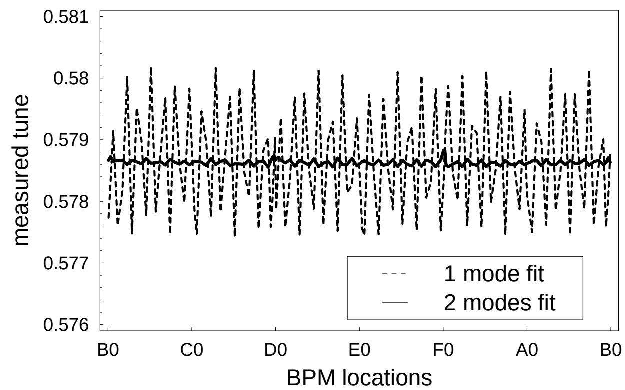

FIG. 8: Machine tunes derived from the fits of the amplitude resposes at each BPM locations. Inclusion of the secondary driving term in the fit of each BPM gives a much consistent result for the machine tune (global parameter).

V. CONCLUSION

Under the influence of a sinusoidally oscillating magnetic field of an AC dipole, the beam is driven by two terms. As a result the phase space trajectory of the driven motion is different from that of the free betatron motion. If this difference is simply ignored, interpretation of the linear optics based on the data of the driven motion can have error depending on the driving and machine tunes. For instance, when the difference between the primary driving tune and machine tune is 0.01, the error is 12% for the Tevatron and 6% for the RHIC and LHC. In this paper, we show that this influence on the phase space is formally identical to the influence of a gradient error at the same location as the AC dipole. Just as a gradient error changes the amplitude function around the ring, the expression of the driven motion can be simplified by introducing a new amplitude function for the driven motion.

This paper presents a few examples of the difference between the free and driven motions as observed in the Tevatron. It also shows that the new parametrization of the driven motion clarifies the interpretation of turn-by-turn beam motion data.

With this knowledge, very precise and direct measurements of the true linear optical parameters in a synchrotron can be obtained quickly without degradation of the beam quality, using a small number of data sets obtained at different frequencies of the AC dipole. This technique will be especially useful in the LHC, for example, to adjust the beam envelope at critical locations such as at beam collimation devices.

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[13] This analogy is based on properties of the amplitude function for driven oscillation β_d which is defined in Section II. As the β-function of the free oscillation is well defined only in the linear regime [12], our analogy applies only to the linear regime.

[14] Eq 2 is assuming the amplitude of the AC dipole field is adiabatically ramped up to a constant value. The exact expression of x_d includes transient modes which are inversely proportional to the ramp up time and oscillate with the machine tune ν. If the ramp up is slow enough, all of these modes are very small and decohere before the end of the ramp up. Hence, these ignored modes do not affect the motion of the beam centroid but they may affect the beam size [11].