A Model for Lorentz Detuning in a Superconducting RF Cavity Controlled with a Vector Phase Modulator

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Introduction

This note examines the behavior of a HINS Superconducting Single Spoke Resonator cavity in the presence of Lorentz detuning. This note is based on the model described in Beams Document 2910 "A Simple Model for a Superconducting RF cavity with a Vector Phase Modulator".

Envelope Equations

The cavity voltage and the total cavity current can also be separated into fast varying and slow varying parts:

$$v(t) = Re\{V(t)e^{j\omega t}\} \tag{1}$$

$$i_s(t) = Re\{I_s(t)e^{j\omega t}\} \tag{2}$$

where V(t) and $I_S(t)$ are slowly varying complex phasors. The cavity voltage and current phasors can be separated into real and imaginary parts:

$$V = V_r + jV_i \tag{3}$$

and:

$$I_s = I_{s_r} + jI_{s_i} \tag{4}$$

The response of the cavity voltage to the source current is described by two coupled equations:

$$\frac{2Q}{\omega_o}\frac{dV_r}{dt} + V_r + tan(\varphi_D)V_i = RI_{s_r} \tag{5}$$

and:

$$\frac{2Q}{\omega_o}\frac{dV_i}{dt} + V_i - tan(\varphi_D)V_r = RI_{s_i} \tag{6}$$

where:

$$tan(\varphi_D) = Q\left(\frac{\omega_o}{\omega} - \frac{\omega}{\omega_o}\right) \tag{7}$$

where ω_o is the resonant frequency of the cavity. If the excitation frequency ω is close to the resonant frequency ω_o:

$$\omega = \omega_o + \Delta\omega \tag{8}$$

then Eqn. 7 becomes:

$$tan(\varphi_D) = -\frac{2Q}{\omega_o}\Delta\omega \tag{9}$$

Lorentz Detuning Approximation

For intense electromagnetic fields inside the cavity, the radiation pressure of the fields will push against the walls of the cavity and the resonant frequency of the cavity will shift to a lower value. As an approximation, assume that the shift in resonant frequency is linearly proportional to the magnitude of the cavity voltage:

$$\frac{\Delta\omega}{\omega_o} = -\frac{2\pi}{\omega_o}\mathrm{K_L}|\mathrm{V}| \tag{10}$$

where K_L is a constant describing the Lorentz detuning. The total detuning angle of the cavity can be broken into a static and dynamic part:

$$tan(\varphi_D) = tan(\varphi_{DS}) - 2\pi\frac{2Q}{\omega_o}\mathrm{K_L}|\mathrm{V}| \tag{11}$$

where ϕ_{DS} is the static shift in phase angle due to the initial geometry of the cavity at zero voltage. Equations 5 and 6 become non-linear but can be solved numerically.

Examples

In the following examples, the cavity R/Q of 262 Ohms was used. This value is typical of a superconducting single spoke resonator used in HINS. The cavity voltage and synchronous phase angle were chosen from the design of the first cryomodule in HINS. The cavity voltage of 1472kV with a synchronous angle of 30 degrees was used. It was assumed that the cavity coupling would be optimized for optimum power transfer to the beam at a beam pulse current of 15mA. The loaded Q of the cavity is about 2.2×10^5 and the fill time of the loaded cavity is about 212uS. Beam injection time should be about 147uS after the generator current turns on. The beam pulse length is 1mS.

Example 1

This example is the response of the cavity with no Lorentz detuning. The parameters are listed in Table 1. Cavity voltages, phase angles, and reflection coefficient are shown in Figures 1-4.

Example 2

This example has the same parameters used in Example 1 but with the Lorentz detuning set to 0.1 kHz/MV. The parameters are listed in Table 2. Cavity voltages, phase angles, and reflection coefficient are shown in Figures 5-8. Note that the scale in Figure 6 changes by a factor of 100 from the scale used in Figure 2.

Example 3

This example has the same Lorentz detuning as was used in Example 2 but the klystron power, generator phase, and beam arrival time have been adjusted to compensate for the Lorentz detuning. The parameters are listed in Table 3. Cavity voltages, phase angles, and reflection coefficient are shown in Figures 9-12.

Example 4

This example has a large Lorentz detuning factor of 1.0kHz/MV. The klystron power, generator phase, and beam arrival time have been adjusted to compensate for the Lorentz detuning. Because the generator phase has to be adjusted to -35 degrees, the cavity reflection coefficient is very large (5dB) while the beam is passing though the cavity. Because of the large reflection coefficient, the klystron power has to be increased significantly as compared to the no Lorentz detuning case. The parameters are listed in Table 4. Cavity voltages, phase angles, and reflection coefficient are shown in Figures 13-16.

Example 5

This example also has a large Lorentz detuning factor of 1.0kHz/MV However, in this example, the static cavity detuning has been adjusted to -54 degrees (which shifts the resonant frequency higher) so that the required generator angle when beam is in the cavity is 0 degrees. Keeping the generator in phase with the voltage reduces the reflection coefficient so the required generator power is substantially lower than what is required in the previous example. The parameters are listed in Table 5. Cavity voltages, phase angles, and reflection coefficient are shown in Figures 17-20.

Parameter	Value	Units
Cavity Voltage	1472	kV
Klystron Power	25.57	kW
R/Q	262	Ohms
RF Frequency	325	MHz
Reference Beam Current	15	mA
Actual Beam Current	15	mA
Synchronous Phase Angle	30	degrees
Detuning Angle	0	degrees
Generator phase during filling	0	degrees
Generator phase during beam	16.1	degrees
Vector modulator gain during filling	0.9	
Vector modulator gain during beam	0.9	
Vector modulator slew rate	1	degrees/uS
Beam injection time	0.1384	mS
Vector modulator start time	0.1304	mS
Beam Pulse Length	1	mS
Lorentz Detuning	0.0	kHz/MV

Table 1. Parameters for Example 1.

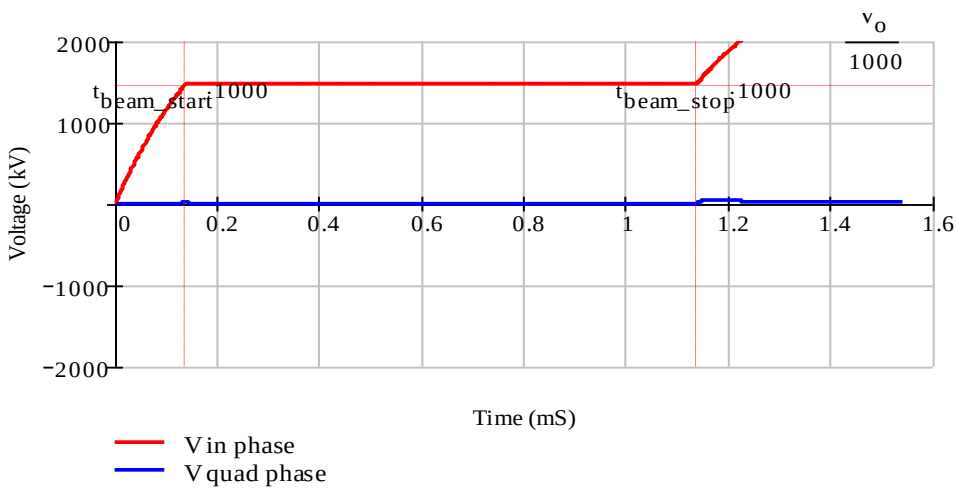

Figure 1. Cavity voltage for Example 1.

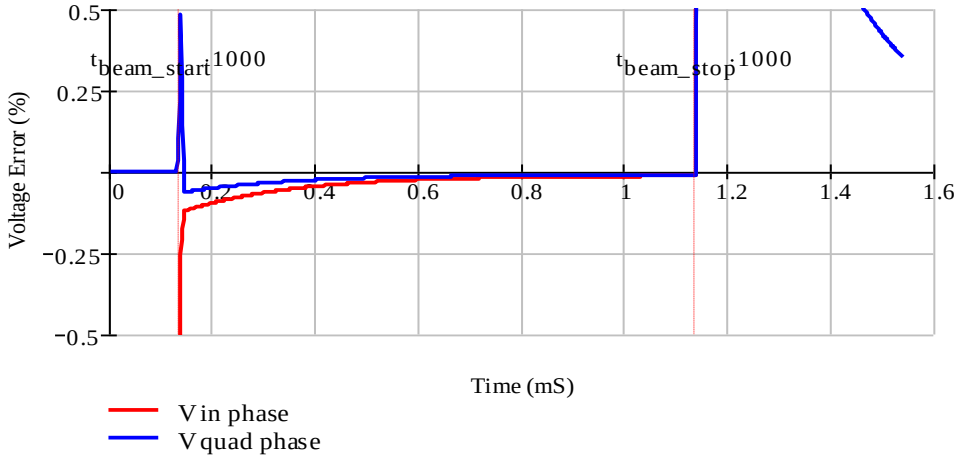

Figure 2. Cavity voltage error for Example 1.

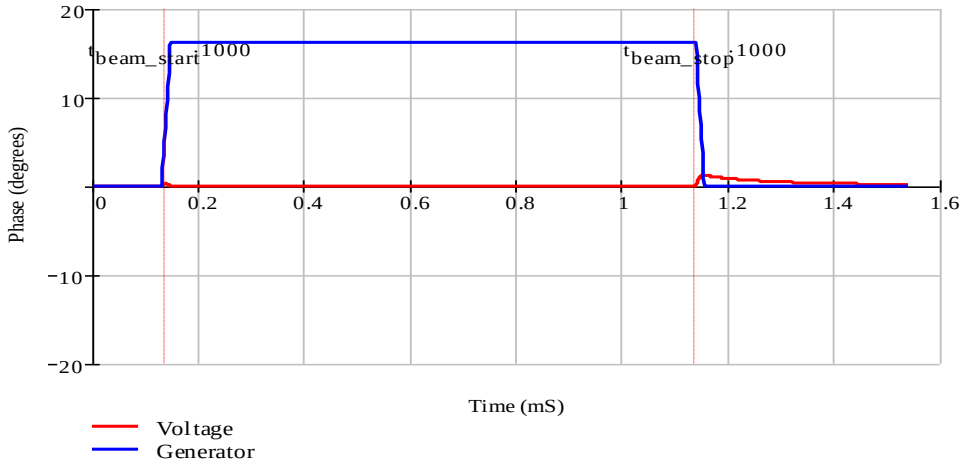

Figure 3. Voltage and Generator phase for Example 1.

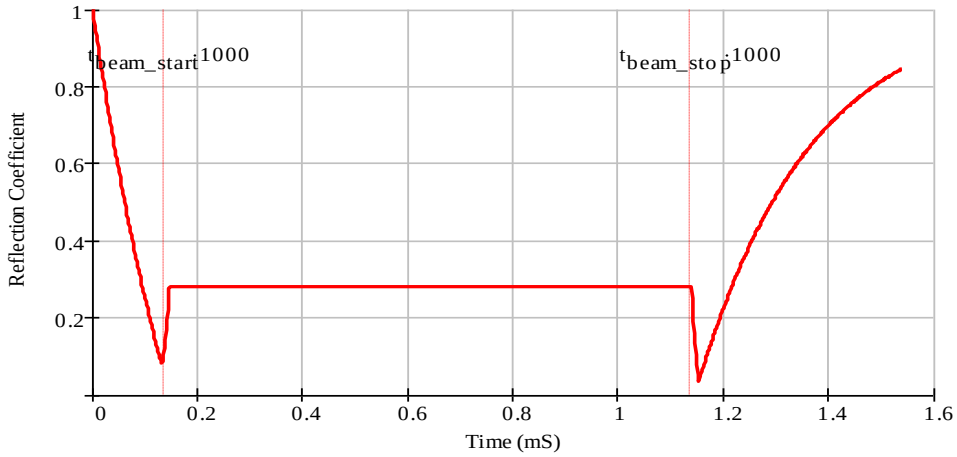

Figure 4. Reflection coefficient for Example 1.

Parameter	Value	Units
Cavity Voltage	1472	kV
Klystron Power	25.57	kW
R/Q	262	Ohms
RF Frequency	325	MHz
Reference Beam Current	15	mA
Actual Beam Current	15	mA
Synchronous Phase Angle	30	degrees
Detuning Angle	0	degrees
Generator phase during filling	0	degrees
Generator phase during beam	16.1	degrees
Vector modulator gain during filling	0.9	
Vector modulator gain during beam	0.9	
Vector modulator slew rate	1	degrees/uS
Beam injection time	0.1384	mS
Vector modulator start time	0.1304	mS
Beam Pulse Length	1	mS
Lorentz Detuning	0.1	kHz/MV

Table 2. Parameters for Example 2.

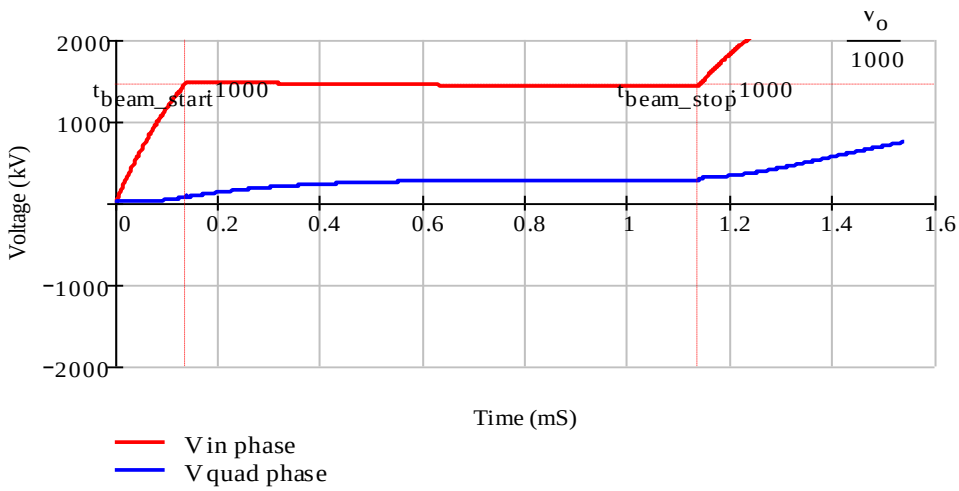

Figure 5. Cavity voltage for Example 2.

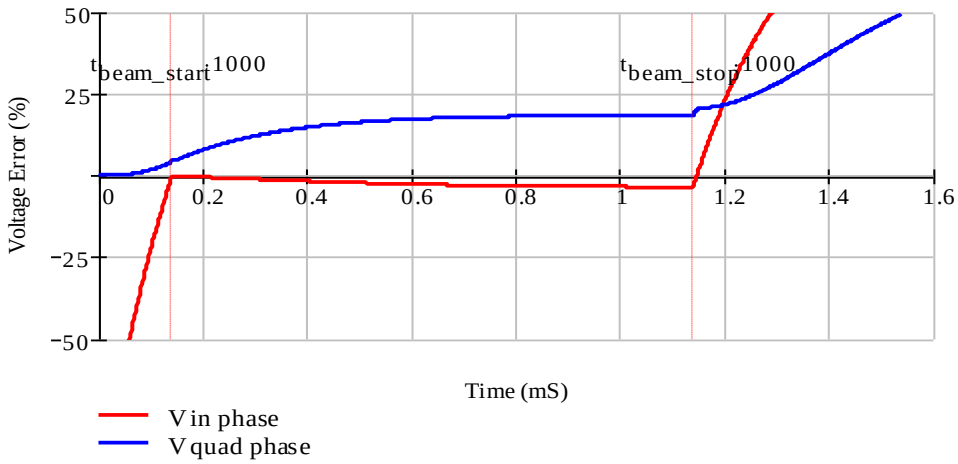

Figure 6. Cavity voltage error for Example 2.

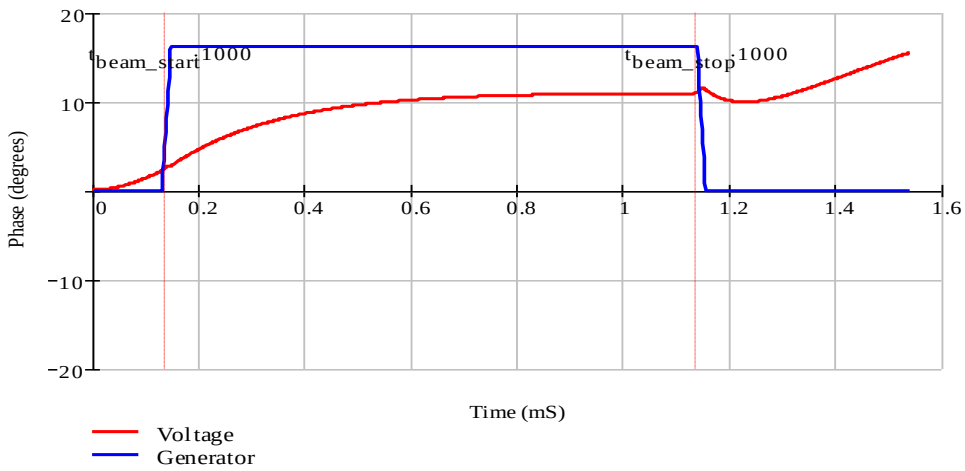

Figure 7. Voltage and Generator phase for Example 2.

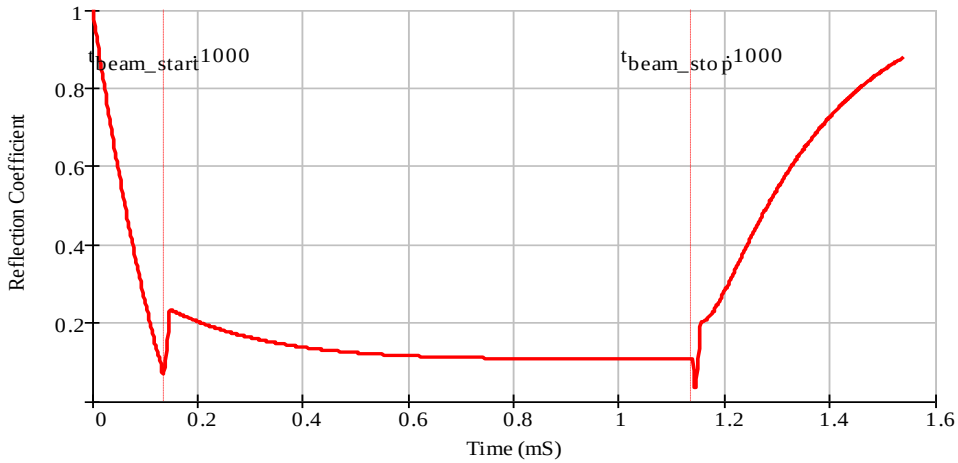

Figure 8. Reflection coefficient for Example 2.

Parameter	Value	Units
Cavity Voltage	1472	kV
Klystron Power	24.46	kW
R/Q	262	Ohms
RF Frequency	325	MHz
Reference Beam Current	15	mA
Actual Beam Current	15	mA
Synchronous Phase Angle	30	degrees
Detuning Angle	0	degrees
Generator phase during filling	-2.42	degrees
Generator phase during beam	10.8	degrees
Vector modulator gain during filling	0.899	
Vector modulator gain during beam	0.9	
Vector modulator slew rate	1	degrees/uS
Beam injection time	0.143	mS
Vector modulator start time	0.137	mS
Beam Pulse Length	1	mS
Lorentz Detuning	0.1	kHz/MV

Table 3. Parameters for Example 3.

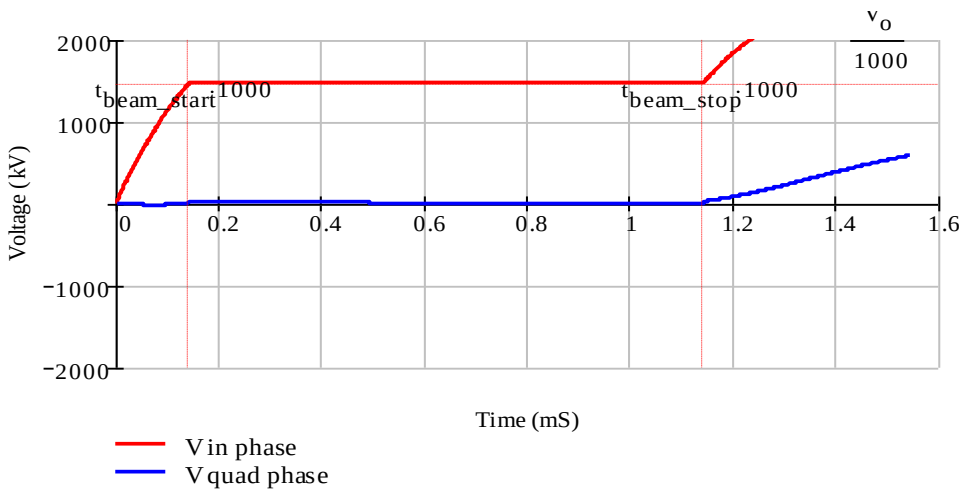

Figure 9. Cavity voltage for Example 3.

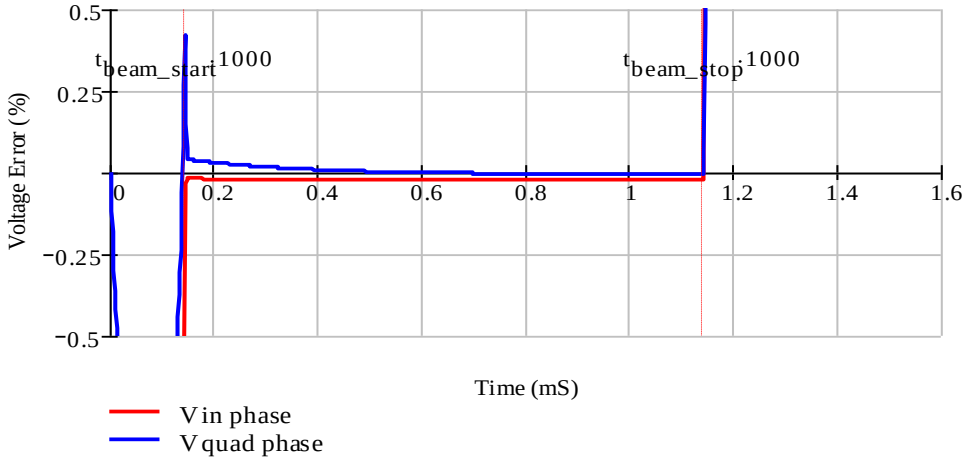

Figure 10. Cavity voltage error for Example 3.

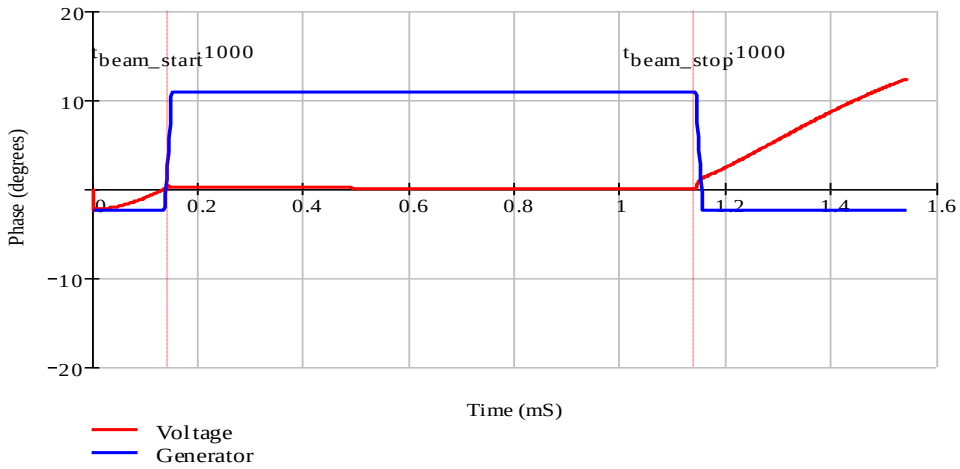

Figure 11. Voltage and Generator phase for Example 3.

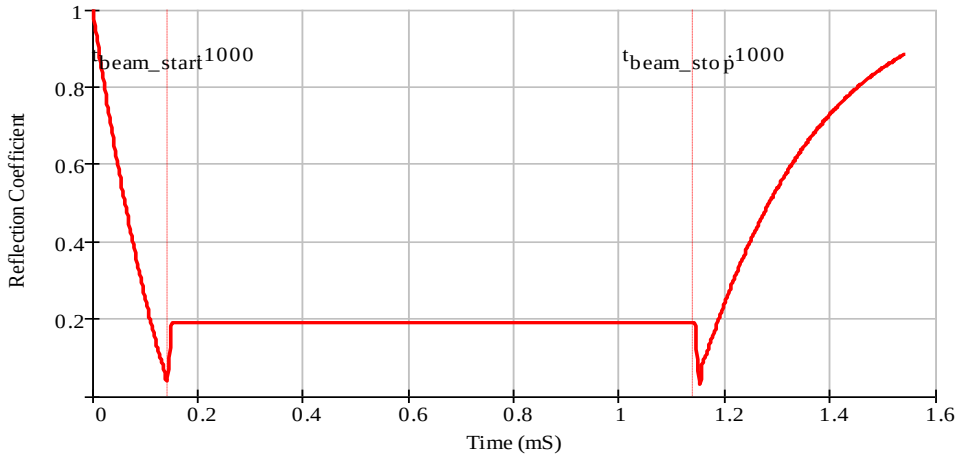

Figure 12. Reflection coefficient for Example 3.

Parameter	Value	Units
Cavity Voltage	1472	kV
Klystron Power	34.8	kW
R/Q	262	Ohms
RF Frequency	325	MHz
Reference Beam Current	15	mA
Actual Beam Current	15	mA
Synchronous Phase Angle	30	degrees
Detuning Angle	0	degrees
Generator phase during filling	-20.3	degrees
Generator phase during beam	-34.6	degrees
Vector modulator gain during filling	0.9	
Vector modulator gain during beam	0.9	
Vector modulator slew rate	1	degrees/uS
Beam injection time	0.1145	mS
Vector modulator start time	0.1085	mS
Beam Pulse Length	1	mS
Lorentz Detuning	1	kHz/MV

Table 4. Parameters for Example 4.

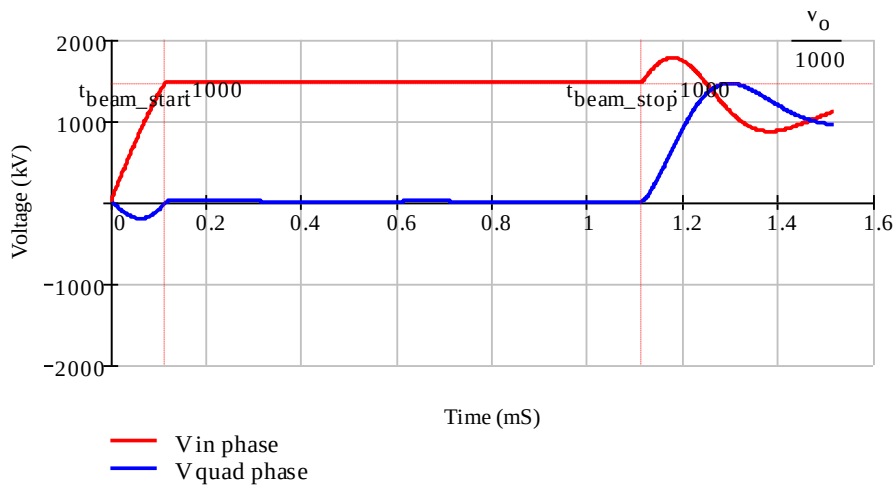

Figure 13. Cavity voltage for Example 4.

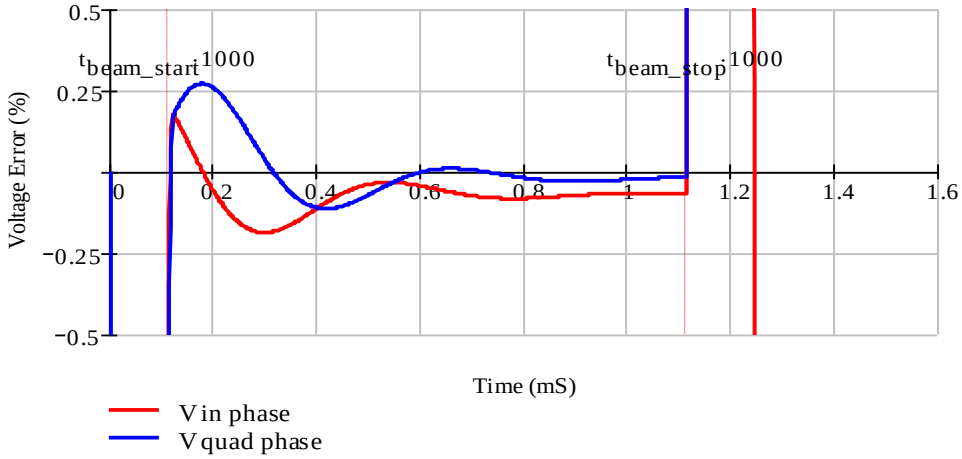

Figure 14. Cavity voltage error for Example 4.

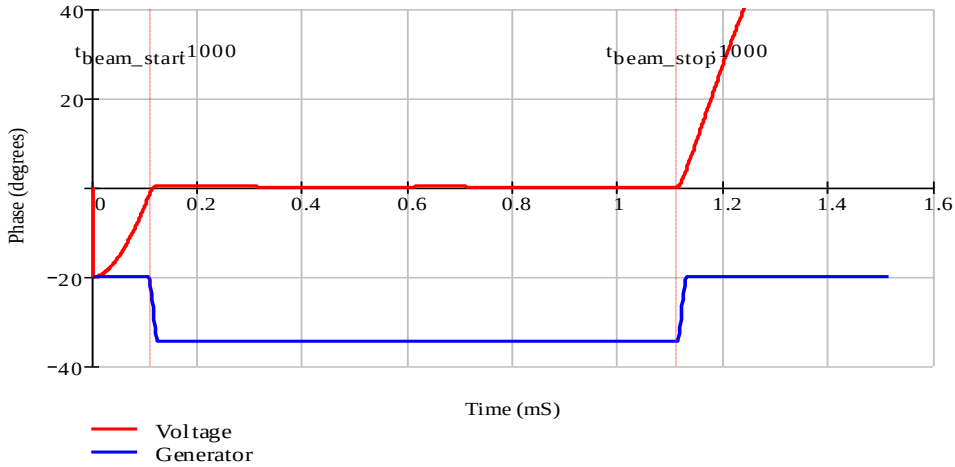

Figure 15. Voltage and Generator phase for Example 4.

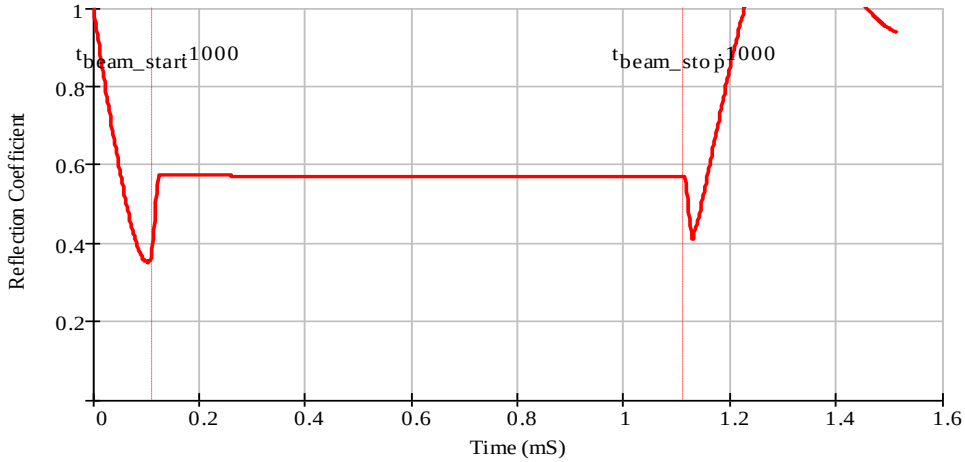

Figure 16. Reflection coefficient for Example 4.

Parameter	Value	Units
Cavity Voltage	1472	kV
Klystron Power	23.6	kW
R/Q	262	Ohms
RF Frequency	325	MHz
Reference Beam Current	15	mA
Actual Beam Current	15	mA
Synchronous Phase Angle	30	degrees
Detuning Angle	-54.1	degrees
Generator phase during filling	-1.3	degrees
Generator phase during beam	0	degrees
Vector modulator gain during filling	0.9	
Vector modulator gain during beam	0.9	
Vector modulator slew rate	1	degrees/uS
Beam injection time	0.147	mS
Vector modulator start time	0.141	mS
Beam Pulse Length	1	mS
Lorentz Detuning	1	kHz/MV

Table 5. Parameters for Example 5.

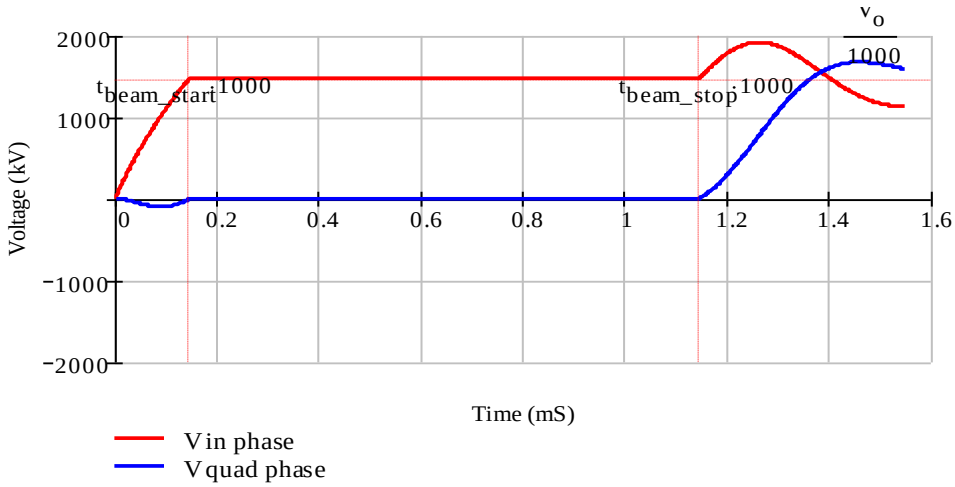

Figure 17. Cavity voltage for Example 5.

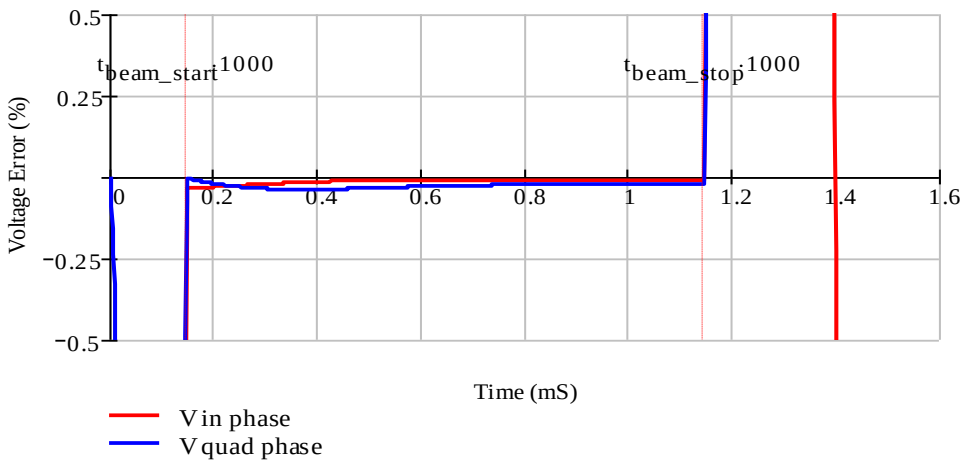

Figure 18. Cavity voltage error for Example 5.

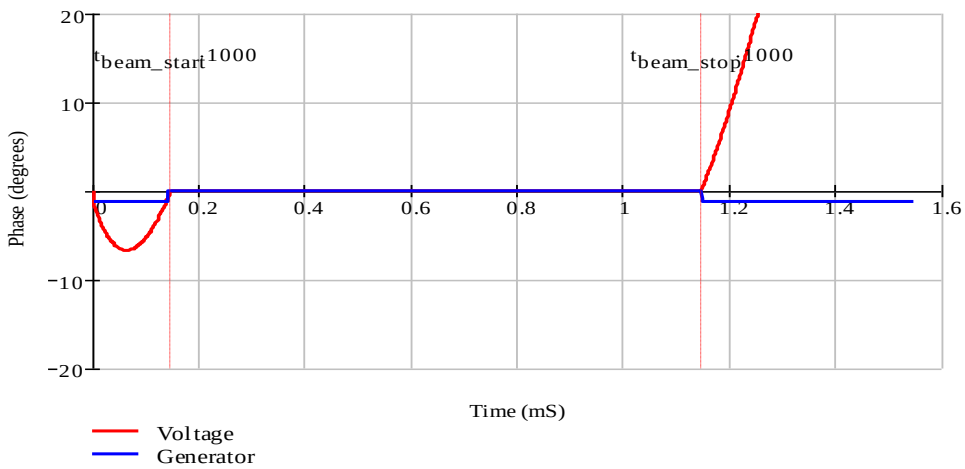

Figure 19. Voltage and Generator phase for Example 5.

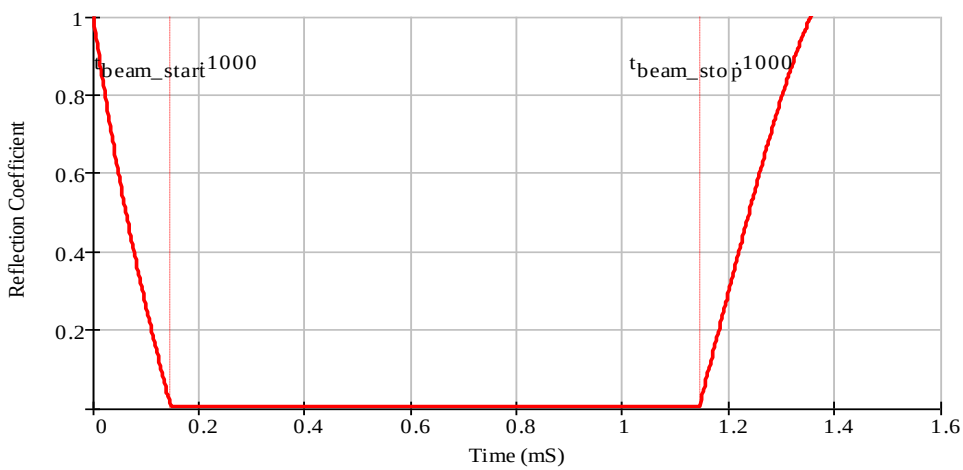

Figure 20. Reflection coefficient for Example 5.