Ring Beam Experiment

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A0 Photoinjector Review 2008 Aug 18

Motivation

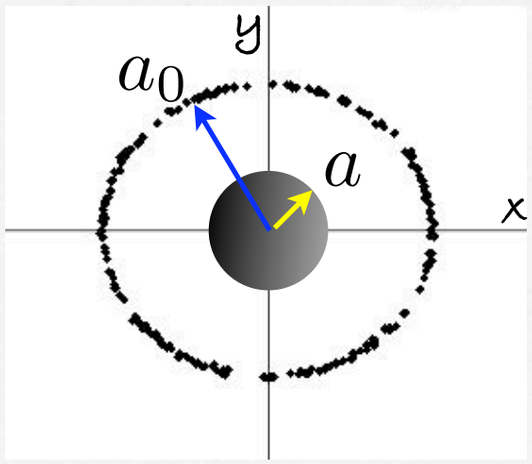
- Has been examined…
  - flat-beam transformation
  - trans-long transformation
- Using another suggestion of Derbenev, can we …
  - make the flat beam smaller in the bigger dimension?
  - ameliorate space charge effects by making a ring at the cathode?

A Possible Scenario

- use photoinjector with masked laser, say, to produce "ring" on the cathode to mitigate space charge
- pass through the flat beam transformer
  - this produces "no emittance" in one plane, and a "ring" in the phase space of the other d.o.f.
- inject into "new device" to reduce the larger dimension
- perform x-z exchange if desired
- ... could give much flexibility to a facility

S.C. Force within a ring

- assume uniform density $F \sim \frac{e^2\rho}{2\gamma^2\epsilon_0}\frac{r^2-b^2}{r}, \qquad b < r < b + da$
- make ring of ave. radius a_o, thickness da, with same area as circle of radius a ($<a_o$); i.e., choose: $2\pi a_0 da = \pi a^2$
- space charge force at radius a of smaller circle is $F \sim \frac{e^2\rho}{2\gamma^2\epsilon_0} a$
- s.c. force at radius ao+da/2 of the "ring" is $F \sim \frac{e^2\rho}{2\gamma^2\epsilon_0} \cdot 2da$
- So, might expect a reduction by factor...



$$\frac{F_{ring}}{F_{circle}} = \frac{2da}{a} = 2\frac{da}{a_0}\,\frac{a_0}{a} = \frac{a^2}{a_0^2}\,\frac{a_0}{a} = \frac{a}{a_0}$$

Flat Beam x-form of a "ring"

Initial Conditions:

$$\vec{X}_0 = \begin{pmatrix} x_0 \\ 0 \\ y_0 \\ 0 \end{pmatrix} = \begin{pmatrix} r_0 \cos\theta_0 \\ 0 \\ r_0 \sin\theta_0 \\ 0 \end{pmatrix}$$

Exiting solenoid field:

$$\vec{X}_s = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{pmatrix} \vec{X}_0 = \begin{pmatrix} x_0 \\ -k\, y_0 \\ y_0 \\ k\, x_0 \end{pmatrix}.$$

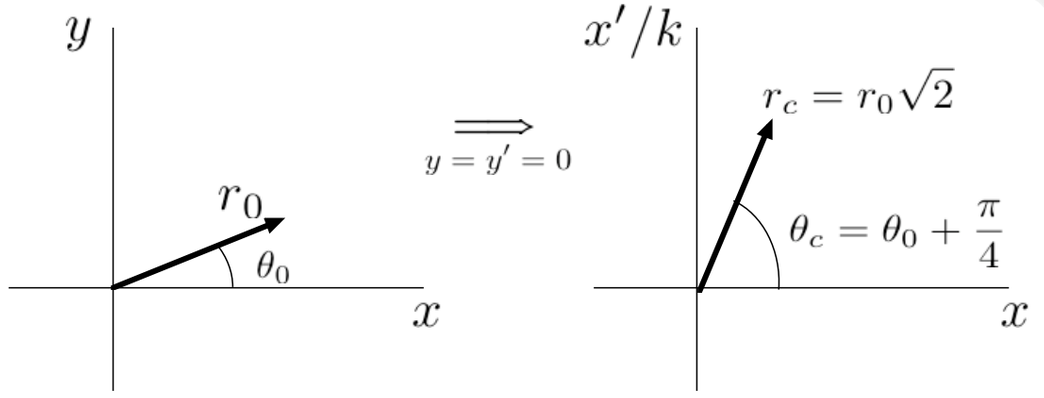
Through a skew quad channel:

$$\vec{X}_f = R^{-1} M_c R \vec{X}_s$$

$$R = \frac{\sqrt{2}}{2} \cdot \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \qquad M_c = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 \\ 0 & 0 & -1/\beta_0 & 0 \end{pmatrix}$$

Final result:

$$\vec{X}_f = \begin{pmatrix} x_0 - y_0 \\ k(x_0 + y_0) \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} r_0(\cos\theta_0 - \sin\theta_0) \\ k r_0(\cos\theta_0 + \sin\theta_0) \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} r_c \cos\theta_c \\ k r_c \sin\theta_c \\ 0 \\ 0 \end{pmatrix}.$$



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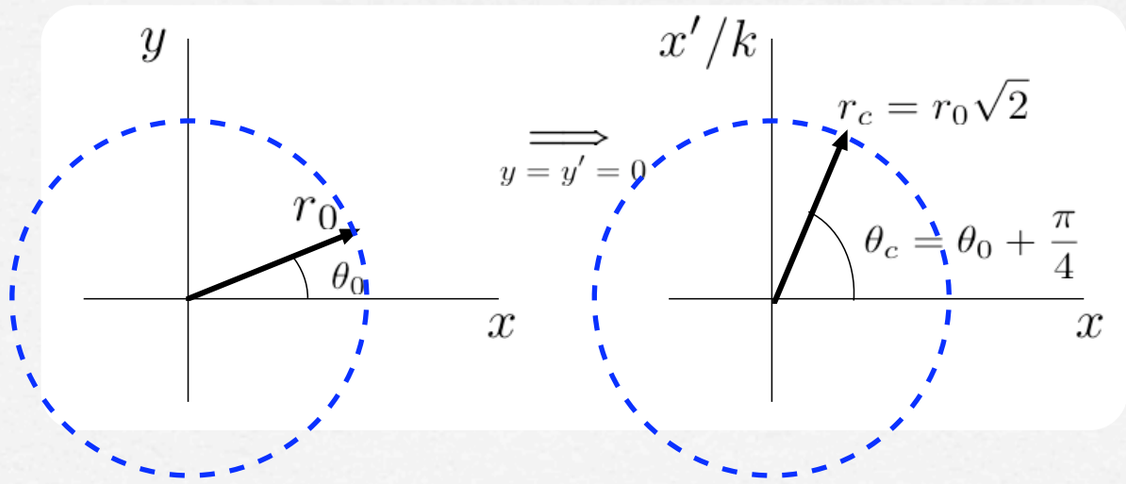


Turn "ring" into "bunch"

- Derbenev noted the process in NIM-A ***441*** (2000) *221-233*.
- Akin to adiabatic capture in synchrotron RF systems (h=1)
- Assume a focusing channel or ring, with a repeat period of length L that has phase advance near, but just below, 2π;
  - use octupole fields within the channel to give an amplitude dependent tune shift; adjust octupole to produce phase advance of exactly 2π at an amplitude of a_o (which will be the radius of the initial ring distribution in phase space).
  - Then, introduce a gradually increasing dipole field at the end of each repeat period (could be distributed, w/ proper phasing)

Parameterization

- small amplitude phase advance of channel: $2\pi - \Delta\phi$
- phase shift with amplitude due to octupole field...
  - do not want octupole field to contribute to Δa; only $\Delta\psi$; then, $\Delta B_8 = \left(\frac{B'''(s)}{6}\right) x^3 = \left(\frac{\beta_0}{\beta}\right)^3 \left(\frac{B'''}{6}\right)_0 x^3$ yields $\Delta a = 0$
  - define $\Theta_8 \equiv (B'''/6)_0 \, a_0^3 L/(B\rho)$, then $\Delta\psi_8 = \frac{3}{4}\frac{\pi\beta_0}{L}\frac{\beta_0\Theta_8}{a_0^3}(a^2 - a_0^2)$
- dipole kick: $\Theta_2 \equiv B_0 L/(B\rho)$
- total kick (depicting ramped field): $\Theta_2 \cdot b$, where $0 < b < 1$
- resulting Hamiltonian: $\mathrm{H} = \mathrm{k}\left[\frac{3\Theta_8}{16}(r^2-1)^2 - \Theta_2 br\cos\psi\right]$

  (See details in Edwards/Syphers note) $\quad \mathrm{k} \equiv \frac{\pi\beta_0}{L}\frac{\beta_0}{a_0} \quad \mathrm{r} \equiv a/a_0$

Resulting Hamiltonian

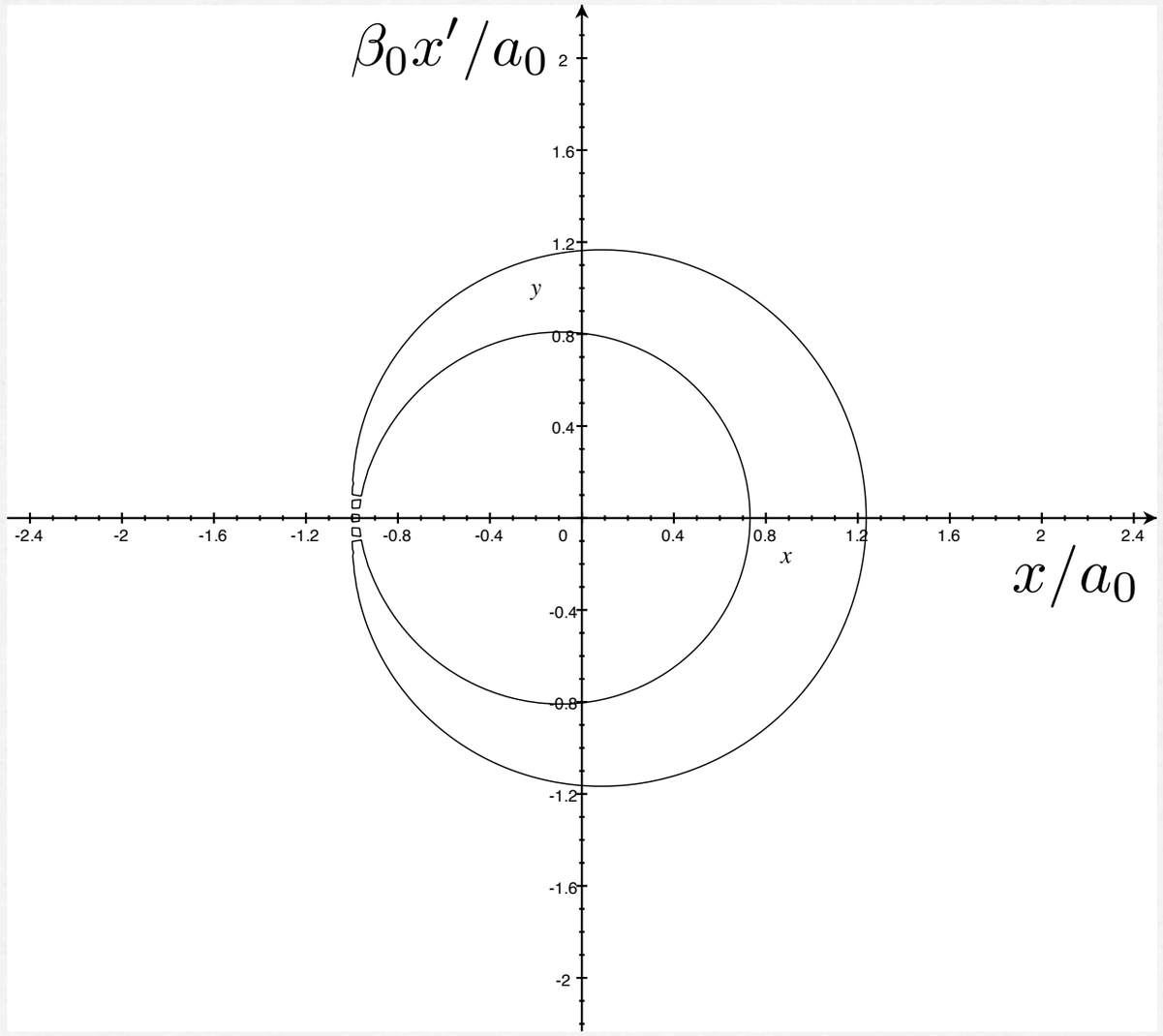
- fixed points at $\psi = 0$ (stable), $\psi = \pi$ (unstable)
- for full "b", island at $r \sim 1$ (i.e., $a \sim a_o$) of width:

$$\mathrm{w} = 2\left(\frac{8}{3}\frac{\Theta_2}{\Theta_8}\right)^{1/2}$$

- island "tune": $\nu_s \equiv \left[\frac{3}{32\pi^2}k^2\Theta_2\Theta_8 b\right]^{1/2}$
  - must be sufficiently small, compared with unity, for adiabatic process

Expected Phase Space

- use as input parameters:
  - given L, determine β_0
  - scale dimensions w/ a_0
  - choose $\Delta\phi$, w
  - from which:
    - $\Theta_8 = \frac{4La_0}{3\pi\beta_0^2}\Delta\phi$
    - $\Theta_2 = \frac{3}{32}\Theta_8 w^2$



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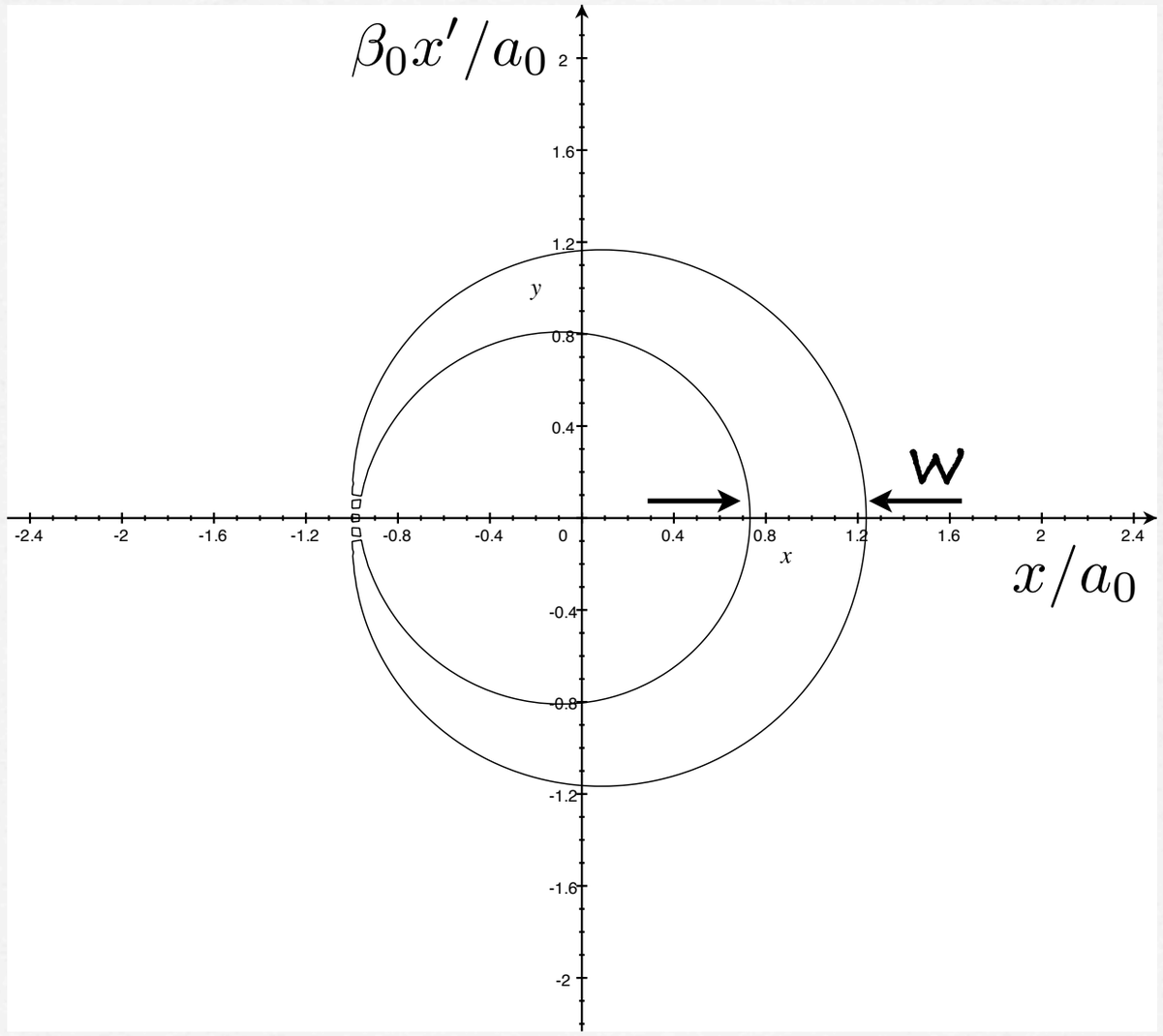

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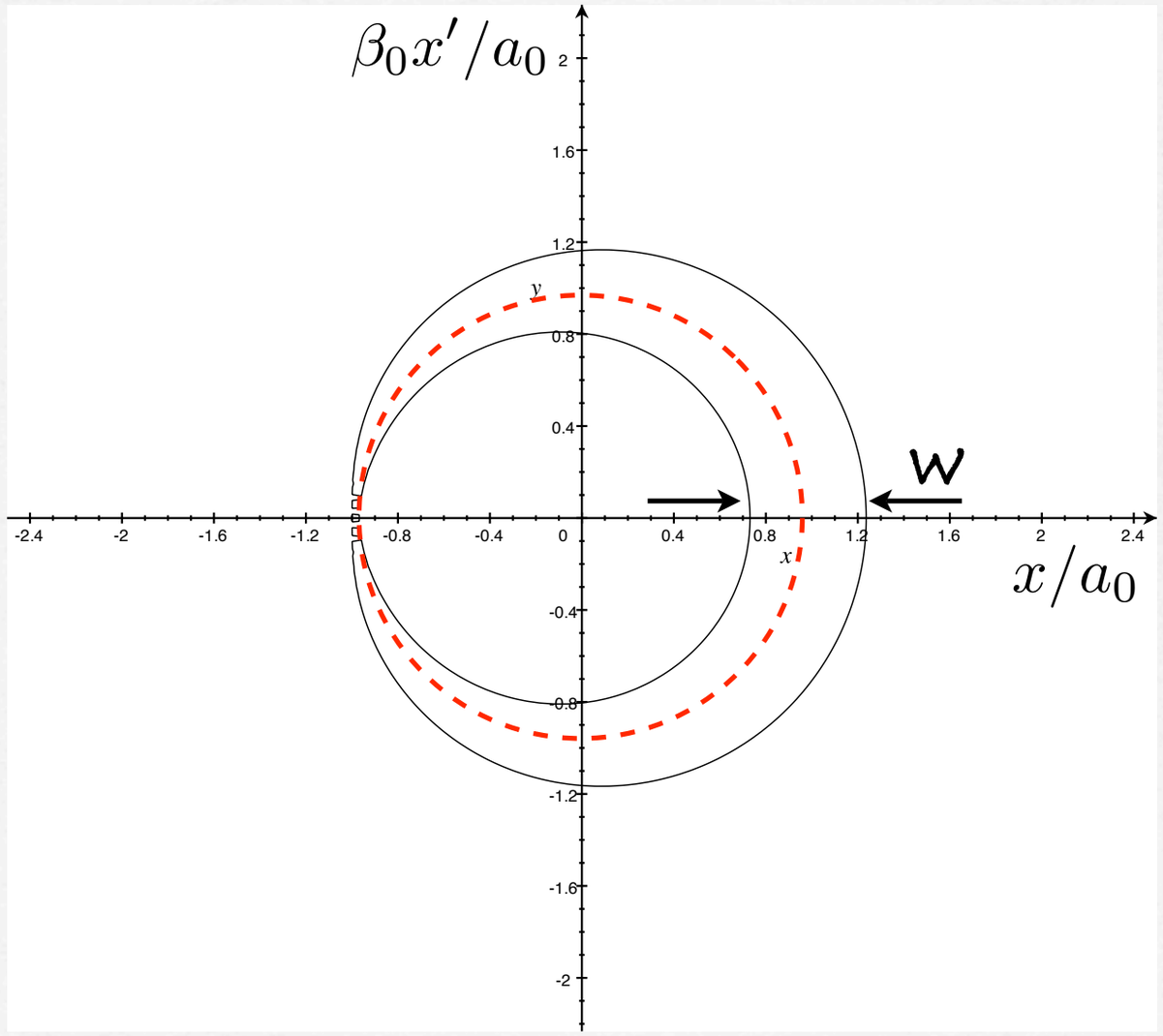

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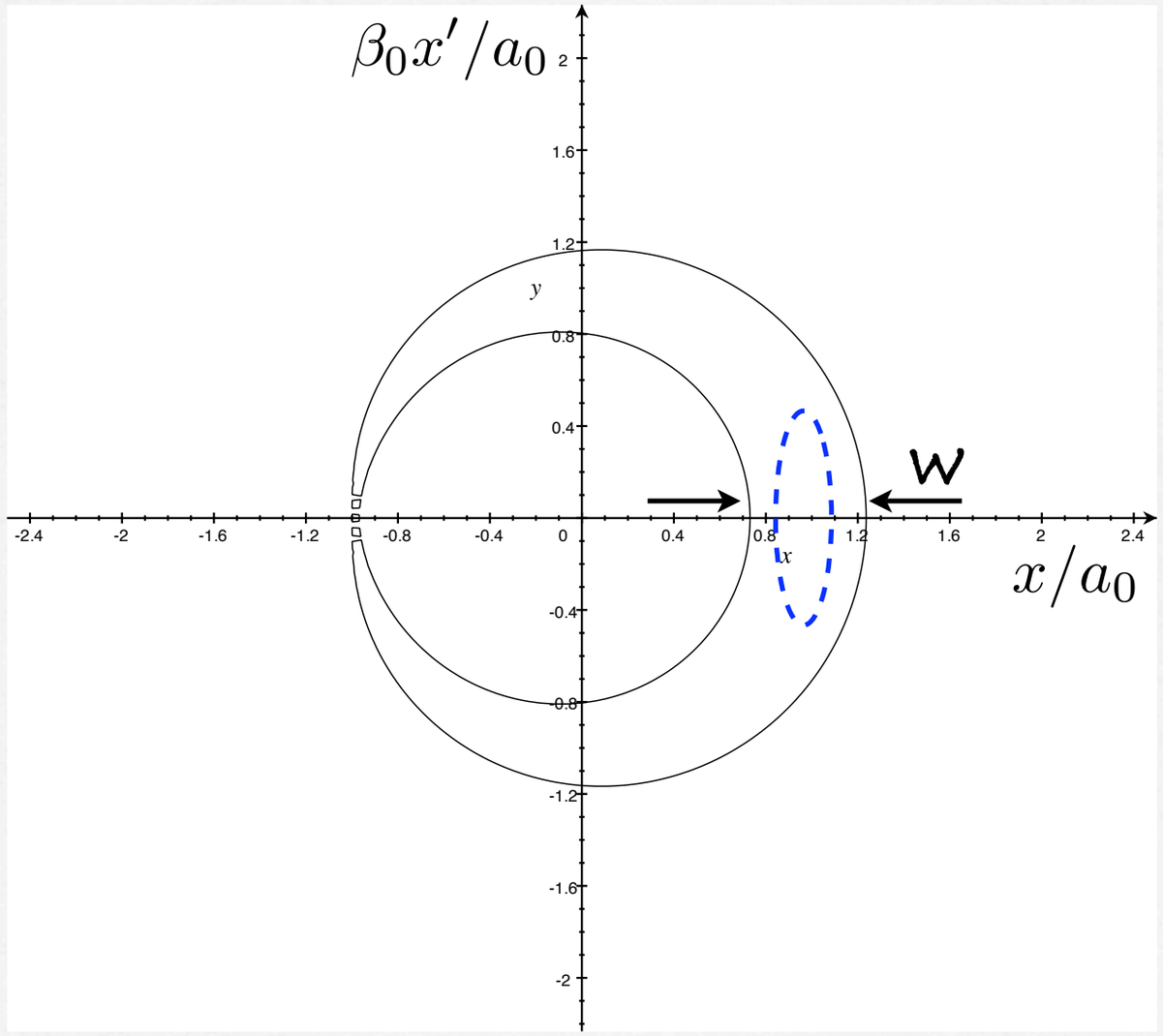


A Device?

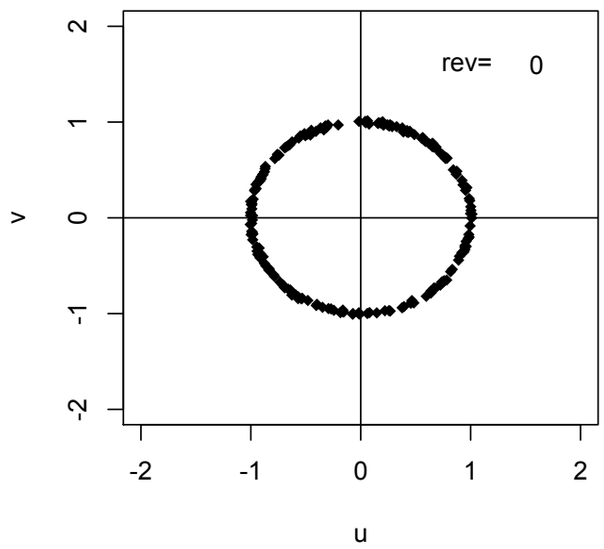
- How would one make a device?
  - FODO channel -- how long?
    - communications from others who have looked at this -- "process takes long time"
  - use FODO ring? -- how big?
  - try weak focus ring?
    - easiest to quickly examine...

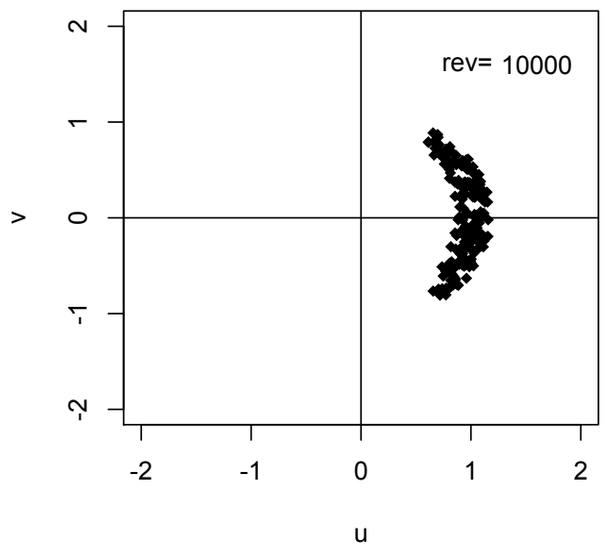
Numerical Simulation

- assume weak focusing, circumference L
  - tune $= \sqrt{(1-n)}$; as an example, choose n such that tune $= 15/16$, $\Delta\phi = 2\pi/16$
- take $u = x/a_0$, $v = \beta_0 x'/a_0$; choose $w = 0.5$
- divide ring into m sections, at the center of which an octupole kick is received:
  - $\Delta v = -3/8\ \Delta\phi/m\ u^3$
- after m sections, $s = L$ and dipole kick is provided:
  - $\Delta v = 1/4\ \Delta\phi\ w^2\ b$, $0<b<1$ (ramped dipole)

Result

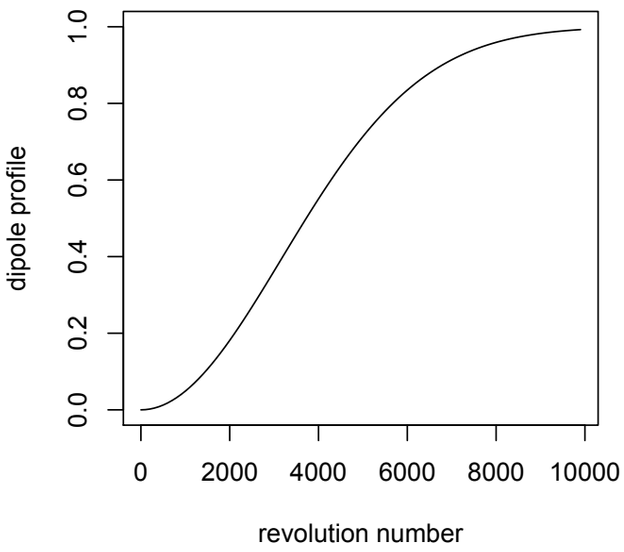
initial distribution

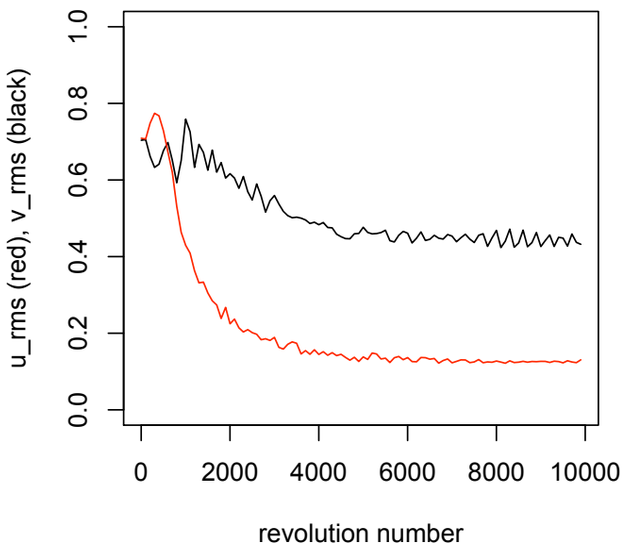




"final" distribution

adiabatically ramp the dipole





rms of distribution

position (red)
angle (black)

Animation

Parameters of presumed ring:

$L = 2\ m$

$B_0 = 0.314\ T$

$\nu = \Delta\phi/2\pi = 15/16$

$\beta_0 = 0.34\ m$

$pc = 30\ MeV$

$a_0 = 4\ mm$

$w = 0.5$

$\Theta_8 = 12.3\ mr$

$\Theta_2 = 0.289\ mr$

$B_2L = 0.03\ T\text{-}mm$

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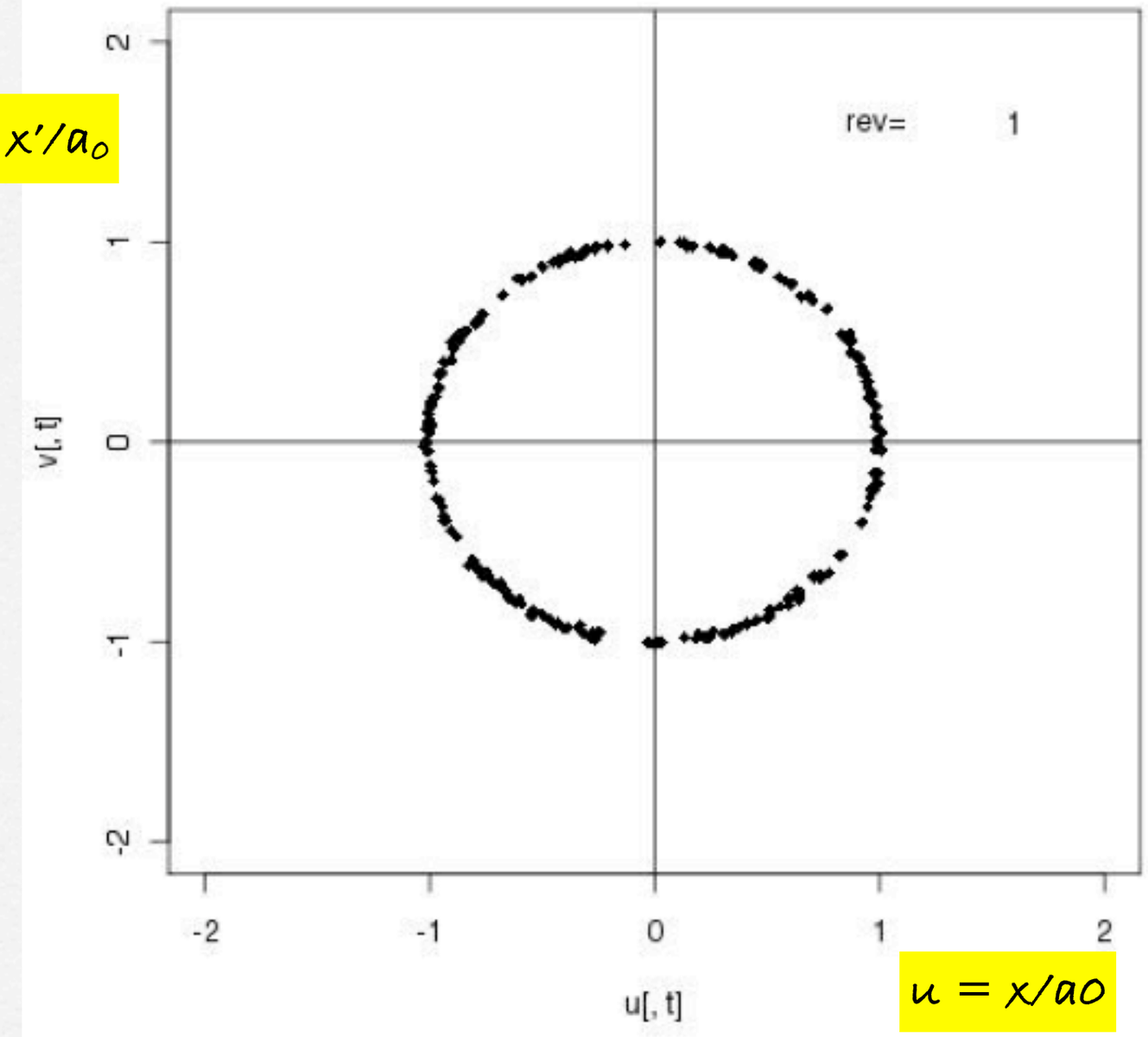

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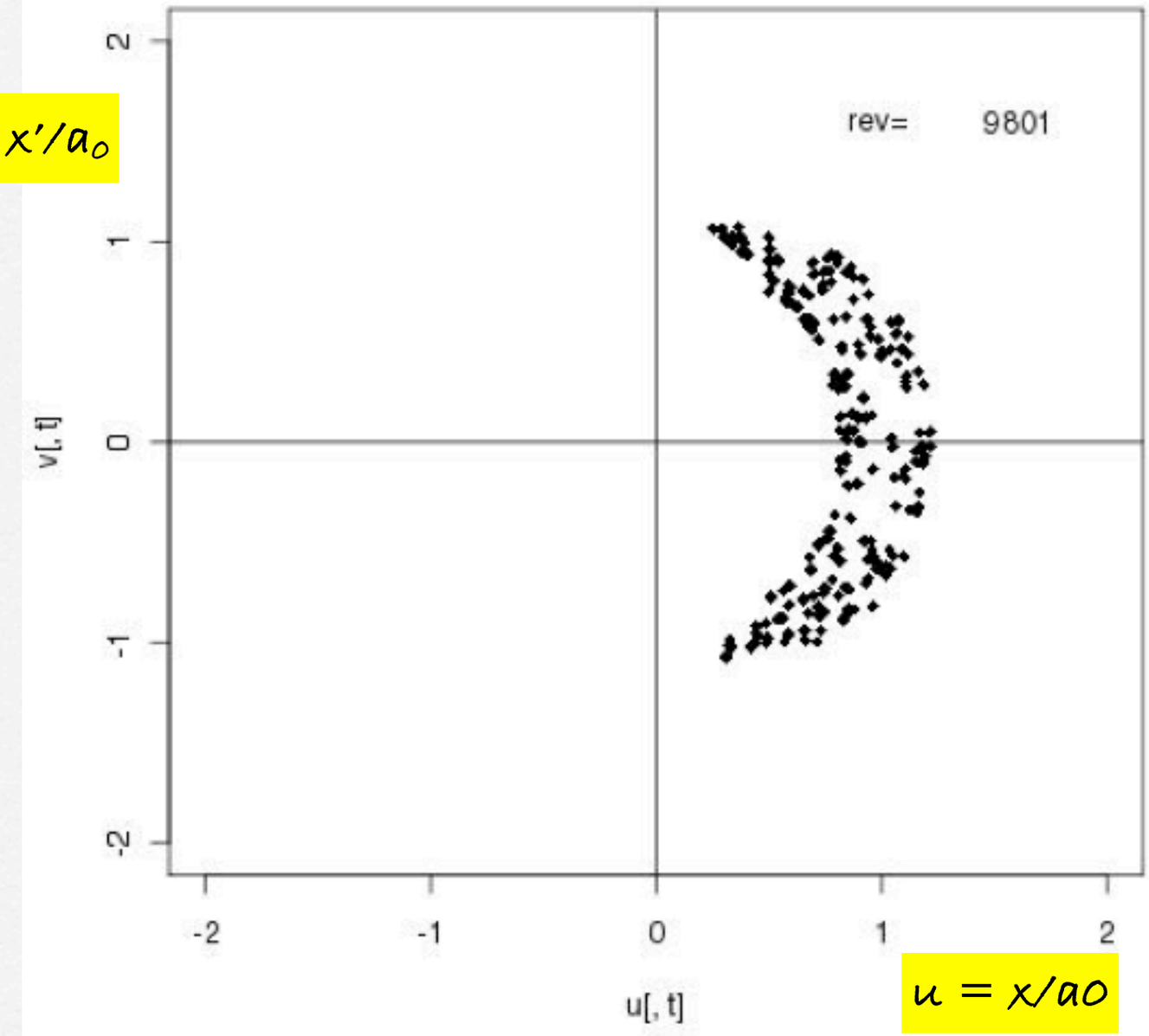

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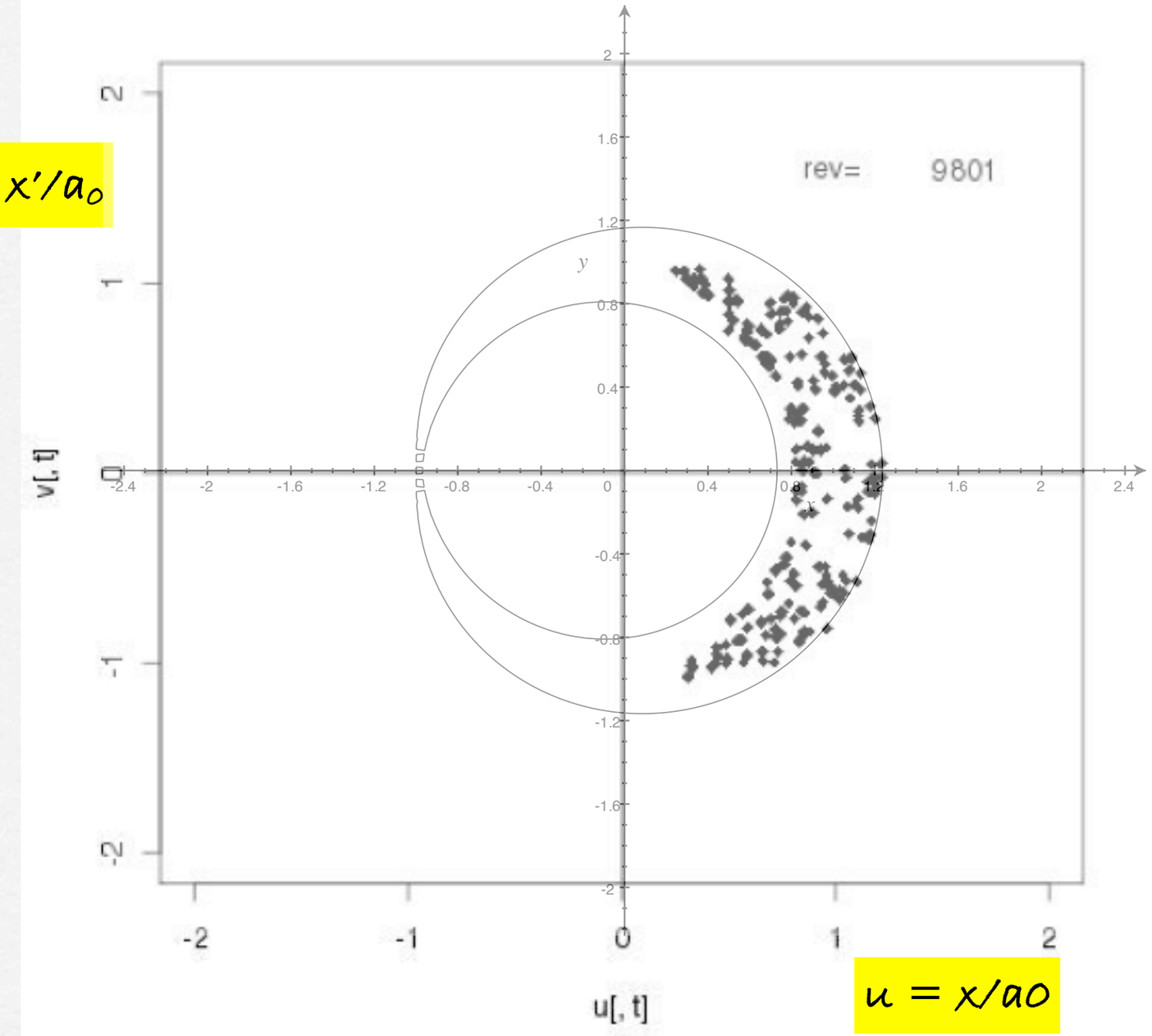

$u = x/a0$

Remarks

- Type of device
  - if channel, very very long
  - if ring, worry about bunch lengthening (isochronous ring?), chromatic effects, etc.
- Possible Further Studies
  - distribute dipole field along channel/ring
  - how far from adiabaticity can be tolerated?
- With Liouville in mind, only makes sense if a "ring beam" off the cathode helps with space charge effects

Remarks

- Could make nice student project; "too large" (?) for one student, if large-scale ring or channel is involved; could be enough work for several students
  - needs further thought -- can be scaled down?

References

[1] Ya. S. Derbenev, "Advanced optical concepts for electron cooling", Nucl. Instr. and Meth. A **441** (2000) 221-233. See the discussion beginning at the bottom of page 226.

[2] D. A. Edwards, M. J. Syphers, "Introduction to the Physics of High Energy Accelerators", J. Wiley and Sons, 1993. The material used here begins on page 116 of this textbook.

[3] Private communications from R. Brinkmann, K. Flöttmann, S. Nagaitsev

[4] D.A. Edwards, M.J. Syphers, "Comments on an Experiment on Transverse Coalescing," unpublished.