Summary of the new options in BETACOOL code

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1. Initial distribution from file

If one chooses "Beam | Parameters | Injection | Initial distribution From File" then the emittances and momentum spread will be recalculated in accordance with the particle distribution from "injection file". Note: this option only works if one pushes 'Open' on 'Draw distribution'. If one directly starts the calculation with the Model Beam algorithm the emittances and momentum spread will not be recalculated in the "Beam | Parameters | Emittance" window but the correct values will be displayed on the plots.

2. Output plot files

The format of the output plot files has been completely changed. Now, "*.cur" files can include several curves. In the Graph dialog, one can choose which columns one wishes to use for a certain curve (parameters X-axis and Y-axis on Fig.1). This option essentially decrease the number of "*.cur" files. For instance, the "Space.cur" file includes the 6-dimension distribution of the model particles. It means that one can easy save a particle distribution after a calculation and start a new calculation with the results of a previous run, i.e. one can use "Space.cur" as an initial distribution on Beam | Parameters | Injection | Initial distribution "File with initial distribution".

Also one can manage the absolute scale for the axes. It helps to output the plots in the units one wishes with the parameter "Scale" (Fig. 1). One can manually change the title of the axes. Of course, for example, if for otherwise identical parameters, the energy of the particles is changed, the scale parameters will have to be recalculated to obtain the same absolute units.
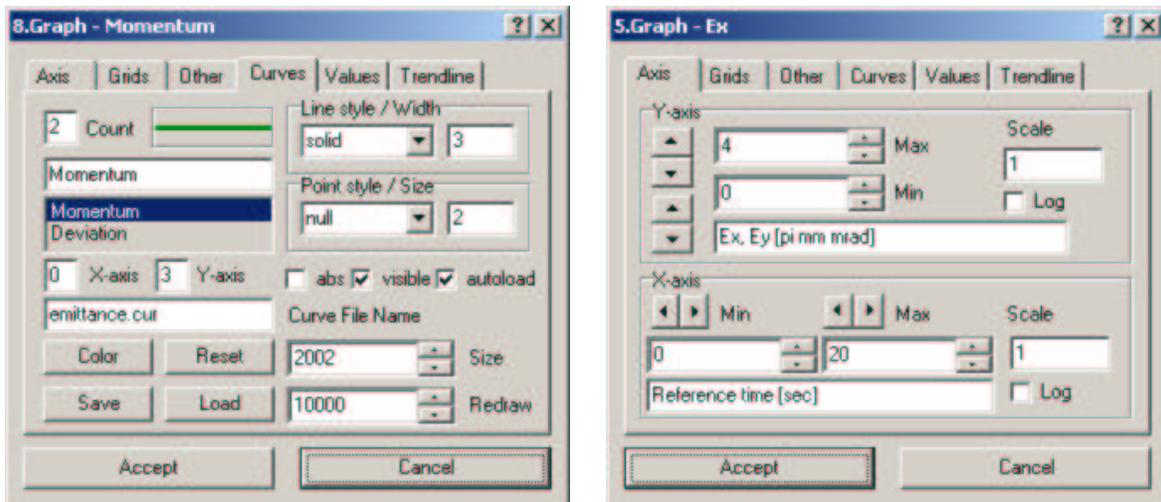

Fig. 1 Graph dialog windows

3. Parabolic model of the electron beam

The parabolic distribution of the electron beam density was introduced in the BETACOOL code for simulation of electron cooling at the Recycler. The density in the center of the electron beam is calculated as

$$n_0 = 2 \frac{I_e}{\pi a^2 e \beta c},$$

(1)

where I_e – electron current, a electron beam radius, e – electron charge, βc – electron velocity. The dependence of the electron density on the radius for a parabolic distribution is defined as

$$n_e(r) = n_0 \left(1 - \frac{r^2}{a^2} \right)$$

(2)

where r – current coordinate of the ion. The BETACOOL window with the parameters for the parabolic distribution of the electron beam is shown on Fig. 2. The gradient $G_\perp$ was introduced for the description of the dependence of the relative transverse velocity on the radius:

$$V_\perp^2 = T_\perp / m_e + G_\perp^2 r^2$$

(3)

where $V_\perp$ - relative transverse velocity, $T_\perp$ - transverse temperature, m_e – electron mass.
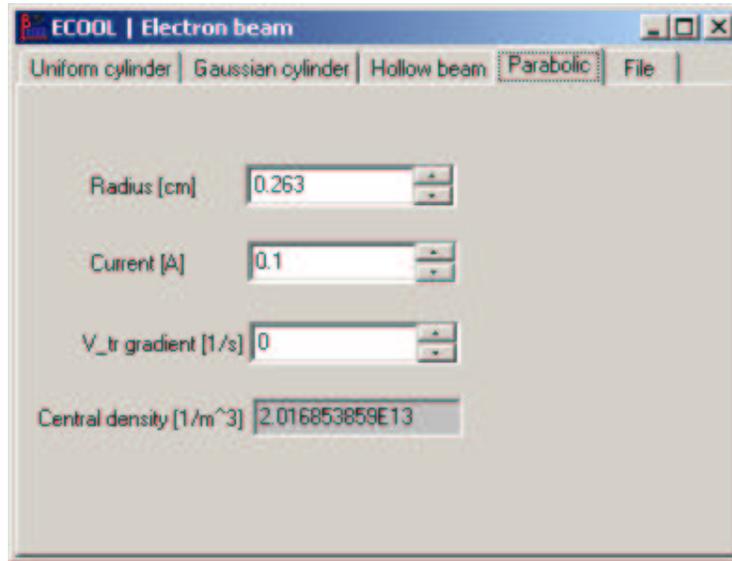

Fig. 2 Parameters of parabolic distribution of the electron beam.

4. Distribution of electron density from file

The distribution of the electron beam on the radius can be calculated in other programs and the results from these calculations can be read into the BETACOOL program as text files. The BETACOOL window for reading the external files with radial distribution of the electron density and transverse gradient is shown on Fig. 3. The first column of the file is the radial coordinate in

mm, the second column is the electron density in A/cm^2, and the third column is the transverse gradient G$_\perp$ in 1/sec. If the third column is absent in the file then the constant transverse gradient will be used from the parameter window (Fig. 3). An example of a text file with the electron beam density is shown on Fig. 4.
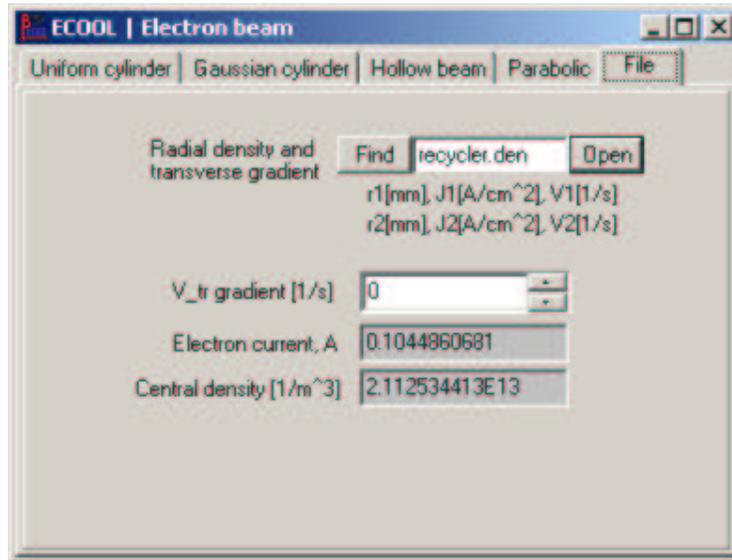

Fig. 3 Window for reading the electron beam density and
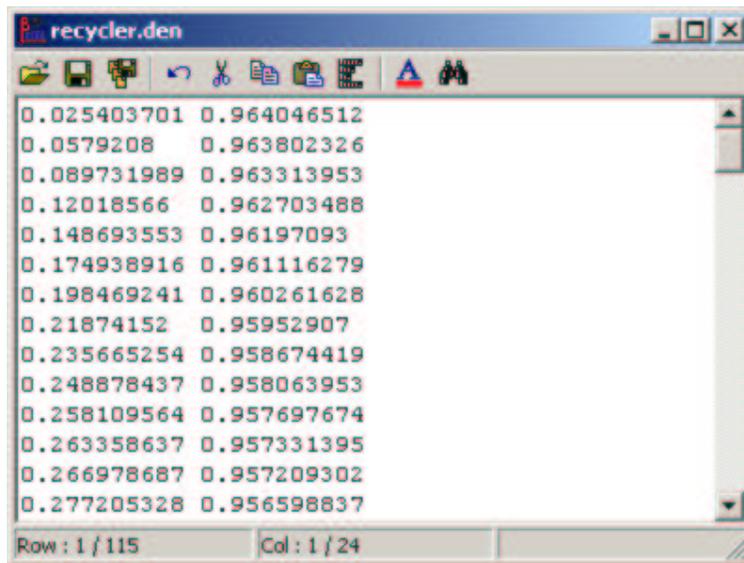transverse gradient from external text file

Fig. 4 Example of a text file with electron beam density.

5. Normalized emittance and energy spread

In the BETACOOL program, the transverse emittance is defined as one sigma un-normalized emittance and momentum spread is defined in the relative value $\Delta P/P_0$. At the Recycler, the transverse emittance is defined as a 95% normalized emittance and the momentum spread is defined in absolute value MeV/c. Now, the user can input the transverse emittance and momentum

spread in the format of the BETACOOL definition and check these parameters in the other units (Fig. 5). The normalized emittance is calculated from the one sigma un-normalized emittance as:

$$\varepsilon_{norm} = \beta\gamma n_\sigma \varepsilon_{rms}, \quad (4)$$

where $\beta\gamma$ - relativistic factors, $n_\sigma = -2 \cdot \ln(1 - \alpha/100)$ - number of sigma calculated from the percentage of the enclosed particles number α [%] for a Gaussian distribution. For the Recycler measurements α=95% which corresponds to n_σ =5.991 rms sigma. The momentum spread in absolute value can be calculated from the relative momentum spread as:

$$\left(\frac{\Delta P}{P_0}\right)_{abs} = \left(\frac{\Delta P}{P_0}\right)_{rel} M \, [\text{GeV/c}], \quad (5)$$

where M – is the particles momentum in GeV/c (Fig.1.4). The energy spread is defined as:

$$\left(\frac{\Delta E}{E_0}\right)_{abs} = \frac{\gamma+1}{\gamma}\left(\frac{\Delta P}{P_0}\right)_{rel} K \, [\text{GeV}], \quad (6)$$

where K – is the particles kinetic energy. If the kinetic energy was defined per nucleon then the energy spread will also be defined as [GeV/u].
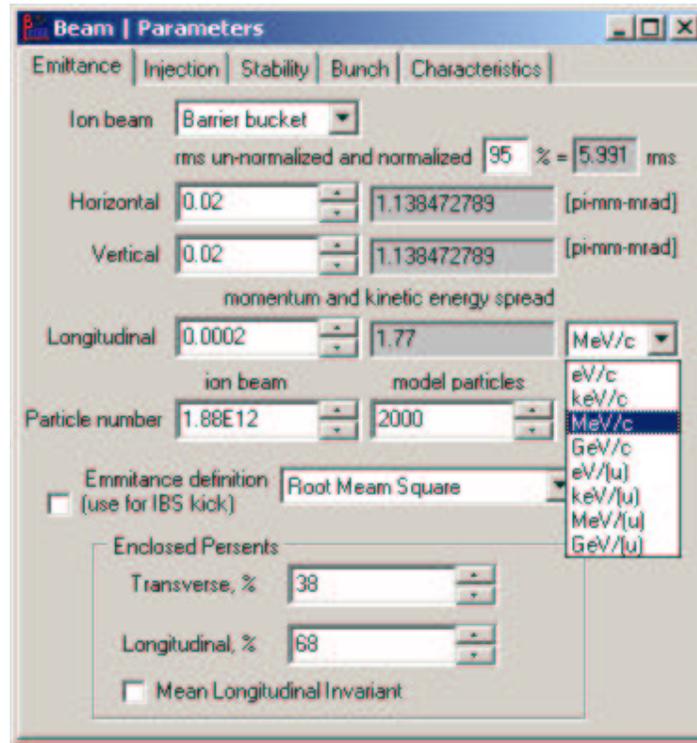

Fig. 5 Beam parameters window.

The evolution of transverse emittances and momentum spread can also be plotted with different definitions (Fig. 6): transverse emittance – one sigma un-normalized or normalized with certain percent value of the particle number; momentum spread – relative or absolute value. These definitions correspond to the parameters chosen on the Beam Parameters window (Fig. 5).
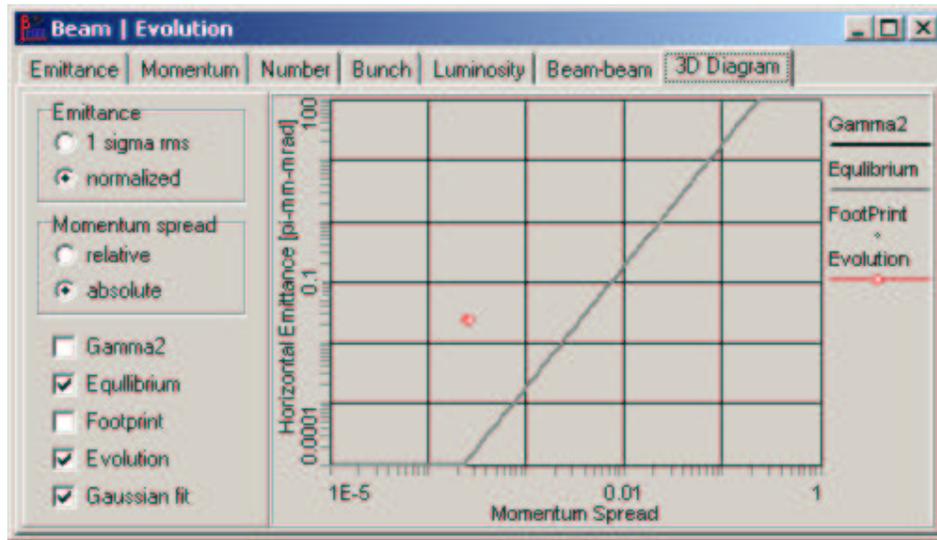
Fig. 6 Control of beam evolution graphics.

5. Barrier bucket models

5.1. Two barriers

The Ring Parameters window with barrier buckets is shown on Fig. 7. Option **2 barriers** is used for the simple model of two barriers which has an analytical solution and a numerical approximation. **Analytic** solution is valid for the region between barriers only. If the particle momentum spread becomes larger than the barrier height then the particle does not move in the longitudinal direction. The numerical solution simulates the longitudinal particle dynamics around the whole ring. If the initial distribution of particles is not chosen from a file then the parameters of the **2 barriers** model are used for the initial longitudinal distribution of particles and will be fitted in accordance with barriers parameters – particles can not have a momentum spread larger than the barrier height.
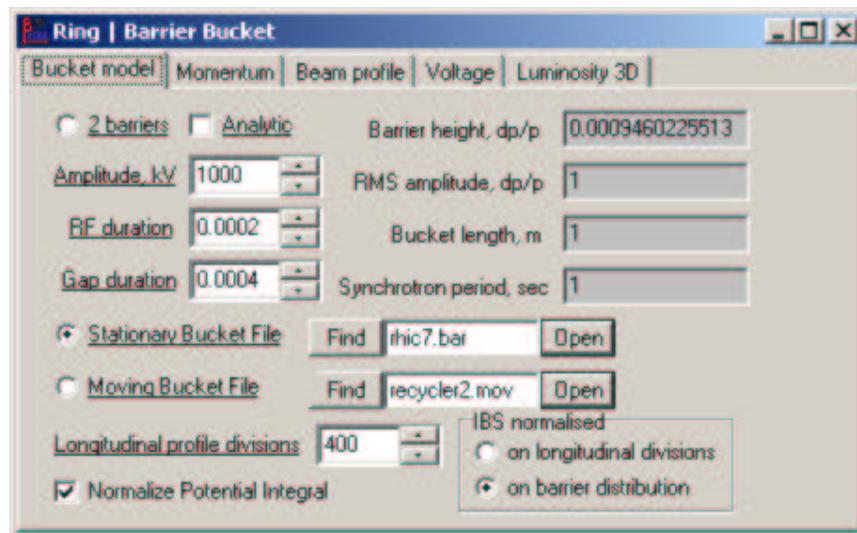

Fig. 7 Window with barrier bucket parameters.

Amplitude, kV defines the amplitude of the barrier. **RF duration** – barrier width in circumference units; **Gap duration** – distance between barriers in circumference units. **Barrier**

height, dp/p calculates the barrier height in relative momentum spread units. **RMS amplitude, dp/p** – momentum spread after fitting in accordance with barriers parameters (can be substantially less than the input momentum spread); **Bucket length, m** – RMS bucket length; **Synchrotron period, sec** – synchrotron period of RMS particle. The **RMS amplitude** can be less than in the **Beam Parameters** window if the **Barrier amplitude** is too small and the initial distribution is not Gaussian.

5.2. Stationary bucket

The numerical stationary model of the barrier bucket can include any number of barriers with any integration step in BETACOOL. The position and voltage stay constant during the simulation. The **Stationary Bucket File** (Fig.8a) has two columns: first is the position of the barriers in units of circumference; second is the amplitude in volts. The first barrier always starts from position -0.5 and the file can not include this position. If barrier position 0.5 is not included in the file it means that last barrier has position 0.5 with zero amplitude. The red line (Fig.8b) is the amplitude distribution; the blue line is the integral of barrier amplitudes [kV*m]. It can be normalized to the maximum amplitude of the barriers if **Normalize Potential Integral** option is chosen (Fig.7).
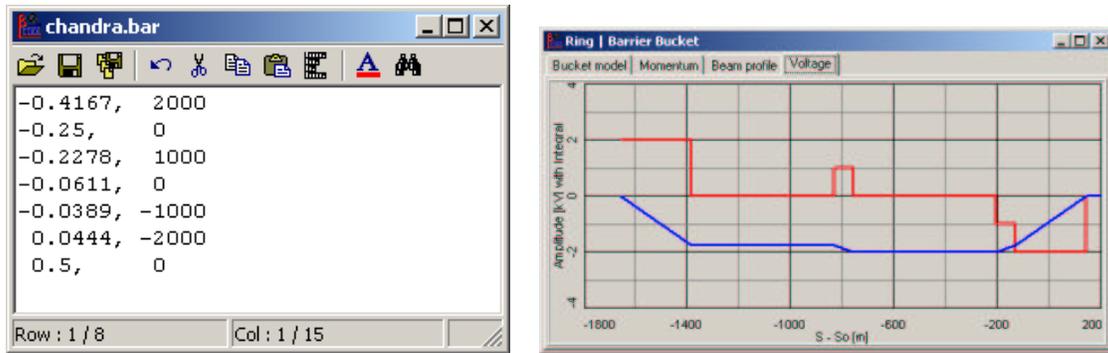

Fig.8. (a) File for definition of stationary BB, (b) amplitude and integral distributions.

The particle distribution and barrier positions in the longitudinal phase space are shown on Fig.9a. The longitudinal profile (Fig9b, green line) is calculated in accordance with the parameter **Longitudinal profile divisions** (Fig.7); the red line is the average particle density in each barrier; black line – integration of particles along the longitudinal coordinate. Intrabeam scattering heating rates are calculated via the longitudinal density of particles which can be chosen in parameter **IBS normalized** (Fig.7): **on longitudinal divisions** means that IBS is calculated in accordance with the particle density within each longitudinal division; **on barrier divisions** – in accordance with the particle density in each barrier region.
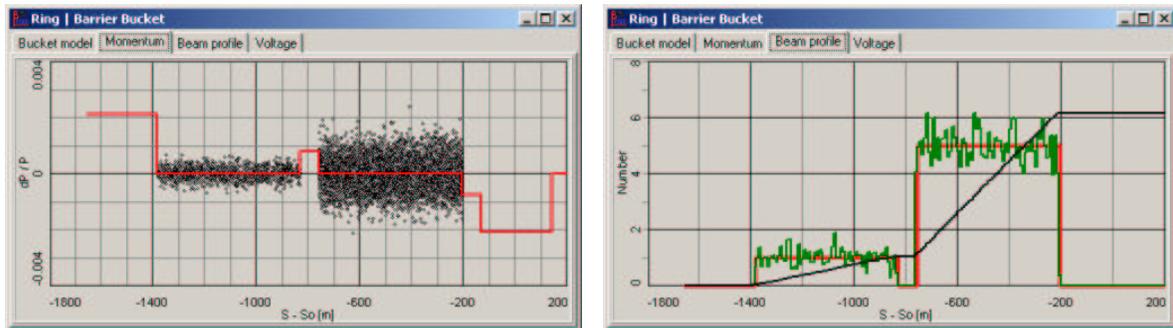

Fig.9. (a) Particle and barrier distribution in the longitudinal phase space
and (b) particle density along the longitudinal coordinate.

The parameter **Number** (Fig.9b) is an arbitrary value of the number of particles normalized to the flat (coasting) distribution. As such for a coasting beam the **Number** parameter everywhere will be equal to unity on Fig.9b. Then, the IBS kick is calculated in accordance with the **Number** parameter for the longitudinal beam profile.

5.3. Moving barrier bucket

The moving barrier can be used for manipulation of particles in the longitudinal phase space. The numerical model for the moving barriers can include any number of barriers. The position and amplitude of the barriers are changed during the simulation through the use of an input file (Fig.11). Note that the integration step for this task should be much smaller than the synchrotron tune and the option **Random synchrotron** (Fig.10) should be switched off.
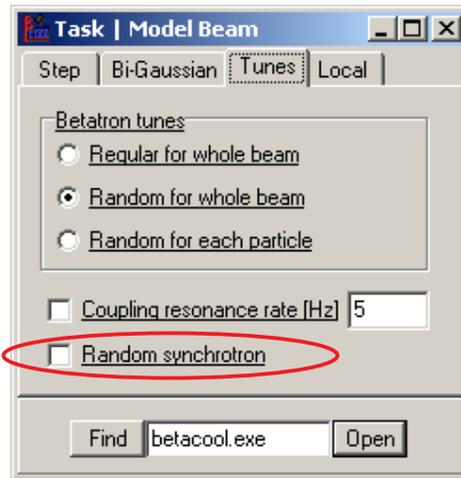

Fig.10. Options for betatron and synchrotron tunes.

The **Moving Bucket File** (Fig.11) has any number of columns: the first column is the time in seconds; the next pairs of columns describe the positions of the barriers in circumference units and the amplitudes in volts. During the simulation process a linear fit of the barrier positions and amplitudes is performed between each input lines. When the reference time has reached the final input line, the parameters of the barriers do not change anymore. An example of the input file is shown on Fig.11. The initial distribution of particles and momentum spread profile with two barriers are shown on Fig.12; the particle distribution and momentum spread profile after 60 sec are shown on Fig.13.

Fig. 11 Example of a moving bucket file.

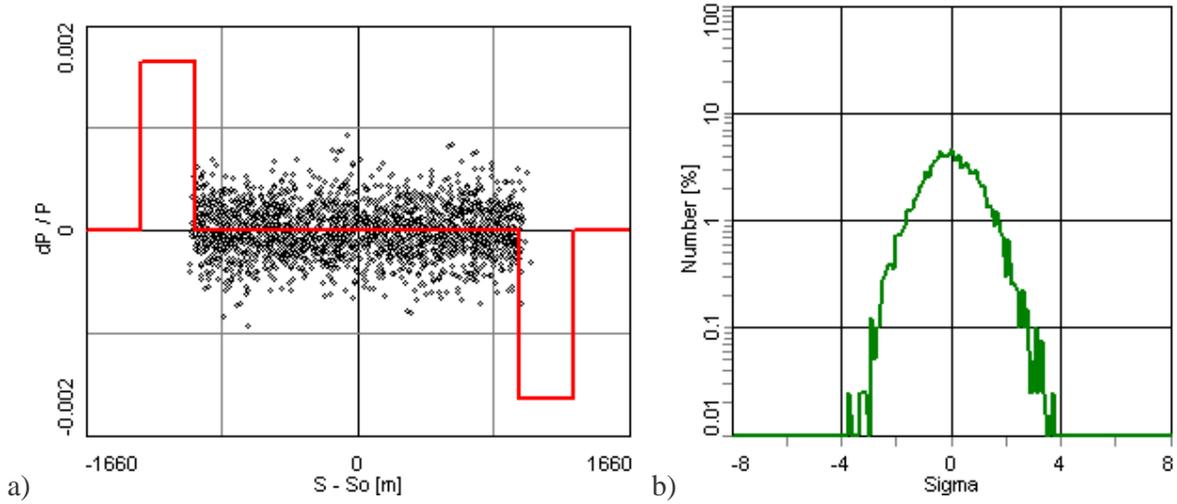a)   b)

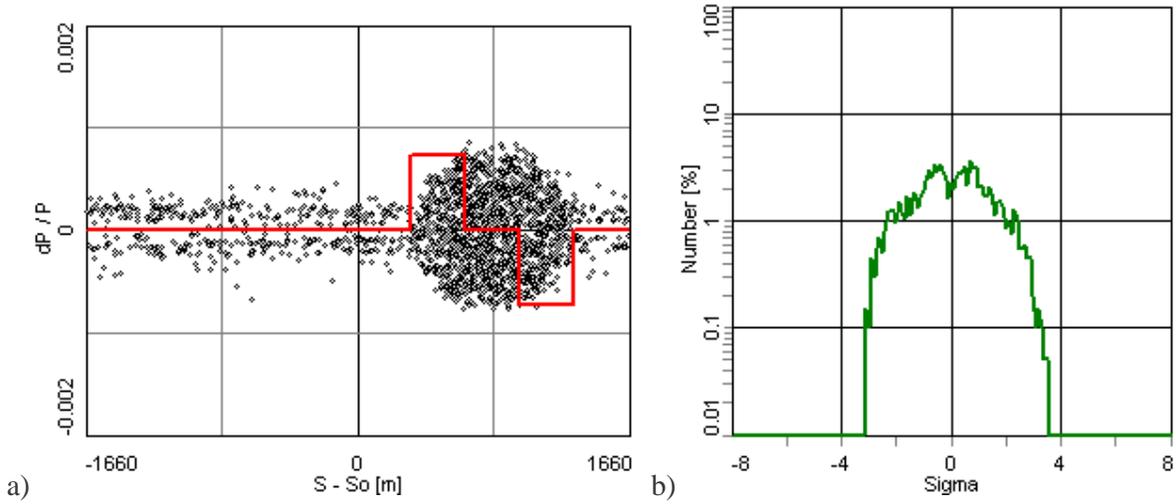Fig.12. Initial distribution (a) of particle and momentum spread profile (b) with two barriers.

a)   b)

Fig.13. Particle distribution (a) and momentum spread profile (b) after 60 sec.

6. Gated cooling

6.1. Definition of stochastic cooling rates for gated cooling

Stochastic cooling can be interpreted as the composition of the coherent single particle effect, when each particle is cooled by means of its self-generated signal and the incoherent noise signal generated by all its neighbor particles. The basic formula, Eq. (7), shows the relation between these two processes and determines the cooling rate for the transverse emittance $1/\tau_{tr} = (1/\varepsilon_{tr})d\varepsilon_{tr}/dt$ or the longitudinal momentum spread $1/\tau_l = (1/\Delta p)d\Delta p/dt$:

$$\frac{1}{\tau_{tr,l}} = \frac{W}{N}\left[\underbrace{2g(1-1/M_{pk}^2)}_{\substack{coherent \\ effect(cooling)}} - \underbrace{g^2(M_{kp}+U)}_{\substack{incoherent \\ effect(heating)}}\right], \tag{7}$$

where N is the number of particles in the ring, $W = f_{max} - f_{min}$ is the cooling system bandwidth, g is the gain parameter, U is the noise to signal ratio, M_{pk} is the mixing factor from pick-up to kicker, and M_{kp} – from kicker to pick-up. The mixing factor is

$$M_{pk,kp} = \frac{1}{2W\left|\eta_{pk,kp}\right|T_{pk,kp}\dfrac{\Delta p}{p}}, \tag{8}$$

where $\eta_{pk,kp}$ – slip factor from pick-up to kicker and from kicker to pick-up, $T_{pk,kp}$ – time flight from pick-up to kicker and from kicker to pick-up, $\Delta p/p$ – momentum spread.

The gated stochastic cooling has four identical **Sections** (Fig.14) with the same input parameters: **Type** (horizontal, vertical or longitudinal), **Bandwidth [GHz]**, **Initial position** and **Final position** of the cooling section in circumference units (like for the position Barrier Buckets). The check-enabled option **More parameters** is used in simulations for which the mixing factors are calculated from Eq. (8). The additional parameters are then: **Time (pick-kick)** –time of flight from pick-up to kicker (the time of flight from kicker to pick-up is calculated via the revolution time), **Slip factor** – slip factors between pick-ups and kickers. The Noise is assumed to be equal 0. The user can define the **Gain** manually or use the **Optimal gain** which will be calculated from the formula $g_{opt} = \dfrac{1 - 1/M_{pk}^{2}}{M_{kp} + U}$.

Fig.14. Parameters for gated stochastic cooling.

For optimal gain, assuming that T_{kp} is equal to the revolution time T_{rev}, η_{kp} is equal to the ring slip factor η_{ring}, and neglecting the term $1/M_{pk}^{2}$ one can use the simple approximation of the cooling rates:

$$\frac{1}{\tau_{tr,l}} = \frac{W}{N}\cdot\frac{1}{M_{kp}} = \frac{2W^{2}\eta_{ring}T_{rev}\dfrac{\Delta p}{p}}{N}. \tag{9}$$

To use this approximation, switch off option **More parameters** (Fig.14).

The position of each active cooling section is indicated by a black horizontal line with square points at its ends (Fig.15).
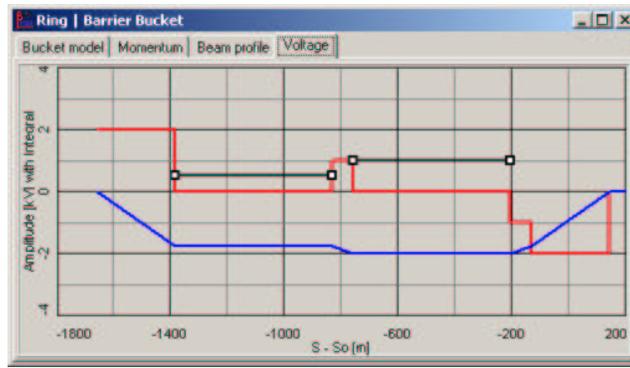
Fig.15. Barrier Bucket voltage distribution and positions of gated cooling sections.

6.2. Simulation with gated cooling

As an example, a simulation using longitudinal gated cooling (Fig.16) was run for the same initial parameters (Fig.9) as in the first example for stationary barrier buckets in the report "BETACOOL simulations and comparison to ESME calculations". Parameters for longitudinal stochastic cooling are shown on Fig.14. For the calculation of the cooling rate for each model particle, the average distribution of the momentum spread (Fig.16a, blue line) and the average density of particles (Fig.16b, red line) are used in Eq. (9).

The simulation shows that the particles in the section where stochastic cooling is applied cool down (Fig.16) and lead to a larger density of particles in this section (Fig.16b) than for the example mentioned above. The core of the momentum spread distribution (Fig.17a) is also much denser than for the case without cooling. The momentum spread evolution reaches an equilibrium which is mainly defined by the particles which are in the section of the ring that is not being stochastically cooled (Fig.17b).
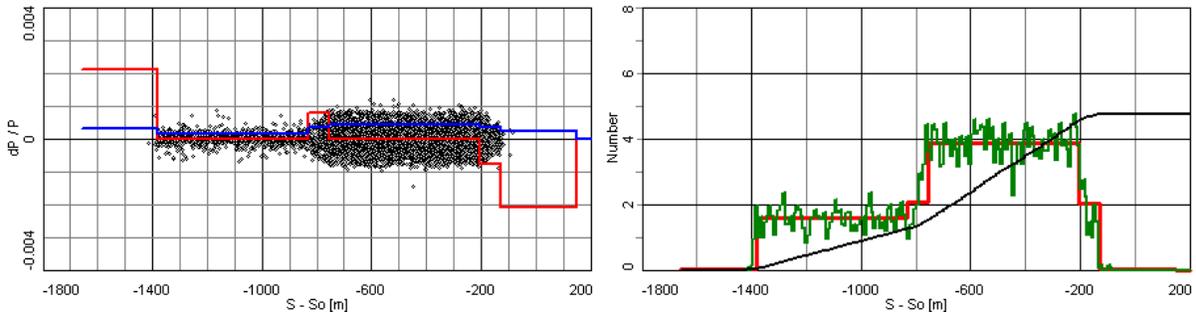

Fig 16 (a) Longitudinal distribution of model particles and barrier bucket in momentum space;
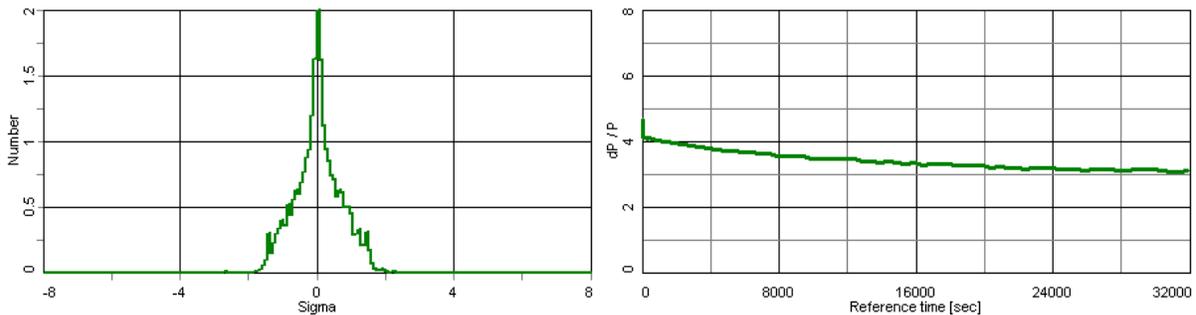(b) beam profile along the longitudinal coordinate.

Fig. 17 (a) Final longitudinal beam profile for momentum spread and
(b) momentum spread evolution during cooling process.