Note on solving Q_L for flat gradients at FLASH ACC6 and ACC7

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Feb 2011

1 The current ratio approach

The starting point is the usual cavity voltage equation with a voltage step during beam time:

$$V_{Ci} = 2R_{Li}I_{gi}(1 - e^{-\frac{t\omega_0}{2Q_{Li}}}) - 2R_{Li}(1-r)I_{gi}(1 - e^{-\frac{(t-t_0)\omega_0}{2Q_{Li}}}) - 2R_{Li}I_b(1 - e^{-\frac{(t-t_0)\omega_0}{2Q_{Li}}}) \quad (1)$$

In this equation, t_0 is the fill time which is assumed to coincide with the beam arrival time, r is the fill-time-to-flat-top voltage ratio (i.e. $V_{\text{flat top}} = r \times V_{\text{fill time}}$), Q_{Li}, R_{Li}, I_{gi} and V_{ci} are the loaded Q, the loaded resistance, the forward current and the voltage of cavity i and are specific to each cavity.

The usual approach to solving Q_L for flat gradient during flat top consists of nulling the time derivative of the equation above. This results in the following equation for Q_{Li}

$$Q_{Li} = \frac{t_0\omega_0}{2\ln(\frac{I_{gi}}{I_b+(1-r)I_{gi}})} \quad (2)$$

The problem with this equation is that I_{gi} is a function of Q_{Li}. Indeed, the forward power to cavity i can be calculated as:

$$P_{Ki} = \frac{1}{2}R_{Li}I_{gi}^2 \quad (3)$$

or

$$I_{gi} = \sqrt{\frac{2P_{Ki}}{R_{Li}}} \quad (4)$$

Making use of the fact that $R_{Li} = \frac{1}{2}Q_{Li}R/Q$, the equation above can be re-written as

$$I_{gi} = \sqrt{\frac{4P_{Ki}}{Q_{Li}R/Q}} \quad (5)$$

This shows that even if the forward power to cavity i is kept constant, changing Q_{Li} will change the current going to cavity i. From an RF point of view, this is simply explained by the fact that changing the loaded Q will determine how much power gets reflected.

As a consequence, a fixed waveguide power distribution determines the power ratios between total klystron power and cavity forward powers but it is wrong to assume that it translates into a fixed current distribution, independent of Q_L settings.

2 The power ratio approach

To solve for Q_L without making the assumption that the current ratios are independent of Q_L settings, one can replace I_{gi} by its definition from Eq. 4 in Eq. 1. The cavity voltage equation becomes:

$$V_{Ci} = 2R_{Li}\sqrt{\frac{2P_{Ki}}{R_{Li}}}(1-e^{-\frac{t\omega_0}{2Q_{Li}}}) - 2R_{Li}(1-r)\sqrt{\frac{2P_{Ki}}{R_{Li}}}(1-e^{-\frac{(t-t_0)\omega_0}{2Q_{Li}}}) - 2R_{Li}I_b(1-e^{-\frac{(t-t_0)\omega_0}{2Q_{Li}}})$$
$$= \sqrt{8P_{Ki}R_{Li}}(1-e^{-\frac{t\omega_0}{2Q_{Li}}}) - (1-r)\sqrt{8P_{Ki}R_{Li}}(1-e^{-\frac{(t-t_0)\omega_0}{2Q_{Li}}}) - 2R_{Li}I_b(1-e^{-\frac{(t-t_0)\omega_0}{2Q_{Li}}})$$

Making use of the following identity $R_{Li} = \frac{1}{2}Q_{Li}R/Q$, the cavity equation becomes:

$$V_{Ci} = \sqrt{4P_{Ki}\frac{R}{Q}Q_{Li}}(1-e^{-\frac{t\omega_0}{2Q_{Li}}}) - (1-r)\sqrt{4P_{Ki}\frac{R}{Q}Q_{Li}}(1-e^{-\frac{(t-t_0)\omega_0}{2Q_{Li}}}) - \frac{R}{Q}Q_{Li}I_b(1-e^{-\frac{(t-t_0)\omega_0}{2Q_{Li}}}) \quad (6)$$

Zeroing the time derivative to get a flat gradient gives

$$\sqrt{4P_{Ki}\frac{R}{Q}Q_{Li}} - (1-r)e^{\frac{t_0\omega_0}{2Q_{Li}}}\sqrt{4P_{Ki}\frac{R}{Q}Q_{Li}} - \frac{R}{Q}Q_{Li}I_b e^{\frac{t_0\omega_0}{2Q_{Li}}} = 0 \quad (7)$$

which can be rewritten as

$$1 - r + I_b\sqrt{\frac{\frac{R}{Q}Q_{Li}}{4P_{Ki}}} = e^{-\frac{t_0\omega_0}{2Q_{Li}}} \quad (8)$$

One needs to solve for Q_{Li} using the equation above. This can be done using a solver but a solution cannot always be found. When a solution exists, typically, 4 solutions are produced, two real positive and two complex.

3 Numerical example

Starting from Katalev's power distribution [1], for a beam current I_b=4.5 mA, R/Q=1036 Ω, a fill time t_0=500 μsec and a ratio r =0.82 (including beam compensation), one can find the following Q_L solutions for ACC6 and ACC7:

ACC6	cav1	cav2	cav3	cav4	cav5	cav6	cav7	cav8
P_{Ki} [kW]	313.1	336.2	291.5	281.6	113.2	119.6	178.9	175.3
Q_{Li} [$\times10^6$]	2.02	1.98	2.07	2.10	4.69	3.98	2.61	2.64

ACC7	cav1	cav2	cav3	cav4	cav5	cav6	cav7	cav8
P_{Ki} [kW]	250.7	250.7	268.3	268.3	365.2	365.2	201.5	201.5
Q_{Li} [$\times10^6$]	2.20	2.20	2.14	2.14	1.93	1.93	2.43	2.43

Note that changing the cavities loaded Q will also affect their operating gradient. No consideration was made in this analysis to assess whether the new operating gradients exceed the cavity quench limits after changing Q_L. Solving Eq.8 only guaranties a flat gradient, not a safe one.

References

[1] V. Katalev. flash_wg_20100214-1.xls, Feb. 2010. MS Excel sheet.