PBAR NOTE NO. 593
A FEEDBACK NOTCH FILTER

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INTRODUCTION

Large coherent signals can saturate sensitive front-end detectors with limited dynamic range. These coherent signals can be removed by inserting a passive notch filter before the first amplifier of the detector. However, any insertion loss of the filter outside the notch band will reduce the signal to noise performance of the system. Building such a filter over a wide frequency band is difficult. In addition, if more coherent signals are added to the input, then the entire filter must be redesigned to add more notches. A low insertion loss multi-notch filter is even more difficult to design than a single notch filter.

FEEDBACK NOTCH FILTER

An alternative to building a passive notch filter is to remove the coherent signal using feedback. A simple schematic showing a feedback filter is shown in Figure 1. The feedback signal is injected into the pre-amplifier (labeled G_p in Figure 1) via a directional coupler (labeled C_1 in Figure 1) inserted in front of the pre-amp. To preserve the signal to noise of the system, the coupling of the directional coupler must be low. (For an antenna system, instead of a directional coupler, the feedback signal could be injected into a sidelobe of the antenna pattern.[1])

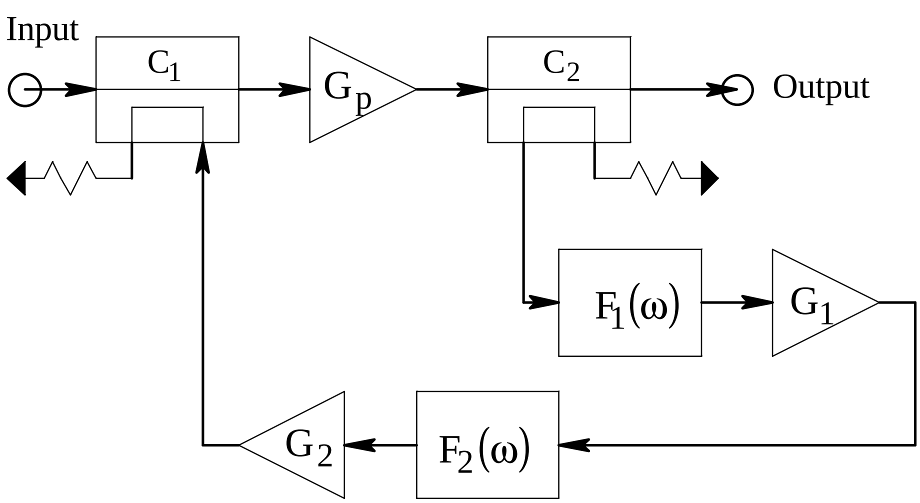

Figure 1. Schematic of Feedback notch filter.

The feedback signal is created by siphoning off a portion of the output with another directional coupler placed on the output of the pre-amp. The feedback signal is processed through a series of filters and amplifiers. The first filter (labeled $F_1(\omega)$ in Figure 1) is a bandpass filter that is centered at the notch frequency. The job of this filter

[1] Private Communication with David Peterson

is to remove most of the wideband signal so that the downstream amplifiers will not be saturated.

The next amplifier (labeled G_1 in Figure 1) should contain most of the gain of the feedback loop. This feedback signal is filtered again by another bandpass filter (labeled $F_2(\omega)$ in Figure 1). The main purpose of this filter is to remove the noise power created by the G_1 outside the notch band. Because of this filter, the amplifier G_1 can be a fairly noisy (on the order of 250 K). With such a relaxed noise temperature constraint on G_1, this amplifier can be high power (30 dBm) which allows the feedback system to have a large dynamic range. To keep the feedback system stable, the poles of the filter $F_1(\omega)$ should not be close to the poles of $F_2(\omega)$. It is better to have the bandwidth of $F_2(\omega)$ much less than the bandwidth of $F_1(\omega)$.

The filter $F_2(\omega)$ is followed by another amplifier (labeled G_2 in Figure 1). Since there is no filter following this amplifier, the noise generated by this amplifier will show up in the output outside the notch band. Therefore, this amplifier must have good noise properties (or be omitted altogether). If the filter $F_2(\omega)$ is an active filter, the noise properties of this filter can be lumped into the noise properties of the amplifier G_2.

<u>SYSTEM GAIN AND NOISE PROPERTIES</u>

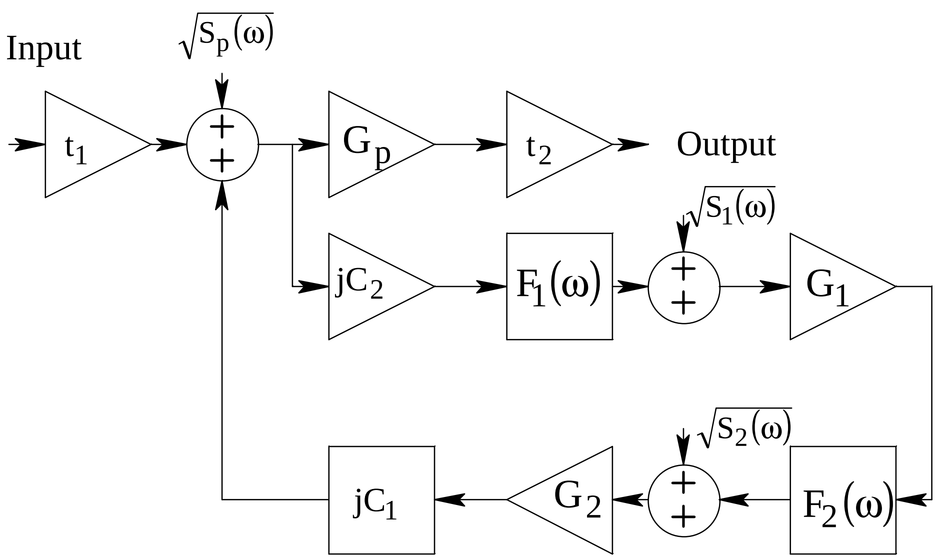

Figure 2. Block diagram of the feedback notch filter with noise sources.

Figure 2 shows the block diagram of the notch filter with the noise power spectral density of the amplifiers shown explicitly as sources. Since this is a vector block diagram, the units of the noise sources are in root power. The gain blocks $\mathbf{t_1}$ and $\mathbf{t_2}$ designate the forward transmission coefficient of the directional couplers. The 90 degree phase shift of the directional couplers are given by multiplying the coupling coefficient by $\mathbf{j}$ (square root of –1). (The phase shift due to the loop delay is not shown explicitly in Figure 2, but the implications of the loop delay on loop stability will be discussed later.) If the directional couplers are lossless and perfectly directional, then the transmission coefficient of the directional couplers is:

$$t_n = \sqrt{1 - C_n{}^2} \tag{1}$$

The open loop gain of the feedback loop is:

$$L(\omega) = -C_1 C_2 F_1(\omega) F_2(\omega) G_1 G_2 G_p \tag{2}$$

The closed loop gain of the feedback loop excluding the pre-amp gain is:

$$H(\omega) = \frac{t_1 t_2}{1 - L(\omega)} \tag{3}$$

The output power of the notch filter is:

$$\begin{aligned} P_{out} &= \left| H(\omega) G_p \right|^2 P_{in} \\ &+ \left| \frac{1}{t_1} H(\omega) G_p \right|^2 S_p(\omega) \\ &+ \left| \frac{1}{jC_2 F_1(\omega) G_p t_1} L(\omega) H(\omega) G_p \right|^2 S_1(\omega) \\ &+ \left| \frac{jC_1 G_2}{t_1} H(\omega) G_p \right|^2 S_2(\omega) \end{aligned} \tag{4}$$

The power at the output of the first amplifier in the feedback loop is:

$$\begin{aligned} P_{G_1} &= \left| jC_2 F_1(\omega) G_1 G_p \frac{t_1}{1 - L(\omega)} \right|^2 P_{in} \\ &+ \left| jC_2 F_1(\omega) G_1 G_p \frac{1}{1 - L(\omega)} \right|^2 S_p(\omega) \\ &+ \left| G_1 \frac{1}{1 - L(\omega)} \right|^2 S_1(\omega) \\ &+ \left| \frac{1}{F_2} \frac{L(\omega)}{1 - L(\omega)} \right|^2 S_2(\omega) \end{aligned} \tag{5}$$

The power at the output of the second amplifier in the feedback loop is:

$$P_{G_2} = \left| \frac{t_1}{jC_1} \frac{L(\omega)}{1-L(\omega)} \right|^2 P_{in}$$
$$+ \left| \frac{1}{jC_1} \frac{L(\omega)}{1-L(\omega)} \right|^2 S_p(\omega)$$
$$+ \left| F_2 G_1 G_2 \frac{1}{1-L(\omega)} \right|^2 S_1(\omega) \qquad (6)$$
$$+ \left| G_2 \frac{L(\omega)}{1-L(\omega)} \right|^2 S_2(\omega)$$

Each band pass filter can be modeled as a simple 2 pole filter:

$$F_i(x\omega_c) = \frac{j\frac{A_i}{Q_i}x}{1-x^2+j\frac{1}{Q_i}x} \qquad (7)$$

where ω_c is the center frequency of the filter, ω_c/Q_i is the bandwidth of the filter, and A_i is the gain of the filter.

For low coupling values and high loop gain, the depth of the notch is approximately:

$$\text{depth} = \frac{1}{C_1 C_2 A_1 A_2 G_1 G_2 G_p} \qquad (8)$$

The noise of the amplifiers outside of the notch bandwidth can be converted to effective noise sources at the front end of the pre-amp. The noise temperature due to the first feedback amplifier at the front end of the pre-amp is:

$$T = \left| C_1 F_2(\omega) G_1 G_2 \right|^2 T_{G_1} \qquad (9)$$

where T_{G1} is the noise temperature of the first feedback amplifier. Since the second filter response is included in this expression, the noise contribution due to this amplifier outside of the notch band is essentially zero. The noise temperature due to the second feedback amplifier at the front end of the pre-amp is:

$$T = \left| C_1 G_2 \right|^2 T_{G_2} \qquad (10)$$

Because there is no filter after the second amplifier, this amplifier can contribute a significant amount of noise outside the notch band. It is best if there is no noise added after the second filter.

The last issue of concern is how much power does the first feedback amplifier have to provide for a coherent input signal centered at the notch frequency. The gain from the input to the output of the first feedback amplifier at the notch frequency is approximately:

$$P_{G1}\Big|_{notch} = \frac{1}{\left|C_1 A_2 G_2\right|^2} P_{in}\Big|_{notch} \tag{11}$$

To keep the output power of the first feedback amplifier low it is best to have the insertion loss of the second filter small. While increasing the coupling of the first coupler also helps keep the output power of the first feedback amplifier low, increased coupling will drop the signal to noise at the output for signals outside the notch band.

AN EXAMPLE DESIGN

The following is an example of a feedback notch filter. The design parameters are shown in Table 1. The open loop gain of the feedback path at the center frequency is about 10 which gives a 20 dB notch as shown in Figure 3. Figure 4 shows the effective noise temperature at the front of the pre-amp. The red trace is the contribution by the pre-amp alone, the blue trace is the contribution due to the first feedback amplifier alone and the green trace is the sum of both amplifiers. The slight peaking of the red-trace at the edges of the notch band are due to the 5 nS of loop delay and the poles of the first filter ($F_1(\omega)$).

First Coupler coefficient	C_1	25 dB
Second Coupler coefficient	C_2	15 dB
Pre-amp gain	G_p	35 dB
First feedback amplifier gain	G_1	25 dB
Second feedback amplifier gain	G_2	0 dB
Gain of first filter	A_1	0 dB
Gain of second filter	A_2	0 dB
Q of first filter	Q_1	100
Q of second filter	Q_2	5000
Center frequency of filter	f_c	1.5 GHz
Feedback loop delay	τ_d	5 nS
Pre-amp noise temperature	T_p	10K
First feedback amplifier noise temperature	T_{G1}	300K

Table 1. Example Design Parameters

Figure 5. Shows the transfer gain from the input of the system to the output of the first feedback amplifier. This plot was done to check on how much power would be required of the feedback amplifiers for a given coherent signal applied at the notch frequency at the input of the system. At the notch frequency, this gain is 24 dB. Therefore, if G_1 had a 1 dB compression point of +30dBm (1Watt), a +6 dBm signal could be applied to the input of the system with saturating the feedback loop.

The above discussion in the preceding section did not include any delay in the loop gain calculations. Adding delay for a second order feedback system can make the system unstable. The example design included a 5 nS loop delay. To check the stability of the loop, the imaginary part of $L(\omega)$ is plotted against the real part of $L(\omega)$ as shown in Figure 6. Since the trace does not encircle the +1 real point, then the system will be stable for this delay. The effective of loop delay can also be seen in the slight peaking of the response in Figures 3 and 4 around the notch edges.

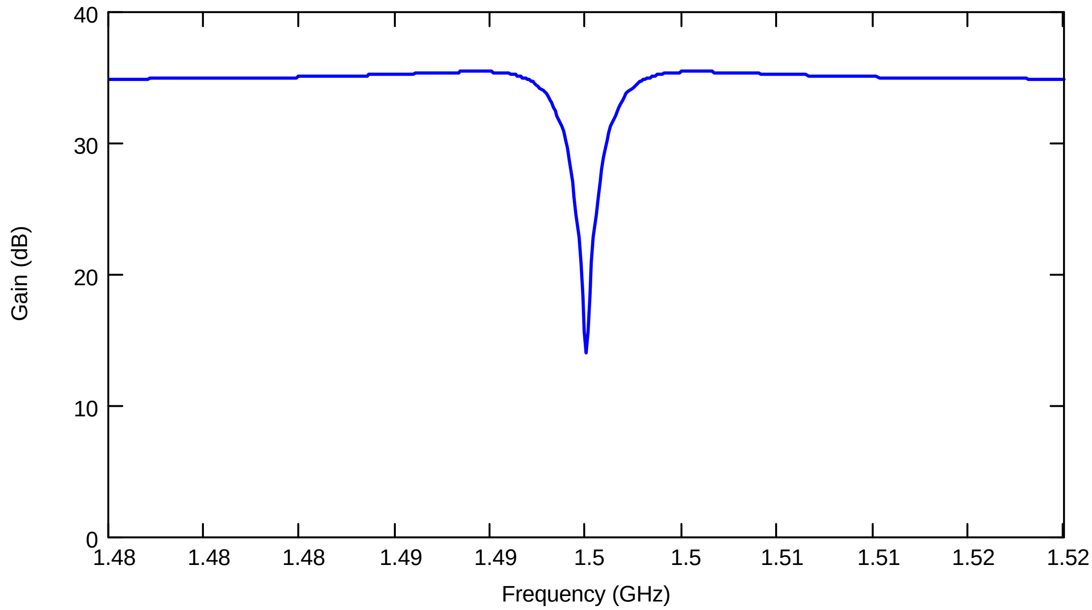

Figure 3. Total notch filter response.

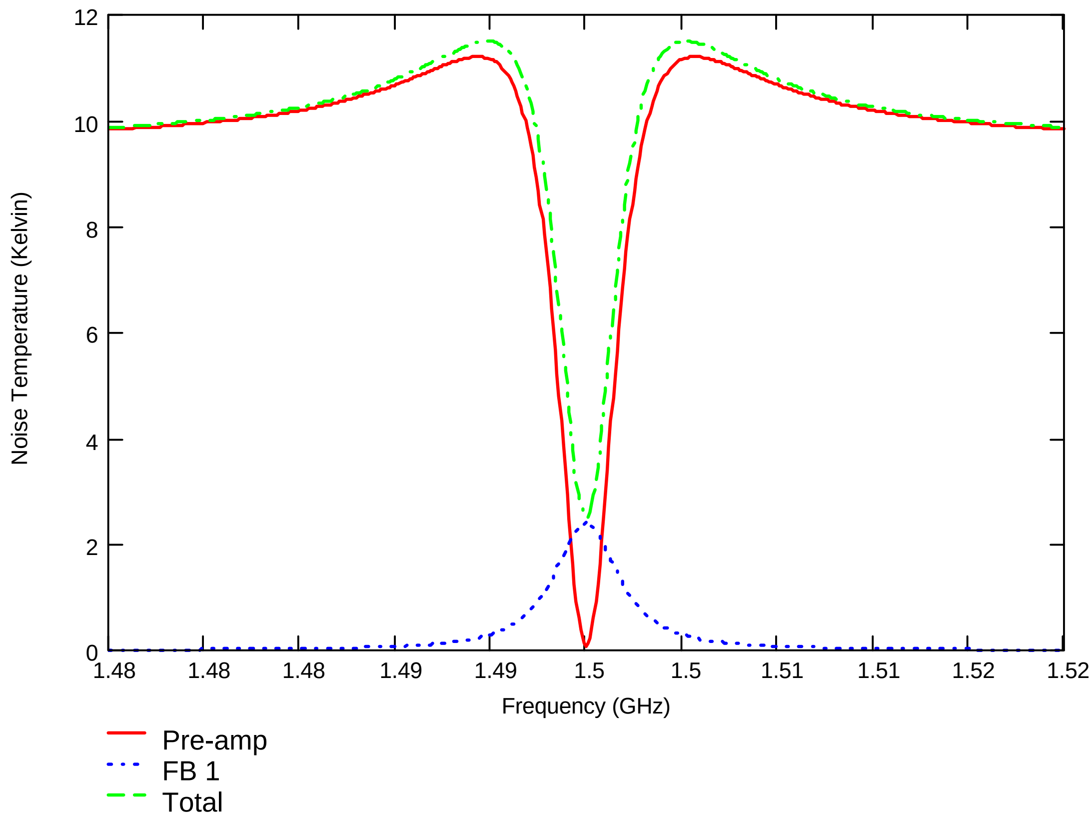

Figure 4. Effective noise temperature at the input of the pre-amp.

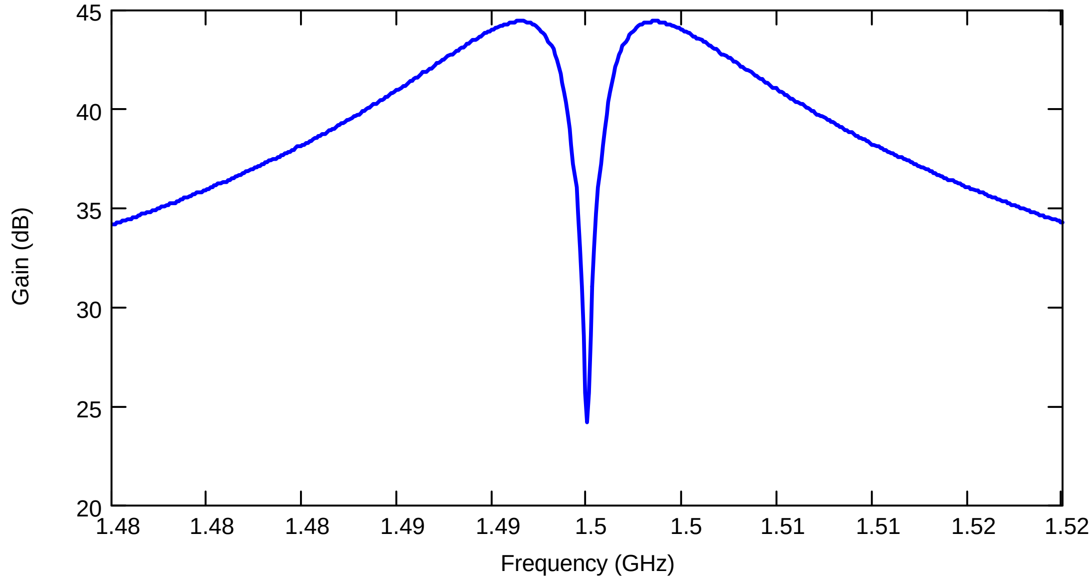

Figure 5. The transfer gain from the input of the system to the output of the first feedback amplifier.

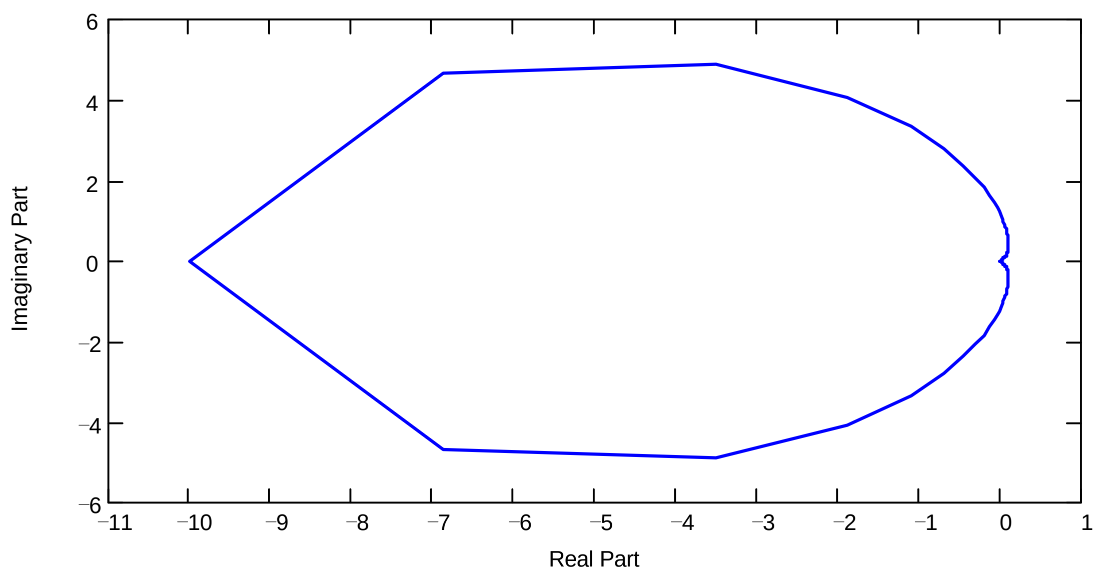

Figure 6. Nyquist plot of the feedback loop.

I-Q FILTERS

The preceding discussion did not specify what type of filters where used in the feedback path. For a passive filter, a Q value over 100 usually has to be obtained with a resonant cavity configuration. For a passive resonator with a high Q value, the resonant frequency can change dramatically (as compared to the bandwidth of the resonator) for small variations in temperature, size and loading conditions. This means that the notch of the feedback filter could drift significantly away from the desired frequency. This frequency drift problem can be reduced by using an In phase–Quadrature (I-Q) filter as shown in Figure 7. For an I-Q filter, the input signal is split in two and down converted against a local oscillator (ω_{rf}). The upper path local oscillator is 90 degrees out of phase with the bottom path local oscillator. The base-band signals are then filtered, rotated (if needed, and then up-converted with the same local oscillator. The transfer function of the filter is:

$$\frac{V_{in}(\omega)}{V_{out}(\omega)} = \left|G(\omega - \omega_{rf})\right| e^{j \cdot \arg(\omega - \omega_{rf})} e^{j\delta} \tag{12}$$

Since the filtering is done at base-band ($G(\Delta\omega)$), the bandwidth of this IF filter can be made extremely small. The quadrature leg of the filter is only needed if the phase relationship between the input signal and the local oscillator (ω_{rf}) is unknown. If the input signal is phase locked to the local oscillator signal then only the in-phase path is needed.

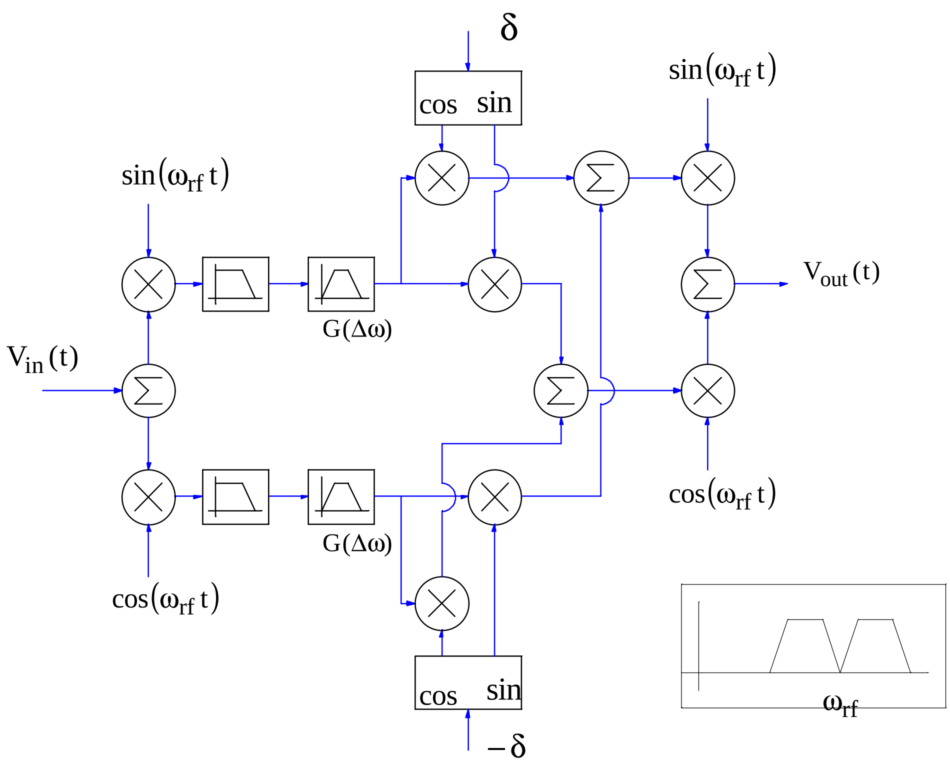

Figure 7. Schematic of I-Q filter.

The main advantages of the IQ filter are better frequency stability and smaller bandwidth. The main drawback of the IQ filter is the noise performance. The noise can come from two sources; coherent "noise" due to local oscillator bleed-through from the mixers (or DC offsets in the mixers) or incoherent noise due to the kT noise of the mixers and the IF filter. For a local oscillator bleed-through level of P_{Blo} Watts at the output of the I-Q filter ($F_2(\omega)$), the amount of power detected at the output of the total system due to solely to this signal would be:

$$P_{out} = \frac{P_{B_{lo}}}{\left|C_2 A_1 A_2 G_1\right|^2} \tag{13}$$

where it has been assumed that the local oscillator frequency is at the notch frequency. For the example design, this attenuation given by the denominator of Equation 13 would be 10dB. If the I-Q filter has a total noise temperature of T_{F2} measured at the output of $F_2(\omega)$, the effective noise temperature outside the notch band as seen at the output of the total system would be:

$$T_{F2_{eff}} = \left|C_1 G_2\right|^2 T_{F2} \tag{14}$$

In the example design, C1 had a value of 25 dB, which would give a reduction in effective noise due to the I-Q filter by a factor of about 300. Thus a 300K noise temperature of the I-Q filter would contribute about 1 K onto the output of the entire system.