PBAR NOTE 597
ATTENUATION OF WAVEGUIDE MODES WITH RESISTIVELY COATED DIELECTRIC WALLS

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INTRODUCTION

Microwave modes in the beam pipe of the 4-8 GHz Debuncher Upgrade pickup and kicker arrays are attenuated with absorbing slabs placed on the sides of the beam pipe walls. Placing a thin resistive film on the surface of a ferrite can enhance the absorbing characteristics of the ferrite absorber design. This note describes how to calculate the attenuation of rectangular waveguide modes with resistively coated dielectric slabs absorbers placed on the side walls of the waveguide.

ABSORBER ON THE SIDE-WALLS OF THE WAVEGUIDE

The geometry of the problem is shown in Figure 1. Without the absorber, the dominant mode in the waveguide will be the Transverse Electric to Z 1,0 mode (TE^Z_{10}). With the introduction of the absorber, the waveguide modes can no longer be classified as transverse to Z (TE^Z, TM^Z) but can be classified as transverse to X (TE^X, TM^X).[1] Since the TE^X_{10} mode is the same as the TE^Z_{10} mode in the absence of absorber, and the incident mode on the absorbing section of waveguide will be TE^Z_{10} , we will consider TE^X modes which are even in X only. TE^X modes can be derived from a x-directed electric vector potential:

$$\vec{F}(x,y,z) = \hat{x}F_x(x,y)e^{-\gamma z} \tag{1}$$

The electric field is given as:

$$\vec{E} = -\frac{1}{\varepsilon}\vec{\nabla}\times\vec{F} \tag{2}$$

The magnetic field is given as:

$$\vec{H} = -\frac{1}{j\omega\mu\varepsilon}\vec{\nabla}\times\vec{E} \tag{3}$$

From Equations 1-3, electric field is:

$$\vec{E} = \left(0\hat{x} + \frac{\gamma}{\varepsilon}F_x\hat{y} + \frac{1}{\varepsilon}\frac{\partial F_x}{\partial y}\hat{z}\right)e^{-\gamma z} \tag{4}$$

The magnetic field is:

[1] Time Harmonic Electromagnetic Fields, R.F. Harrington, McGraw-Hill, Inc., 1961, pg. 158

$$\vec{H} = \frac{1}{j\omega\mu\varepsilon}\left(-\left(\frac{\partial^2 F_x}{\partial y^2} + \gamma^2 F_x\right)\hat{x} + \frac{\partial^2 F_x}{\partial x \partial y}\hat{y} - \gamma\frac{\partial F_x}{\partial x}\hat{z}\right)e^{-\gamma z} \tag{5}$$

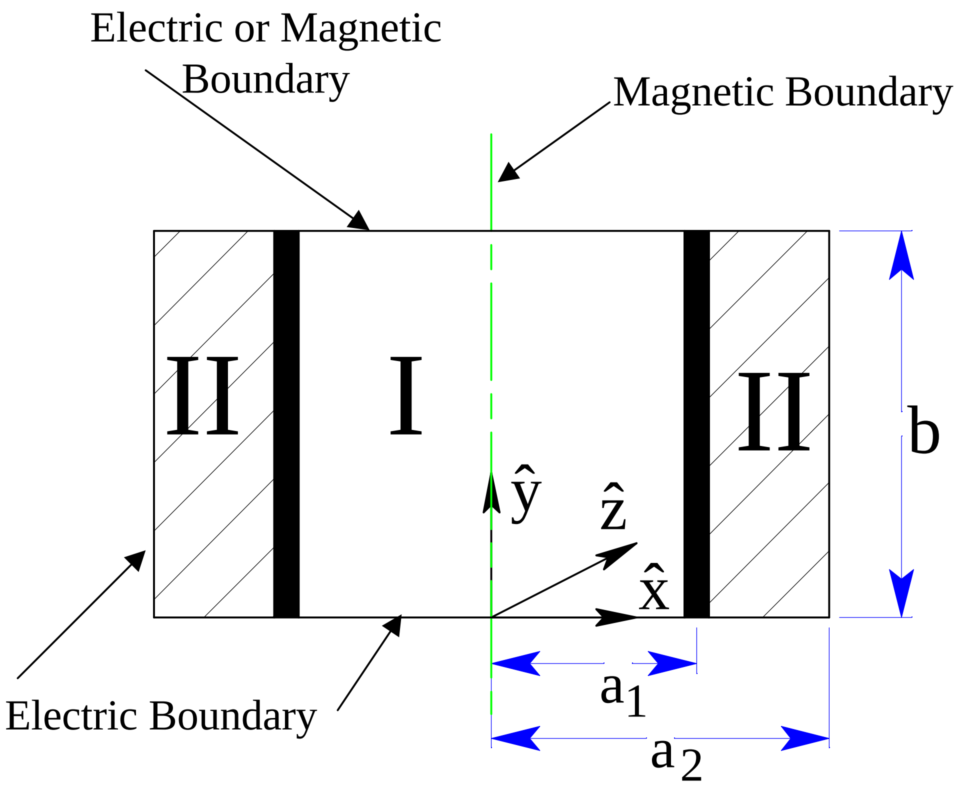

Figure 1. Geometry for sidebar absorbers.

To meet the boundary conditions in Region I of Figure 1, the electric vector potential must be of the form:

$$F_x(x, y) = F_1 \cos(f_1 x)\cos(k_y y) \tag{7}$$

The electric field in Region I is:

$$E_x(x, y) = 0 \tag{8}$$

$$E_y(x, y) = \frac{\gamma F_1}{\varepsilon_1}\cos(f_1 x)\cos(k_y y) \tag{9}$$

$$E_z(x, y) = -\frac{\gamma F_1}{\varepsilon_1}\frac{k_y}{\gamma}\cos(f_1 x)\sin(k_y y) \tag{10}$$

The magnetic field in Region I is:

$$H_x(x, y) = \frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{\gamma^2 - k_y{}^2}{\gamma^2}\cos(f_1 x)\cos(k_y y) \tag{11}$$

$$H_y(x,y) = -\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{f_1 k_y}{\gamma^2}\sin(f_1 x)\sin(k_y y) \tag{12}$$

$$H_z(x,y) = -\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{f_1}{\gamma}\sin(f_1 x)\cos(k_y y) \tag{13}$$

To meet the boundary conditions in Region II of Figure 1, the electric vector potential must be of the form:

$$F_x(x,y) = F_2 \sin(f_2(a_2 - x))\cos(k_y y) \tag{14}$$

The electric field in Region III is:

$$E_x(x,y) = 0 \tag{15}$$

$$E_y(x,y) = \left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\frac{\gamma F_1}{\varepsilon_1}\sin(f_2(a_2 - x))\cos(k_y y) \tag{16}$$

$$E_z(x,y) = -\left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\frac{\gamma F_1}{\varepsilon_1}\frac{k_y}{\gamma}\sin(f_2(a_2 - x))\sin(k_y y) \tag{17}$$

The magnetic field in Region III is:

$$H_x(x,y) = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{\gamma^2 - k_y{}^2}{\gamma^2}\sin(f_2(a_2 - x))\cos(k_y y) \tag{18}$$

$$H_y(x,y) = -\left(\frac{\mu_1}{\mu_2}\right)\left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{f_2 k_y}{\gamma^2}\cos(f_2(a_2 - x))\sin(k_y y) \tag{19}$$

$$H_z(x,y) = -\left(\frac{\mu_1}{\mu_2}\right)\left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{f_2}{\gamma}\cos(f_2(a_2 - x))\cos(k_y y) \tag{20}$$

For electric boundaries at both the top and bottom of the waveguide:

$$k_y = n\frac{\pi}{b} \tag{21}$$

where n is an integer. For an electric boundary on the bottom of the waveguide and a magnetic boundary on the top of the waveguide:

$$k_y = \left(n + \frac{1}{2}\right)\frac{\pi}{b} \tag{22}$$

At the interface of $x=a_1$, E_y, E_z must be continuous:

$$\cos(f_1 a_1) = \left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\sin(f_2(a_2 - a_1)) \tag{23}$$

For the magnetic field boundary condition, an effective voltage and current in the y direction can be defined:

$$dV_y = E_y dy \tag{24}$$

$$dI_y = \left(H_z^{II} - H_z^{I}\right)dz \tag{25}$$

Ohm's law requires:

$$dV_y = R dI_y \tag{26}$$

where

$$R = \rho \frac{dy}{dxdz} \tag{27}$$

where ρ is the resistivity of the interface between Region I and II. The boundary condition of Equation 26 becomes:

$$E_y = R_{sq}\left(H_z^{II} - H_z^{I}\right) \tag{28}$$

where R_{sq} is given as the resistivity of the interface divided by the thickness of the interface:

$$R_{sq} = \frac{\rho}{dx} \tag{29}$$

The boundary condition on E_z can also be specified as:

$$E_z = -R_{sq}\left(H_y^{II} - H_y^{I}\right) \tag{30}$$

From Equations 28 and 30:

$$\cos(f_1 a) = R_{sq}\left(\frac{j\gamma}{\omega\mu_1}\right)\left[\frac{f_1}{\gamma}\sin(f_1 a) - \frac{\mu_1}{\mu_2}\left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\frac{f_2}{\gamma}\cos(f_2(a_2 - a_1))\right] \tag{31}$$

Using Equation 23 in Equation 31:

$$\cos(f_1 a) = j\frac{R_{sq}}{\eta}\frac{f_1}{\omega/c}\left[\sin(f_1 a) - \frac{1}{\mu_r}\frac{f_2}{f_1}\frac{\cos(f_1 a)}{\tan(f_2(a_2 - a_1))}\right] \tag{32}$$

where it has been assumed that Region I is free space so that η is the impedance of free space, c is the velocity of light and μ_r is the relative permeability of Region II.

The solution to the Helmholtz wave equation requires:

$$\gamma^2 - f_1^2 - k_y^2 + \omega^2\mu_1\varepsilon_1 = 0 \tag{33}$$

$$\gamma^2 - f_2{}^2 - k_y{}^2 + \omega^2\mu_2\varepsilon_2 = 0 \tag{34}$$

Subtracting Equation 34 from Equations 33 results in:

$$f_2{}^2 = f_1{}^2 + \omega^2\mu_1\varepsilon_1\left(\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1} - 1\right) = 0 \tag{35}$$

The attenuation is:

$$\alpha = \left|\mathrm{IM}\left(\sqrt{\omega^2\mu_1\varepsilon_1 - k_y{}^2 - f_1{}^2}\right)\right| \tag{36}$$

The ratios of the vector potential amplitudes are:

$$\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1} = \frac{\cos(f_1 a_1)}{\sin(f_2(a_2 - a_1))} \tag{37}$$

A SAMPLE DESIGN

The design parameters of a sidebar waveguide absorber (as shown in Figure 1) operating between 5-7 GHz in the difference mode will be explored in this section. The width of Region I is 40 mm. A parameter sweep (similar to the multi-layer absorber design[2]) for different thicknesses and resistances was done for two types of material: ferrite TT2-111R[3] and Alumina (which has a dielectric constant of 10). Figure 2 is the attenuation of the fundamental mode as a function of frequency for the three different absorber designs. Figures 3 through 5 are the electromagnetic fields for the fundamental mode at 6 GHz as a function of position in the waveguide.

[2] PBAR Note 596, Attenuation of Waveguide Modes with Multi-Layer Absorbing Walls, Dave McGinnis, July, 1998

[3] PBAR Note 594, Analysis of Microwave Properties for Various Absorbing Materials, Dave McGinnis, June, 1998

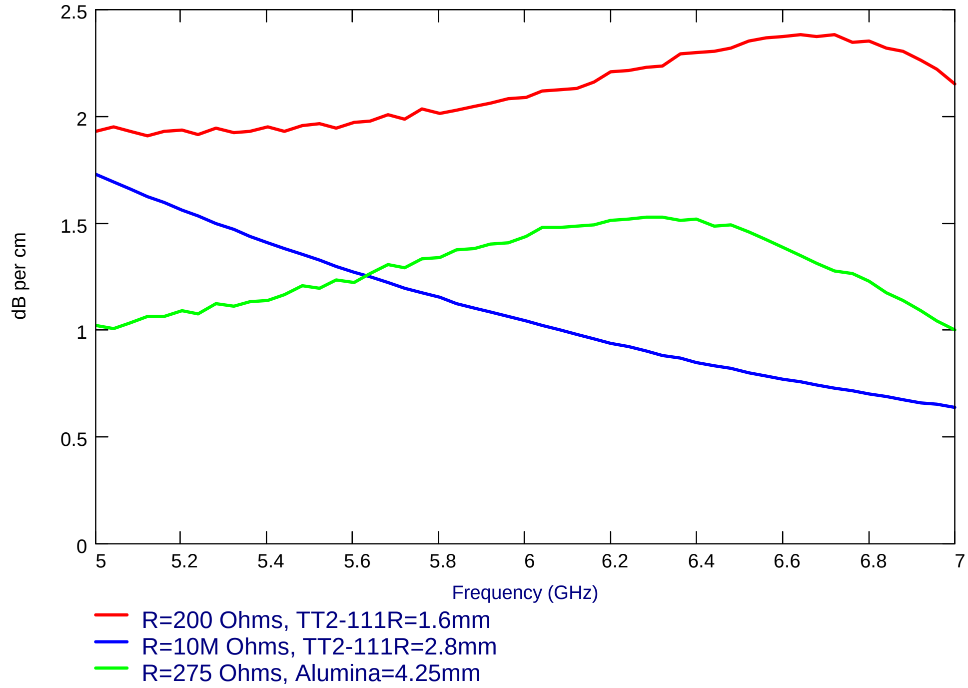

Figure 2. Attenuation as a function of frequency for three different sidebar absorber designs.

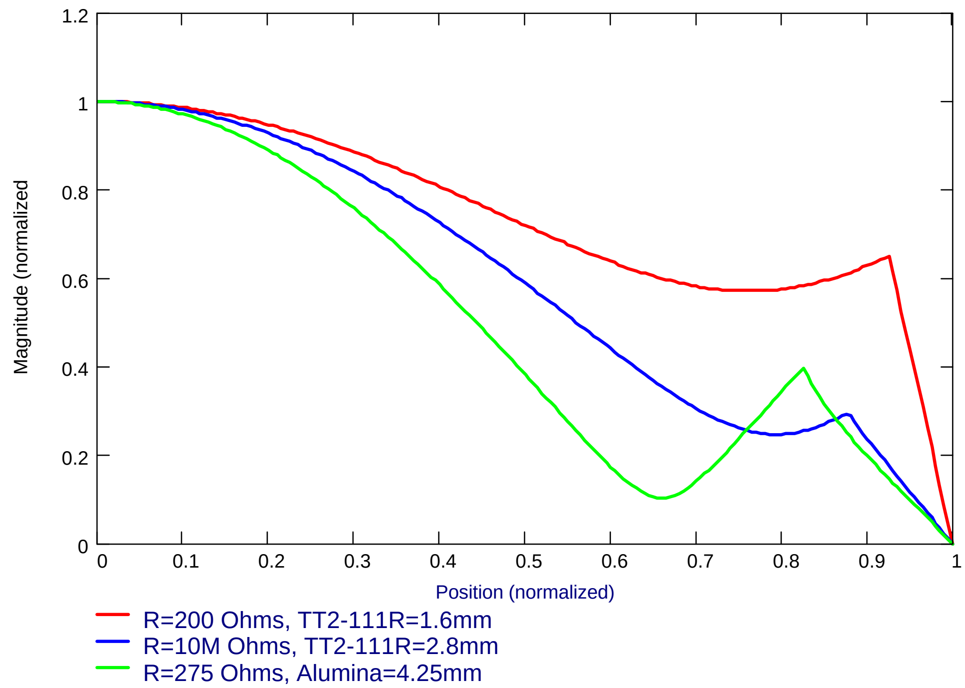

Figure 3. Magnitude of the electric field in the y direction at 6 GHz as a function of position in the waveguide. X=0 is the center of Region I.

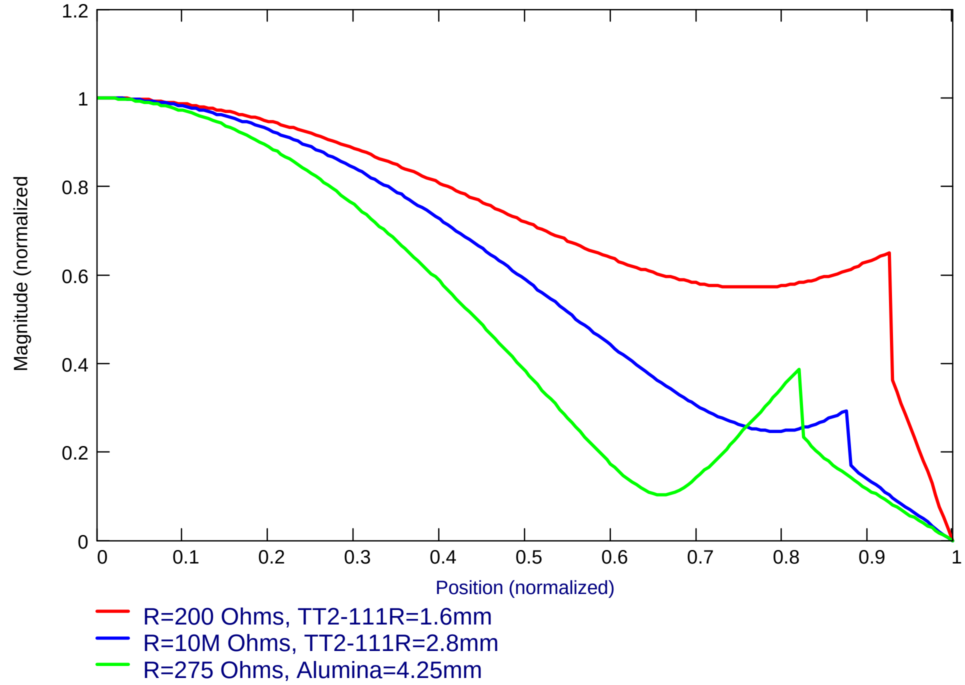

Figure 4. Magnitude of the magnetic field in the x direction at 6 GHz as a function of position in the waveguide. X=0 is the center of Region I.

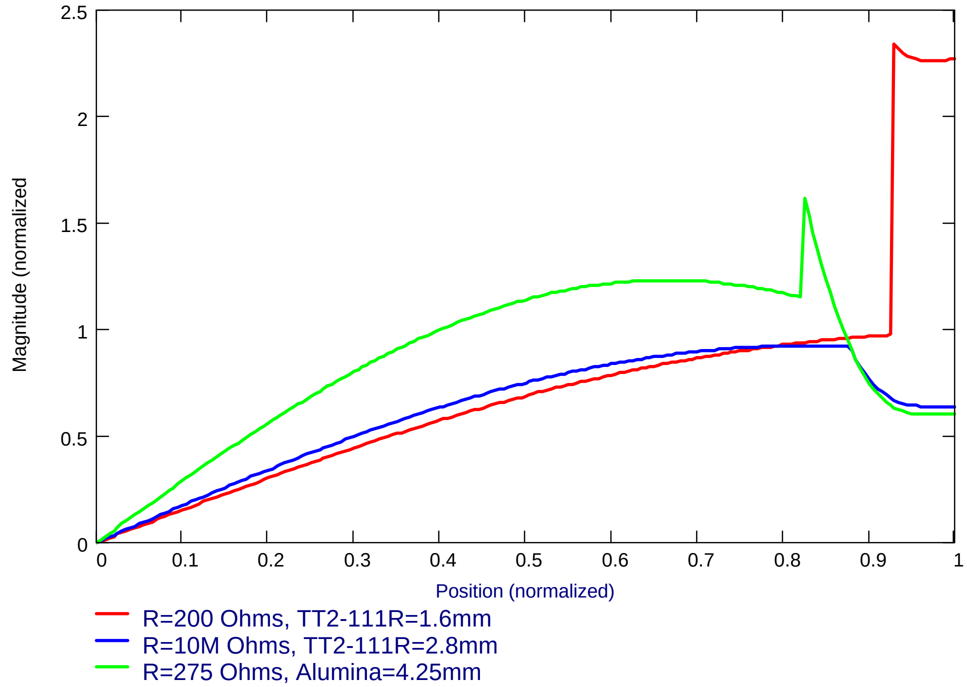

Figure 5. Magnitude of the magnetic field in the z direction at 6 GHz as a function of position in the waveguide. X=0 is the center of Region I.