PBAR NOTE 636

EXTRACTION FROM THE ACCUMULATOR WITH RECTANGULAR MOMENTUM DISTRIBUTIONS

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May 7, 2000

INTRODUCTION

Antiprotons for the Collider will be bunched in the Accumulator using a RF system (ARF4) that operates at a frequency that is four times the revolution frequency of the Accumulator. Four bunches can be extracted from the Accumulator in a single transfer. Since the TEVATRON will operate with 36 antiproton bunches, nine extractions from the Accumulator will be needed. During Run 1, the momentum distribution of the Accumulator core was shaped by the core cooling systems and was approximately gaussian. After an antiproton bunch was extracted from the core, the momentum spread of the beam was increased because of RF displacement. The beam would then be re-cooled which would delay shot setup (or the phase space density of subsequent extractions would be diluted).

Since the number of extractions per shot for Run II is increasing by 50% (from 6 to 9), the extra time due to re-cooling (or the alternative of momentum phase space dilution) might not be acceptable. This note will outline a scheme that will shape the momentum distribution into a rectangle and will extract the beam from the edges of the distribution so that RF displacement is minimized.

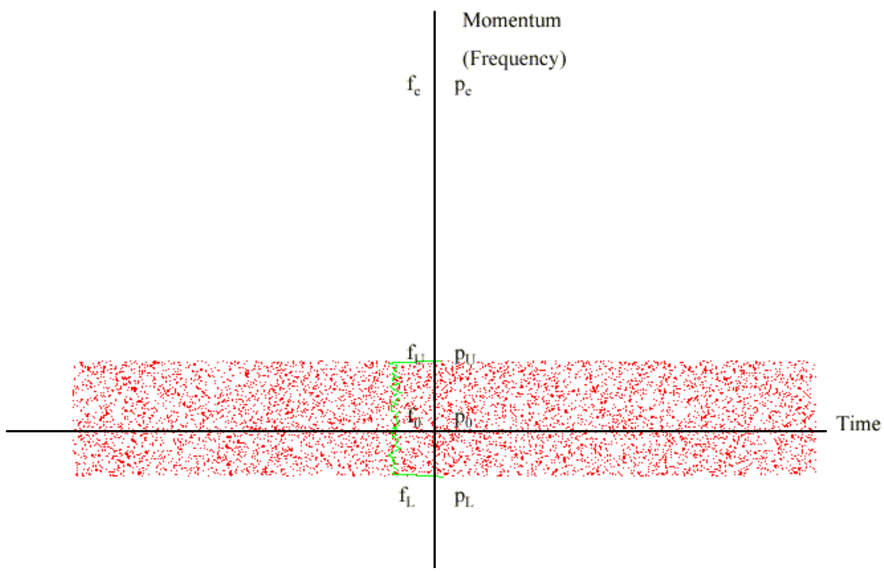

Figure 1. Phase space sketch of a rectangular momentum distribution

THE RECTANGULAR MOMENTUM DISTRIBUTION

So that the phase space density is a constant for all extractions, the momentum distribution of the unbunched beam must have the shape of a rectangle as shown in Figure 1. The revolution frequency distribution of the beam is as shown in Figure 1 is given by:

$$\frac{f_U - f_L}{f_0} = -\eta \frac{p_U - p_L}{p_0} \tag{1}$$

η is +0.0122 for the Accumulator. Note that because $\eta>0$, $f_U<f_L$.

To show that making a rectangular momentum distribution is possible in the Accumulator, a noise source spectrum as shown in Figure 2 was applied to the wideband RF system ARF2. The total voltage applied to the beam was about 3.8V for a duration of about 5 minutes. The momentum stochastic cooling was turned off but the transverse cooling was left on. The initial and final beam spectrums are shown in Figure 3. There was no beam loss in the process and negligible tranverse emittance growth. A chirp signal with the same bandwidth as the noise source was also applied to the beam with similar results.

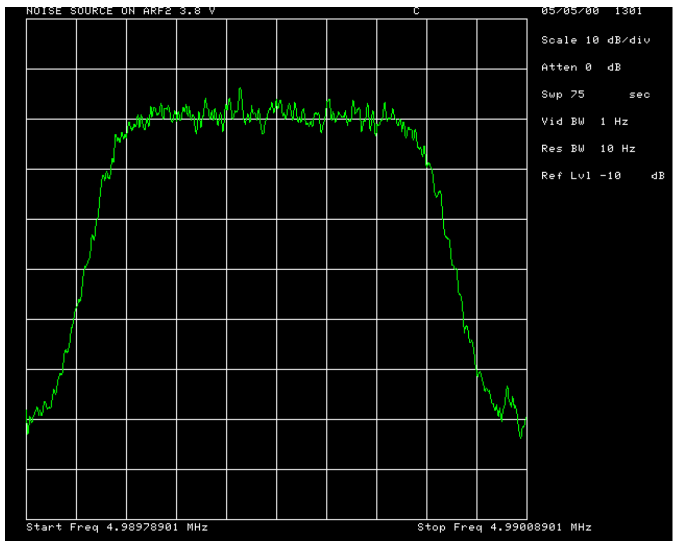

Figure 2. Spectrum of noise source applied to the beam at harmonic 8.

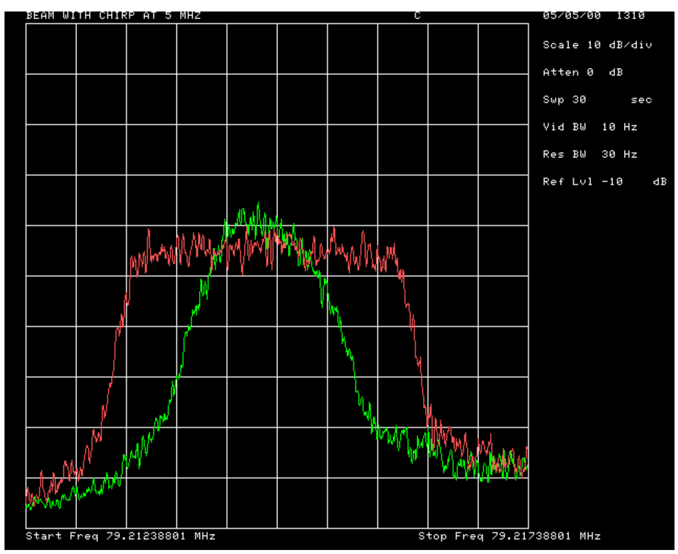

Figure 3. Longitudinal beam spectrum at h=127 before and after the noise source was applied.

VOLTAGE AND FREQUENCY CURVES FOR EXTRACTION

The longitudinal emittance for the unbunched beam is given as:

$$\varepsilon_{beam} = \frac{1}{\eta}\frac{f_U - f_L}{f_0}\frac{1}{f_0} p_0 c \tag{2}$$

The bucket area of one of the h buckets needed to capture a given fraction F of the beam is:

$$\varepsilon_c = \frac{F}{\eta}\frac{f_U - f_L}{f_0}\frac{1}{hf_0} p_0 c \tag{3}$$

In order to avoid RF displacement of the beam, the RF bucket should be placed as close as possible to the high energy edge of the momentum distribution. The frequency of the RF system at the beginning of the capture process should be:

$$f_c = h\left(f_U + \frac{F}{2}(f_L - f_U)\right) \tag{4}$$

The beam is adiabatically bunched at this frequency until the fraction F of the beam is captured. The voltage needed to capture this fraction is:

$$V_c = \frac{2\pi h\eta}{p_0 c/e}\left(\frac{\pi}{8}hf_0\right)^2\left(\frac{\varepsilon_c}{e}\right)^2 \tag{5}$$

The height of this bucket is:

$$\frac{\Delta pc}{p_0 c} = \frac{\pi}{4}hf_0 \frac{1}{p_0 c/e}\frac{\varepsilon_c}{e} \tag{6}$$

Using Eqn. 1, the height of the bucket in units of the RF frequency is:

$$\Delta f_c = \frac{\pi}{4}(hf_0)^2 \frac{\eta}{p_0 c/e}\frac{\varepsilon_c}{e} \tag{7}$$

Substituting Eqn.3 into Eqn. 7:

$$\Delta f_c = \frac{\pi}{4}Fh(f_U - f_L) \tag{8}$$

Since the capture bucket is filled, this bucket area is too small to move the beam without beam spilling out of the bucket. The bucket area must now be increased without gathering anymore beam from the core. This is done by decreasing the frequency (increasing the energy) of the RF while increasing the bucket height so that the low energy edge of the bucket never goes any further into the core.

$$f_{rf} = f_c + \Delta f_c - \frac{\pi}{4}(hf_0)^2 \frac{\eta}{p_0 c/e}\frac{\varepsilon}{e} \tag{9}$$

Tracking Studies

A tracking simulation was done using the frequency and voltage curves shown in Figure 4. The initial momentum spread of the beam is 9.33 MeV which for an η=0.012 corresponds to a revolution frequency spread of 8 Hz around a core frequency of 628,889 Hz. To capture 10% of the beam, an h=4 bucket area of 0.3709 eV-S (4.7 V) is needed. The RF bucket is grown linearly from 0 to 1.5 eV-S (75.24 V) in 5 seconds. The 0.3709 bucket is achieved at 1.24 seconds into the simulation. After 1.24 seconds, the RF frequency is changed according to Eqn. 9. The results are shown in Figures 5a-5f. Using the simple formulas from above 10% of the beam could be extracted with minimal phase spaced dilution.

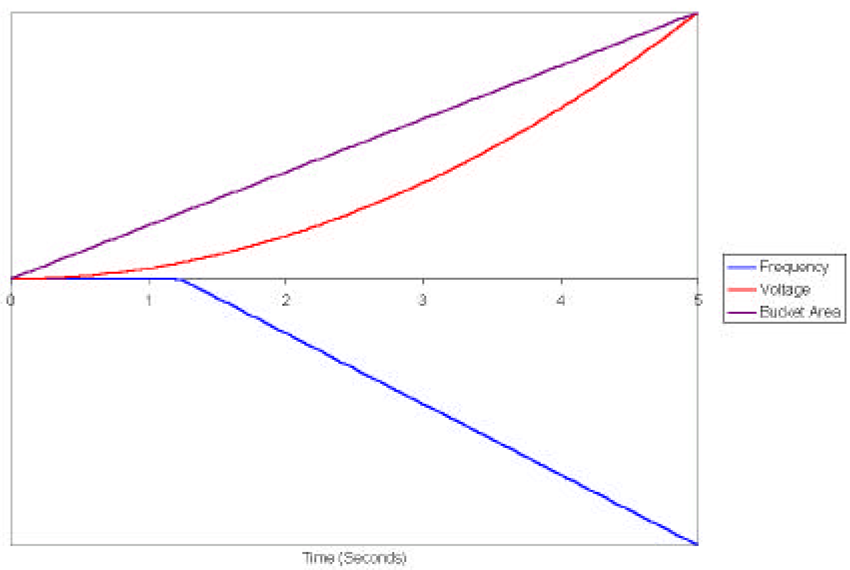

Figure 4. Bucket area, voltage and frequency curves for tracking simulation

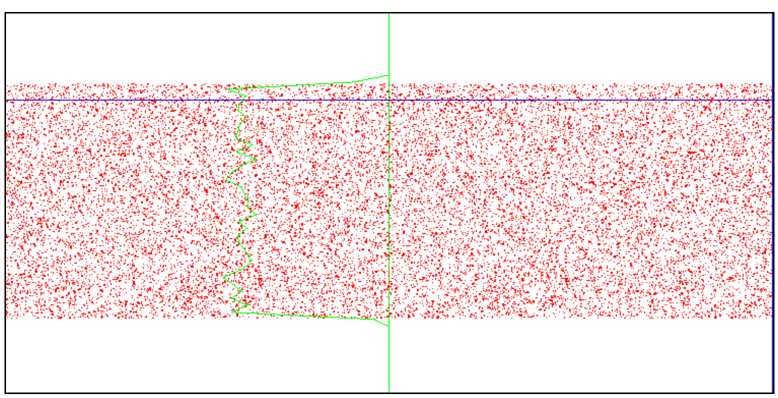

Figure 5a. Initial phase space distribution (t=0 seconds) The green trace is a histogram of the momentum distribution.

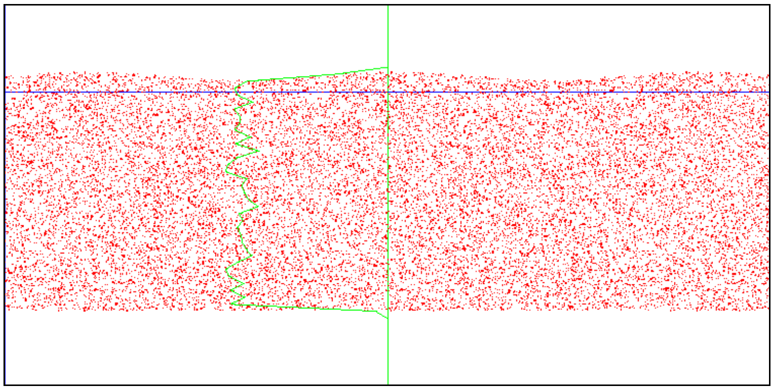

Figure 5b. Phase space distribution at t = 1 seconds.

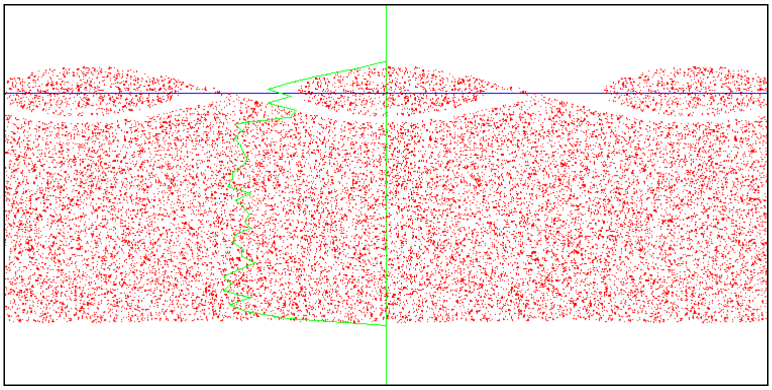

Figure 5c. Phase space distribution at t = 2 seconds.

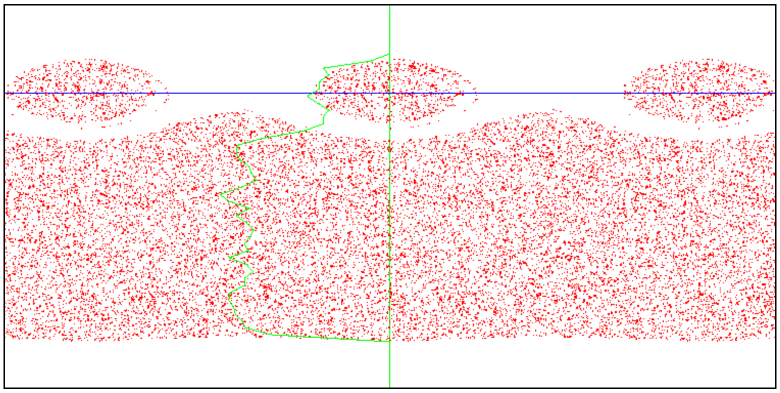

Figure 5d. Phase space distribution at t = 3 seconds.

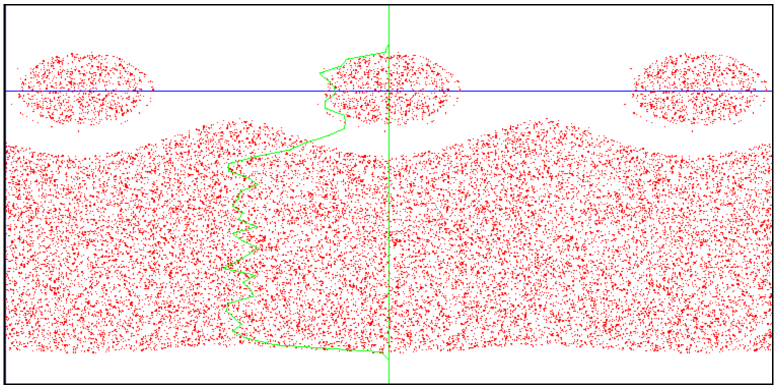

Figure 5e. Phase space distribution at t = 4 seconds.

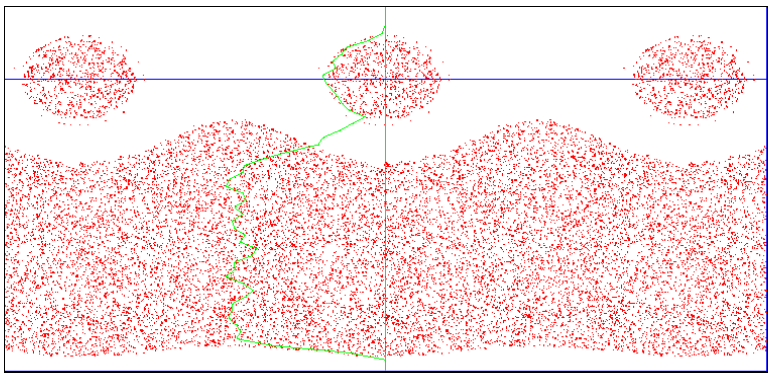

Figure 5f. Phase space distribution at t = 5 seconds.

SUMMARY

Inputs

η	slip factor
p_0c	reference momentum
f_L	revolution frequency of the low energy edge of the beam
f_U	revolution frequency of the high energy edge of the beam
f_0	reference revolution frequency
f_e	extraction revolution frequency
F	fraction of beam to be extracted
ε_M	bucket area of extraction bucket
τ_c	time it takes to bunch the beam
τ_T	total length of extraction process

Definitions

Voltage as a function of bucket area:

$$V = \frac{2\pi h\eta}{p_0c/e}\left(\frac{\pi}{8}hf_0\right)^2\left(\frac{\varepsilon}{e}\right)^2 = \varepsilon_1\left(\frac{\varepsilon}{e}\right)^2 \quad (10)$$

Height of a bucket as a function of bucket area (in units of RF frequency)

$$\Delta f = \frac{\pi}{4}(hf_0)^2 \frac{\eta}{p_0 c / e} \frac{\varepsilon}{e} = \Delta f_1 \frac{\varepsilon}{e} \tag{11}$$

Bucket area needed to capture a given fraction of the beam

$$\varepsilon_c = \frac{F}{\eta} \frac{f_U - f_L}{f_0} \frac{1}{hf_0} p_0 c \tag{12}$$

Bucket height (in units of RF frequency) needed to capture a given fraction of the beam

$$\Delta f_c = \frac{\pi}{4} Fh(f_U - f_L) \tag{13}$$

Starting RF frequency at the beginning of the capture process

$$f_{start} = h\left(f_U + \frac{F}{2}(f_L - f_U)\right) \tag{14}$$

Ending RF frequency when the beam is totally bunched

$$f_{end} = f_c + \Delta f_c - \frac{\pi}{4}(hf_0)^2 \frac{\eta}{p_0 c / e} \frac{\varepsilon_M}{e} \tag{15}$$

For $t < \tau_c$

$$\varepsilon = \varepsilon_M \frac{t}{\tau_c}$$

If $\Delta f_1 \frac{\varepsilon}{e} \le \Delta f_c$

$$f_{rf} = f_{start}$$

If $\Delta f_1 \frac{\varepsilon}{e} > \Delta f_c$

$$f_{rf} = f_c + \Delta f_c - \Delta f_1 \frac{\varepsilon}{e}$$

For $t > \tau_c$

$$\varepsilon = \varepsilon_M$$

$$f_{rf} = (hf_e - f_{end}) \frac{t - \tau_c}{\tau_T - \tau_c} + f_{end}$$