PBAR Note 649
Emittance Growth due to Steering Errors for the 8 GeV AP3-P1 Lattice

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INTRODUCTION

During 8 GeV antiproton transfers between the Accumulator to the Main Injector, the antiprotons must travel through four separate beam lines, AP3, AP1, P2, and P1. The AP1, P1, and P2 transfer lines are also used for 120 GeV antiproton production so that many of the magnet strings must accommodate a large range of excitation current. This note will quantify the relationship between emittance dilution for 8 GeV antiprotons injected into the Main Injector and variations in the excitation currents of the bending magnets.

THEORY

This note will assume that the beam line is matched to the Main Injector. The emittance (95%) dilution in the Main Injector due to a steering error between the transfer line and the Main Injector is:

$$\Delta\varepsilon = 3|A|^2 \tag{1}$$

where **A** is the complex betatron amplitude of the beam centroid. The position and angle error from the ideal trajectory is given as:

$$x(s) = \sqrt{\beta(s)}\,\mathrm{Re}\{A(s)\} \tag{2}$$

$$\frac{dx}{ds} = \frac{-1}{\sqrt{\beta(s)}}\left(\alpha\,\mathrm{Re}\{A(s)\} + \mathrm{Im}\{A(s)\}\right) \tag{3}$$

The contribution to the betatron amplitude coming from a single magnet kick is

$$A_k(s) = j\sqrt{\beta_k}\,\Delta\theta_k e^{j(\phi(s)-\phi(k))} \tag{4}$$

The total betatron amplitude is the sum over all the individual amplitudes

$$A(s) = \sum_k A_k(s) \tag{5}$$

However, if the kicks are uncorrelated, then on average:

$$\left\langle |A(s)|^2 \right\rangle = \sum_k |A_k(s)|^2 \tag{6}$$

This note will assume that the bending angle through a magnet is proportional to the excitation current.

$$\frac{\Delta\theta_k}{\theta_k} = \frac{\Delta I_k}{I_k} \tag{7}$$

The average change in normalized 95% emittance is:

$$\langle \Delta\varepsilon_{norm} \rangle = \sum_k \left(C_k \frac{\Delta I_k}{I_k} \right)^2 \tag{8}$$

This note will designate C_k as the steering sensitivity coefficient where:

$$C_k = \theta_k \sqrt{3\beta_{rel}\gamma_{rel}\beta_k} \tag{9}$$

LATTICE CALCULATIONS

This note will use the lattice described in Pbar Note 648. The lattice functions and bend angles at all the bending magnets are listed in Appendices 1 and 2. The horizontal and vertical steering sensitivity coefficients for each magnet string are listed in Tables 1 and 2. The coefficient for a magnet string was obtained by the coherent summation of each individual magnet in the string. Also listed in Tables 1 and 2 is the excitation current error that would cause 1 π-mm-mrad (95% normalized) emittance dilution. The excitation current errors are also displayed in Figures 1 and 2. The horizontal and vertical steering sensitivity coefficients are displayed in Figure 3.

Power Supply	Current (Amps)	C_k (π-mm-mrad)$^{1/2}$	ΔI/I 10^{-6}	ΔI Amps
D:ELAM	1152.50			
D:V901	681.20			
D:H914	862.50	1923.2	520.0	0.448
D:H926	964.10	492.8	2029.2	1.956
M:V205	58.39			
M:HV202	91.26	2409.2	415.1	0.038
M:HV200	96.80	901.0	1109.8	0.107
I:F17B3	199.10			
I:HVF12	99.68	4016.5	249.0	0.025
I:HVF11	239.78	1280.5	781.0	0.187
I:V714	155.50			
I:HV703	281.73	876.1	1141.4	0.322
I:H703	96.06	269.2	3714.2	0.357
I:V701	211.67			
I:LAM52	268.37	73.3	13647.6	3.663

Table 1. The horizontal steering sensitivity coefficients and the excitation current error that would cause 1 π-mm-mrad (95% normalized) emittance dilution

Power Supply	Current (Amps)	C_k $(\pi\text{-mm-mrad})^{1/2}$	$\Delta I/I$ 10^{-6}	ΔI Amps
D:ELAM	1152.50	1149.3	870.1	1.003
D:V901	681.20	1126.9	887.4	0.605
D:H914	862.50			
D:H926	964.10			
M:V205	58.39	558.2	1791.4	0.105
M:HV202	91.26	145.7	6863.2	0.626
M:HV200	96.80	551.0	1815.0	0.176
I:F17B3	199.10	790.2	1265.6	0.252
I:HVF12	99.68	240.7	4155.3	0.414
I:HVF11	239.78	996.5	1003.5	0.241
I:V714	155.50	167.4	5972.6	0.929
I:HV703	281.73	336.2	2974.6	0.838
I:H703	96.06			
I:V701	211.67	727.3	1374.9	0.291
I:LAM52	268.37	553.4	1806.9	0.485

Table 2. The vertical steering sensitivity coefficients and the excitation current error that would cause 1 π-mm-mrad (95% normalized) emittance dilution

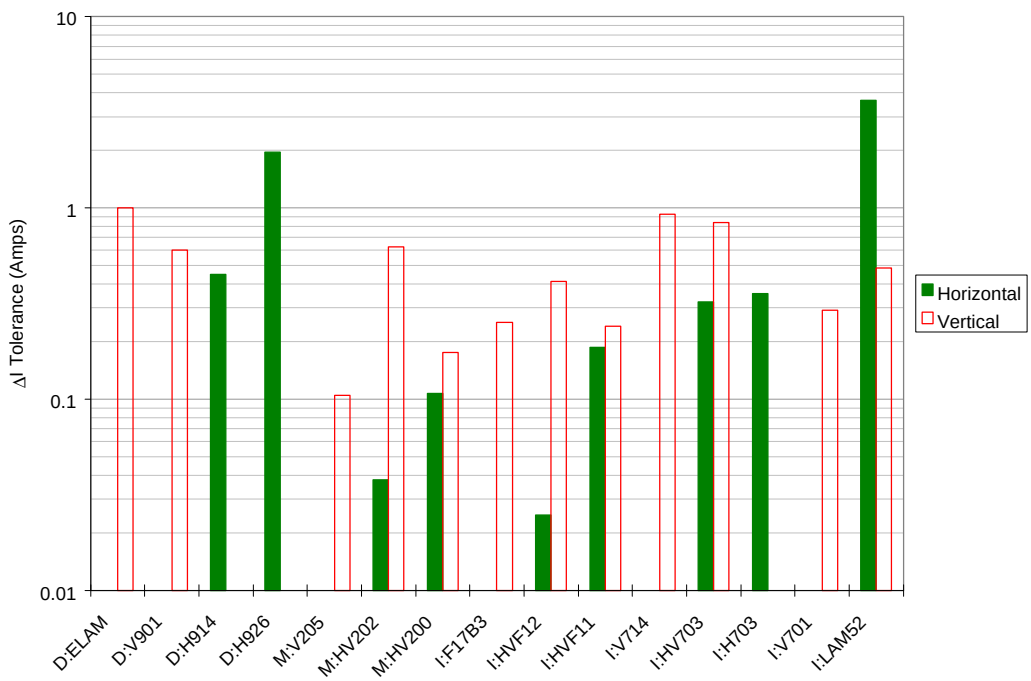

Figure 1. Excitation current error that would cause 1 p-mm-mrad (95% normalized) emittance dilution

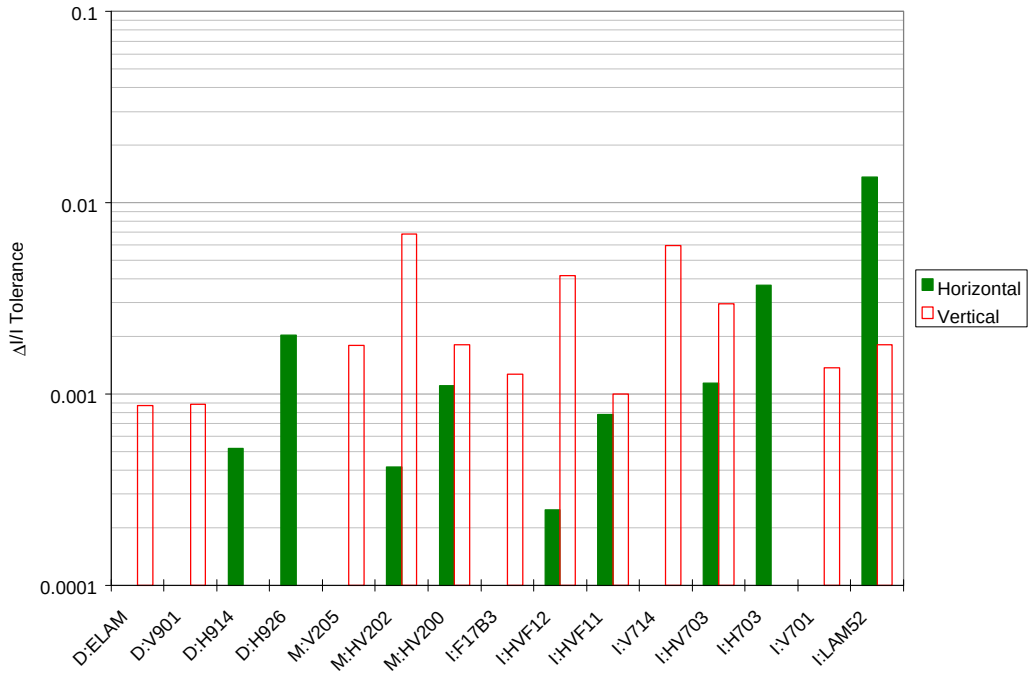

Figure 2. Excitation current tolerance that would cause 1 p-mm-mrad (95% normalized) emittance dilution

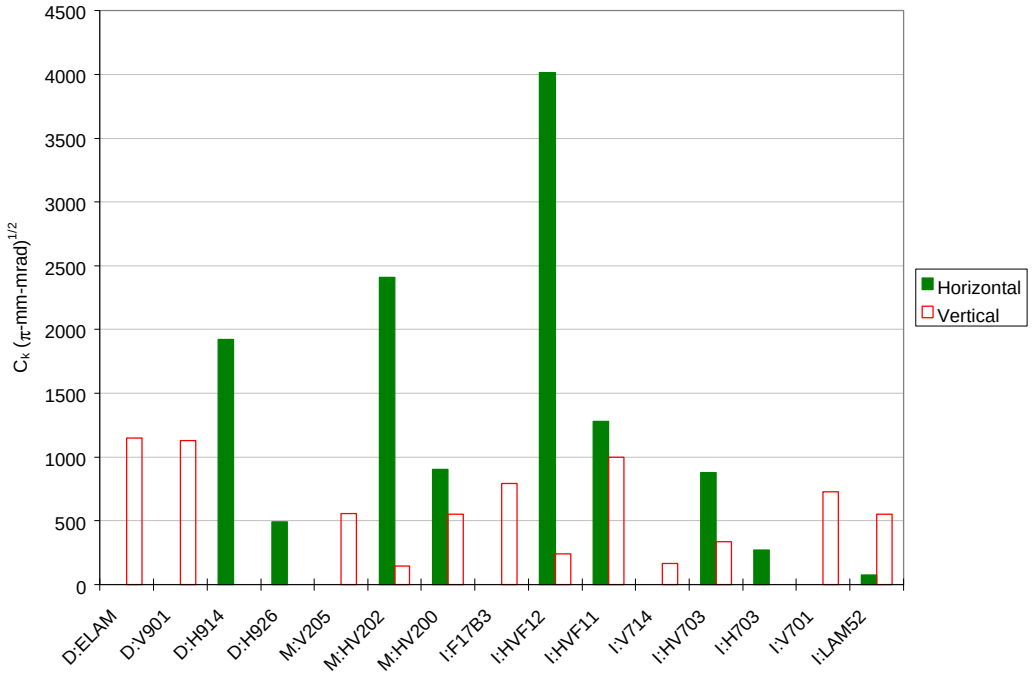

Figure 3. Steering sensitivity coefficients

Appendix 1.
Antiproton Horizontal Lattice Functions at the Bending Magnets of the AP3-P1 Line

Name	Power Supply	s m	Beta X m	Alpha X	Mu X radians	D X m	Dp X	Bend urad
ACCUM		0.0	33.3	-0.534	0.00	0.62	-0.074	0.00
ELAM	D:ELAM	1.5	34.8	-0.544	0.04	0.51	-0.075	0.00
CMAG	D:ELAM	4.0	37.6	-0.568	0.11	0.32	-0.075	0.00
V901	D:V901	9.8	116.4	-27.170	0.24	-0.11	-0.071	0.00
V904	D:V901	18.8	130.0	2.538	0.33	-0.63	-0.031	0.00
H914	D:H914	133.4	32.2	4.963	9.19	-1.26	0.215	-104.41
H915	D:H914	144.2	11.1	-2.060	11.49	0.07	0.045	-104.41
H916	D:H914	154.2	5.0	2.225	12.02	-0.07	-0.076	-104.41
HS925	D:H926	300.9	43.8	-1.250	19.46	-1.99	-0.024	102.19
H926	D:H926	320.4	0.6	-1.534	22.22	0.39	0.498	76.32
H928	D:H926	339.3	48.6	-10.392	25.74	-2.29	-0.450	-76.32
V2052	M:V205	360.8	17.1	0.254	26.30	-1.19	0.055	0.00
V2051	M:V205	364.2	16.1	0.046	26.51	-1.01	0.055	0.00
HV2024	M:HV202	407.8	149.8	-0.520	28.19	2.87	0.024	-9.78
HV2023	M:HV202	411.3	153.4	-0.549	28.21	2.93	0.014	-9.78
HV2022	M:HV202	414.8	157.4	-0.579	28.23	2.96	0.004	-9.78
HV2021	M:HV202	418.1	161.3	-0.606	28.26	2.96	-0.004	-6.91
HV2004	M:HV200	453.6	16.9	-0.200	29.07	-0.06	-0.078	8.13
HV2003	M:HV200	457.0	19.0	-0.406	29.26	-0.31	-0.070	8.13
HV2002	M:HV200	460.3	22.4	-0.612	29.42	-0.53	-0.061	10.32
HV2001	M:HV200	463.7	27.2	-0.818	29.56	-0.72	-0.050	10.32
F17LAM4	I:F17B3	470.8	41.9	-1.255	29.77	-1.04	-0.045	0.00
F17LAM3	I:F17B3	474.4	51.6	-1.472	29.84	-1.20	-0.045	0.00
B3	I:F17B3	480.9	73.6	-1.872	29.95	-1.50	-0.045	0.00
F16D	I:HVF12	492.4	87.3	1.674	30.08	-1.70	0.024	-8.11
F16C	I:HVF12	498.7	67.7	1.396	30.16	-1.57	0.016	-8.11
F16B	I:HVF12	505.1	51.7	1.119	30.27	-1.49	0.008	-8.11
F16A	I:HVF12	511.5	39.2	0.841	30.41	-1.47	0.000	-8.11
F15D	I:HVF12	522.1	37.5	-0.913	30.72	-1.78	-0.070	-8.11
F15C	I:HVF12	528.5	51.2	-1.224	30.87	-2.26	-0.078	-8.11
F15B	I:HVF12	534.9	68.8	-1.535	30.98	-2.78	-0.087	-8.11
F15A	I:HVF12	541.2	90.3	-1.846	31.06	-3.36	-0.095	-8.11
F14D	I:HVF12	551.8	94.2	2.230	31.16	-3.70	0.058	-8.11
F14C	I:HVF12	558.2	68.4	1.826	31.24	-3.36	0.050	-8.11
F14B	I:HVF12	564.6	47.7	1.422	31.35	-3.07	0.041	-8.11
F14A	I:HVF12	571.0	32.1	1.018	31.52	-2.83	0.033	-8.11

Appendix 1.
Antiproton Horizontal Lattice Functions at the Bending Magnets of the AP3-P1 Line

Name	Power Supply	s m	Beta X m	Alpha X	Mu X radians	D X m	Dp X	Bend urad
F13D	I:HVF12	581.6	25.4	-0.507	31.94	-3.01	-0.085	-8.11
F13C	I:HVF12	588.0	33.9	-0.821	32.16	-3.58	-0.093	-8.11
F13B	I:HVF12	594.3	46.4	-1.136	32.32	-4.20	-0.101	-8.11
F13A	I:HVF12	600.7	62.9	-1.451	32.44	-4.87	-0.110	-8.11
F12D	I:HVF12	612.4	66.7	1.360	32.60	-4.97	0.105	-12.90
F12C	I:HVF12	618.9	50.8	1.083	32.71	-4.32	0.095	-7.85
F12B	I:HVF12	625.3	38.7	0.809	32.85	-3.74	0.087	-7.47
F12A	I:HVF12	631.7	30.1	0.534	33.04	-3.20	0.080	-6.39
F11B	I:HVF11	650.4	51.6	-1.561	33.64	-3.03	-0.041	-15.19
F11A	I:HVF11	656.8	74.4	-1.987	33.74	-3.34	-0.056	-15.19
V714	I:V714	699.9	33.3	-0.881	35.32	1.28	0.101	0.00
HV7102	I:HV703	731.6	26.0	-0.952	36.41	2.28	0.090	21.01
HV7101	I:HV703	738.0	41.2	-1.422	36.61	2.92	0.112	22.90
HV7092	I:HV703	749.2	34.2	1.884	36.84	2.80	-0.135	22.61
HV7091	I:HV703	755.7	15.5	1.029	37.13	2.00	-0.113	22.39
HV7082	I:HV703	766.7	17.7	-1.272	38.12	1.61	0.041	22.90
HV7081	I:HV703	773.1	40.1	-2.223	38.36	1.95	0.064	22.61
HV7072	I:HV703	784.2	40.9	2.174	38.57	1.66	-0.116	22.90
HV7071	I:HV703	790.6	18.8	1.275	38.80	0.99	-0.093	22.74
HV7062	I:HV703	801.7	16.2	-0.999	39.72	0.36	-0.023	21.01
HV7061	I:HV703	808.1	34.2	-1.789	39.99	0.28	0.000	22.90
HV7052	I:HV703	819.2	33.7	1.816	40.24	0.25	-0.006	22.61
HV7051	I:HV703	825.6	15.7	0.997	40.53	0.28	0.017	22.39
HV704	I:HV703	843.1	41.0	-2.226	41.72	1.24	0.078	22.61
HV7032	I:H703	854.1	41.0	2.224	41.92	1.39	-0.063	-7.87
HV7031	I:HV703	860.5	18.4	1.292	42.16	0.98	-0.055	22.74
V701C	I:V701	886.3	41.7	1.383	43.58	0.87	-0.061	0.00
V701B	I:V701	890.1	32.0	1.112	43.69	0.63	-0.061	0.00
V701A	I:V701	894.0	24.4	0.840	43.83	0.39	-0.061	0.00
LAM52_3	I:LAM52	907.9	38.5	-1.635	44.42	-0.40	-0.056	-0.55
LAM52_2	I:LAM52	911.3	50.8	-1.956	44.50	-0.59	-0.057	-1.45
LAM52_1	I:LAM52	917.2	37.8	2.020	44.62	-0.69	-0.001	0.00
HC522	I:IKIK	917.9	34.9	1.922	44.64	-0.69	-0.001	0.00
HC522	I:IKIK	918.4	33.2	1.861	44.65	-0.69	-0.001	0.00
P150_START		918.4	33.2	1.861	44.65	-0.69	-0.001	0.00

Appendix 2.
Antiproton Vertical Lattice Functions at the Bending Magnets of the AP3-P1 Line

Name	Power Supply	s m	Beta Y m	Alpha Y	Mu Y radians	D Y m	Dp Y	Bend urad
ACCUM		0.0	5.6	0.493	0.00	0.00	0.000	0.00
ELAM	D:ELAM	1.5	4.6	0.167	0.29	-0.02	-0.037	73.40
CMAG	D:ELAM	4.0	5.2	-0.396	0.83	-0.19	-0.088	28.20
V901	D:V901	9.8	4.7	1.567	1.60	-0.42	0.129	-50.30
V904	D:V901	18.8	15.6	-7.761	6.20	0.01	-0.025	-48.95
H914	D:H914	133.4	16.2	-1.541	10.84	0.02	0.002	0.00
H915	D:H914	144.2	17.9	0.829	11.30	0.02	-0.001	0.00
H916	D:H914	154.2	49.8	-5.888	11.88	0.02	0.002	0.00
HS925	D:H926	300.9	21.4	0.852	17.60	0.02	-0.001	0.00
H926	D:H926	320.4	76.3	-10.187	18.83	0.00	0.000	0.00
H928	D:H926	339.3	16.4	-0.036	19.65	-0.01	-0.001	0.00
V2052	M:V205	360.8	93.2	6.978	20.10	-0.04	0.006	-6.18
V2051	M:V205	364.2	52.5	5.197	20.15	-0.01	0.012	-6.18
HV2024	M:HV202	407.8	34.4	1.330	23.14	0.10	-0.021	0.00
HV2023	M:HV202	411.3	26.2	1.054	23.26	0.02	-0.021	0.00
HV2022	M:HV202	414.8	19.9	0.774	23.41	-0.05	-0.021	0.00
HV2021	M:HV202	418.1	15.6	0.505	23.60	-0.12	-0.018	-6.91
HV2004	M:HV200	453.6	72.6	2.062	24.90	-0.66	0.017	6.35
HV2003	M:HV200	457.0	59.6	1.821	24.95	-0.62	0.010	6.35
HV2002	M:HV200	460.3	48.2	1.579	25.01	-0.59	0.007	0.00
HV2001	M:HV200	463.7	38.4	1.337	25.09	-0.57	0.007	0.00
F17LAM4	I:F17B3	470.8	23.1	0.821	25.33	-0.52	0.003	8.83
F17LAM3	I:F17B3	474.4	18.1	0.563	25.51	-0.52	-0.006	8.83
B3	I:F17B3	480.9	13.9	0.086	25.93	-0.61	-0.020	19.69
F16D	I:HVF12	492.4	28.6	-1.698	26.63	-1.08	-0.064	0.00
F16C	I:HVF12	498.7	55.7	-2.565	26.79	-1.49	-0.064	0.00
F16B	I:HVF12	505.1	94.0	-3.431	26.88	-1.90	-0.064	0.00
F16A	I:HVF12	511.5	143.2	-4.296	26.93	-2.30	-0.064	0.00
F15D	I:HVF12	522.1	176.1	2.807	26.99	-2.50	0.046	0.00
F15C	I:HVF12	528.5	142.4	2.488	27.03	-2.21	0.046	0.00
F15B	I:HVF12	534.9	112.7	2.167	27.08	-1.92	0.046	0.00
F15A	I:HVF12	541.2	87.1	1.846	27.15	-1.63	0.046	0.00
F14D	I:HVF12	551.8	77.7	-1.390	27.29	-1.39	-0.010	0.00
F14C	I:HVF12	558.2	96.9	-1.630	27.36	-1.46	-0.010	0.00
F14B	I:HVF12	564.6	119.2	-1.869	27.42	-1.52	-0.010	0.00
F14A	I:HVF12	571.0	144.6	-2.108	27.47	-1.59	-0.010	0.00

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Antiproton Vertical Lattice Functions at the Bending Magnets of the AP3-P1 Line

Name	Power Supply	s m	Beta Y m	Alpha Y	Mu Y radians	D Y m	Dp Y	Bend urad
F13D	I:HVF12	581.6	133.9	4.054	27.54	-1.40	0.057	0.00
F13C	I:HVF12	588.0	87.5	3.225	27.60	-1.04	0.057	0.00
F13B	I:HVF12	594.3	51.7	2.395	27.69	-0.68	0.057	0.00
F13A	I:HVF12	600.7	26.4	1.564	27.87	-0.32	0.057	0.00
F12D	I:HVF12	612.4	11.9	-0.229	28.69	0.34	0.053	9.84
F12C	I:HVF12	618.9	18.6	-0.801	29.14	0.66	0.049	-2.06
F12B	I:HVF12	625.3	32.5	-1.368	29.41	0.98	0.051	-3.16
F12A	I:HVF12	631.7	53.7	-1.935	29.56	1.32	0.056	-4.99
F11B	I:HVF11	650.4	68.1	0.677	29.82	1.61	-0.006	-11.66
F11A	I:HVF11	656.8	60.3	0.542	29.92	1.60	0.005	-11.66
V714	I:V714	699.9	15.0	0.618	30.82	1.10	-0.059	8.10
HV7102	I:HV703	731.6	43.6	1.158	32.31	-0.31	-0.040	9.10
HV7101	I:HV703	738.0	30.9	0.815	32.49	-0.59	-0.045	0.00
HV7092	I:HV703	749.2	42.8	-2.082	32.88	-1.48	-0.115	3.58
HV7091	I:HV703	755.7	74.7	-2.876	32.99	-2.22	-0.114	-4.78
HV7082	I:HV703	766.7	54.3	4.133	33.12	-2.10	0.134	0.00
HV7081	I:HV703	773.1	14.9	1.994	33.35	-1.24	0.132	3.58
HV7072	I:HV703	784.2	14.5	-1.951	35.27	-0.06	0.083	0.00
HV7071	I:HV703	790.6	53.2	-4.073	35.51	0.47	0.082	2.65
HV7062	I:HV703	801.7	73.7	2.812	35.64	0.93	0.005	-9.10
HV7061	I:HV703	808.1	42.5	2.038	35.76	0.99	0.010	0.00
HV7052	I:HV703	819.2	34.8	-1.196	36.12	1.62	0.104	3.58
HV7051	I:HV703	825.6	53.0	-1.641	36.27	2.28	0.100	4.78
HV704	I:HV703	843.1	8.4	1.113	36.87	1.06	-0.148	3.58
HV7032	I:H703	854.1	23.7	-2.787	38.64	-0.47	-0.128	0.00
HV7031	I:HV703	860.5	74.8	-5.158	38.79	-1.29	-0.127	-2.65
V701C	I:V701	886.3	26.6	-0.256	39.26	-1.42	-0.050	-8.40
V701B	I:V701	890.1	29.2	-0.411	39.40	-1.60	-0.042	-8.40
V701A	I:V701	894.0	33.0	-0.567	39.53	-1.75	-0.034	-8.40
LAM52_3	I:LAM52	907.9	15.1	0.923	40.03	-1.05	0.087	14.80
LAM52_2	I:LAM52	911.3	10.2	0.504	40.31	-0.77	0.072	14.74
LAM52_1	I:LAM52	917.2	15.4	-1.051	40.85	-0.59	0.015	0.00
HC522	I:IKIK	917.9	17.0	-1.150	40.90	-0.58	0.015	0.00
HC522	I:IKIK	918.4	18.0	-1.213	40.92	-0.58	0.015	0.00
P150_START	0	918.4	18.0	-1.213	40.92	-0.58	0.015	0.00